



**SAPIENZA**  
UNIVERSITÀ DI ROMA

**Roma La Sapienza**

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Master degree in Transport systems engineering

MASTER THESIS

**Arterial signal optimization through traffic  
microscopic simulation**

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Academic Year 2016–2017

*A Gaetano Pecoraro:  
Amoris modus est sine modo amare*

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# Introduction

Every day each road user runs into inadequate traffic flow condition, exactly, in the heavy traffic case. Intense traffic flow state determines the origin of the congestion problem, which represents an inconvenience situation for road users, but also a condition, which reduces the performance of transport network severely.

Every day, in fact, many vehicles cross several arteries, which compose road networks. These flows and their distributions on urban roads are very interesting from the engineering point of view because an efficient vehicular flow management would have reduced travel time, it would have improved the road security and the reliability detected by the users in a significant way.

Transport engineering is the field of study, which analysis these factors, exactly, the interaction between transport infrastructures, transport vehicles, transport systems organization and possible effects connected to this interaction. These elements, in general, contribute all together to satisfy transport demand, which defines movement opportunity of goods and people from the origin to the destination node. Every single element is able to define the main functional characteristics and network performances, but it is necessary to evaluate the user role during his movement in the network, analyzing his behavior and his interaction with other road users. These aspects influence not only transport demand, but also the way in which the transport system is used. The transport system, in fact, can be defined as a complex system, because it is made up of multitude components that influence mutually. The reality, therefore, represents a complex scenery and it is necessary to know current state and to study all possible ways to optimize effects corresponding to an interaction between following factors:

- road users;
- several transport vehicles;
- links between urban network components.



This interaction, often, causes an inefficient use of road networks, in particular, the emergence of congestion problem. The problem size and its intrinsic complexity involve using mathematical laws, in order to simplify the analysis of reality. In order to avoid the emergence of both conditions is to necessary take advantage to the mathematical knowledge and tools, so try the solution to the congestion problem, far too widespread both in a large and small urban area. Informatics developments and progress of calculation procedures allow to study this type of problems and to clarify the congestion problem mainly. The aim of this thesis is to solve and to reduce congestion problem in an urban area, trying to optimize cycles, green times and offset, characteristic elements of signalized intersections, through a suitable traffic signal control. The urban area examination, in fact, is equivalent to the study of the optimization problem, in order to try the best compromise between the flow traffic distribution and delays.

Interest field of this study is the analysis of the congestion phenomenon from which it is possible to take advantage of traffic flow theory. Flow theory analyzes mutual influence between every vehicle during their motions along the arteries. The theory above mentioned explains vehicles interaction through specific physical and behavioral laws, that can be described to the following two different approaches:

- macroscopic approach;
- microscopic approach;

The general difference between two approaches is that the first represents the behavior of whole traffic stream, while the second the attitude of single road users along the way on the network arcs.

The first approach is theoretical basis of the most flow models. This approach is able to explain the average characteristics of traffic stream, therefore to simulate the behavior of a given number of vehicles, which it moves as a platoon. From theoretical point of view, this approach doesn't represent the dynamic motion of single vehicle, as it is done to the microscopic approach. In general, the second approach is able to represent the global condition of vehicles stream. The model adopted to study the interaction of the several vehicles in motion individually is the car following model that attempts to reproduce longitudinal conditioning between vehicles in heavy traffic condition.

The car-following models described and kept track of individual car movements, while the fluid models described the movement of the total vehicle population. Specifically, these two models can be regarded as deterministic

flow behavior models, one at the micro-level and the other at the macro-level. The car following models used and implemented in this thesis are:

- Chandler model;
- General motors model;
- IDM (Intelligent driver model).

These models are used to understand and to solve the base problem. In general, these models adopt different equation through which it is possible to describe the movement of the vehicle along the arteries, exactly the models above mentioned describe the motion of the single vehicle as an interaction between leader vehicle. In particular, can be summarized this concept through following relation:

$$Response = sensitive \times stimulus$$

In general, it is simulated reaction of the vehicle when the previous vehicle modifies its behavior. The first model describes the reaction of follower vehicle as an acceleration variation proportionally to the stimulus, which is the difference between the leader vehicle and follower vehicle. In this way, it is possible to simulate the motion of the single vehicle and its reaction when there is a perturbation. Chandler model explains the behavior of several vehicles supposing constant sensitivity. The second model, differently, considers the sensitivity inversely proportional to the distance vehicles. While the third model describes the dynamics of the positions and velocities of single vehicles, fixed the security distance and the desired velocity. In this way the motion of single vehicles has detected by the initial condition imposed by the user, that is user supervises vehicle motion during its movement along the artery, acting on acceleration, speed and distance.

In general, the analyzed conditions can be changed, not only modifying uses and road infrastructure characteristics but also to study specific strategies to apply in general and *ad hoc* to different urban context.

# Chapter 1

## State of the art

### 1.1 Traffic regulation at junctions

### 1.2 Overview

Road junction represents the most critical point of transport network, because in correspondence to this road element it observes the crossing of vehicle and pedestrian flows. These conflict points imply impossibility of simultaneous use of intersection and, at the same time, impose on drivers to make difficult maneuvers. The intrinsic characteristic of intersection involves the following consequences:

- unsafe condition;
- reduced capacity.

The effects of both factors increasing the risk accidents and growth of delays. Unsafe condition and reduced capacity can be decreased through appropriate intersection planning and control. A proper design allows an optimization and an improvement of road performances in terms of:

- delays reduction;
- capacity increase;
- safety increase.

Conventions of traffic controls attempt to try proper compromise between size traffic flow conditions and geometric characteristics of urban artery. The suitable compromise can be reached, through realization and implementation of following traffic system controls:

Table 1.1: System traffic control correspondence to vehicular flow

Traffic flow	Type system controls
Low traffic flow	priority control
Medium-low traffic flow	stop control
High traffic flow	signal intersection or roundabout

- regulation with priority or stop signal;
- roundabout;
- signal controls;
- intersection on one or on two different layers.

### Choice of type regulation

The choice of system control takes on key role in the study of traffic flow condition, because a suitable selection improves management of vehicular flows. System traffic control must be able to adopt a proper strategy in the distribution of traffic flow along main and secondary artery, in order to avoid a non balanced distribution traffic flow in correspondence to conflict point. Traffic control doesn't regulate to specific norm, in general it is possible to follow the guidelines of High capacity manual *HCM 2000*. According to *HCM 2000* the type of system control defined in relation to traffic flow condition of area object of study (figure 1.1 and table 1.1) [15]. The priority and stop control allow to regulate circulation, according to guide priority between stream in correspondence to conflict area. While, roundabout removes the conflict point, and allows input and diversion maneuvers around a central island of round shape, differently, the signalized intersection is able to offset in time conflicting maneuvers, permitting the use of the conflict area in alternate phase to several stream groups, among them compatible.

#### 1.2.1 Planning methodology

The road crossing can be regulated in several ways, in this study, exactly, the signalized intersections are analyzed. Initially, it is studied the single intersection, note in literature as isolate intersection. The study of urban artery, in which it is adopted this system control, requires a suitable planning. Planning methodology, in fact, describes a consolidated scheme, which allows the formulation and resolution of mathematical problem. The scheme explains

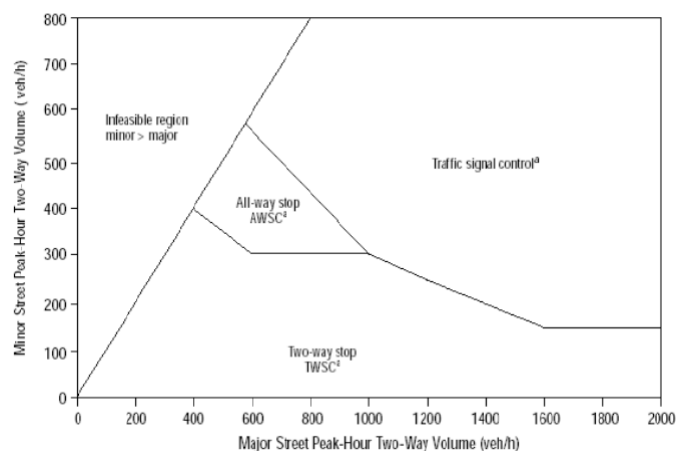


Figure 1.1: Intersection control type and peak hour volumes

the several steps it is necessary to follow in order to built an intersection model. In this analysis it is necessary:

- to acquire geometric and traffic data;
- to organize of vehicular flows in phase of maneuvers;
- to calculate project flow and saturation flow;
- to calculate delay or objective function;
- to try the best solution in terms of greens and cycles.

The proper planning of signal intersection can involve:

- an improving of the road safety;
- a reduction of air and acoustic pollution;
- a reduction of energy consumption;
- a reduction of travel time of vehicles and pedestrians.

All these factors take on a main role in the intersection planning, but in this case it is considered, as a factor to optimize, the travel time and, consequently, the delay of vehicles and pedestrian which collect in the several urban intersection. The delay is the main parameter, through which can be known the level of service of an intersection [5]. The time spent, during the

crossing of the road intersection, describes in a synthetic way the performance of the signal intersection.

This analysis, expected to *HCM 2000*, requires acquisition of following information:

- analysis period;
- acquired data;
- choice of timing;
- calculation of project flow.

### **Analysis period**

A suitable design of a road intersection requires the analysis of spatial and temporal area of interest. In particular, the analysis of the time allows to observe the variability of the traffic flow during the day. In general, because of the absence of a continuous monitoring, the traffic condition can be studied through the partition of the period analysis in four time bands: time peak of morning and evening, soft time of morning and evening. The design of road intersection has realized considering, habitually, observed flow in the quarter of an hour more load of the period of reference. From the spatial point of view, it is studied the interaction between the vehicles of main urban artery and the vehicles of adjacent arteries. This approach allows to know the ways through which the flows of adjacent road have implication for the flows of main artery.

### **Acquired data**

The acquired data allows to realize a good planning of intersection. In general, the requested data are:

- traffic data at the intersection, disaggregated data to single maneuver and to classes (cars, heavy vehicles, mopeds, bicycles, pedestrians);
- geometric data of intersection;
- actual signalized intersection, when the system control is present in the area of study.

All data allows to know the characteristics of study area and to think to the possible strategies to apply in the urban area of interest.

Table 1.2: Choice of timing

Choice of timing	Characteristics
2 phase	Balanced flows with no relevant left turn flows
3 phase	Select the phase sequence and respective critical movements
Phases overlap	One or more movements started are distributed along two subsequent phases.

### Choice of timing

The choice of timing is a complex operation, because it requires a proper design and selection of type of the maneuvers, which can occur in each phase of signal control and of their sequence. The choice, in fact, is based on traffic flows and on the possible maneuver, that can be made in the road intersection. In general, the choice of timing must be carry out as a compromise between the traffic flows and the value of intrinsic parameter of signalized intersection. In the choice of timing is necessary to take into account the following factors:

- start up and clearance lost time;
- the number of phases;
- the dedicated lane;
- the value of red time.

These elements must be selected conveniently in order to reduce, in a significant way, the performance of the road intersection, in terms of reduction of the capacity and increasing the delay.

In order to consider all these factors it attempts to follow this procedure, as shown in the table 1.2.

The criteria for phase selection is to:

- reduce the saturation degree at conflict points;
- reduce number of phases to reduce lost time.

### Calculation of project flow

In the optimization study of the road intersection it is necessary to calculate a specific terms for each lane group:

- geometric condition;
- traffic condition;
- signalization condition;

through which it is possible, at the end, to evaluate the level of service (LOS).

### Geometric condition

Intersection geometry is generally a diagram, which must include all relevant information of the area of interest, exactly: area type, number of lanes, average lane width, grade, existence of exclusive LT or RT lanes, length of storage bay and Ls Parking. These data represent the main information to collect in order to describe geometric characteristics of the urban area.

### Traffic condition

Traffic condition must be specified for each movement on each approach. This factor takes into account all data through which it is possible to conduct an operational analysis in the urban signalized intersection. Therefore are considered the following information: demand volume by movement, base saturation flow rate, peak-hour factor, percent heavy vehicles, approach of pedestrian, local buses stopping at intersection, parking activity, arrival type, proportion of vehicles arriving on green and approach speed.

### Signalization condition

Signalization condition is a final step in order to complete the information related the area of interest. In this section are considered the following data: cycle length, green time, yellow plus all red change and clearance interval, actuated or permitted operation, pedestrian push-button, minimum pedestrian green, phase lane and analysis period. All data have been collected and put into the worksheet of *HCM 2000* in order to conduct the analysis of the current state of the signalized junction. These steps represent a fundamental phases to know the present state of existent road intersection. The collect details can be used as input data for the synchronization of several nodes and to optimize the urban arterial in terms of green, cycle and offset.



## 1.3 Synchronization

### 1.3.1 Overview

The road intersection represents the main cause of delay in urban movement. The synchronization is a reasonable way through which it is possible to improve the congestion state in a road intersection [6]. A feasible strategy is to synchronize, appropriately, the start of green, so that the green begins when the intense flow reaches the signalized intersection. In an ideal conditions would allow to observe which all vehicles to cross the urban artery without to accumulate the delay. This condition, in reality, is very problematic to realize, because in an ideal synchronization it is necessary to respect the following conditions:

- all junctions are equally spaced;
- no entering or exiting traffic along the artery;
- traffic flow lower than a given value.

These condition implicate:

- cycle length and green splits are equal at all junctions;
- all vehicles along the artery run at the same speed and form so a uniform and compact platoon.

The ideal synchronization, though it is an unrealistic, is a significant tool to study the real synchronization problem. In the real condition all junction are not equally spaced and there are entering and exiting traffic along the artery. Therefore a proper phase shift of the green start of signalized junction can reduce the travel times of each road users significantly. This strategy does not allow to progression of all vehicle along the urban artery without any stops at the nodes, so it is possible to reduce the travel time and the vehicles stop along the artery. In general, to solve the realistic problems, two different approaches have been proposed, corresponding to two optimization problems:

- minimizing the delay of the urban artery;
- maximizing the green bandwidth (possible trajectories to cross a constant speed).

The bandwidth synchronization approach, also commonly known as green wave, is used, extensively, in the resolution of this problem. Its advantage, from mathematical point of view, is to be an almost concave problem. This type of problem is solved through specific methods, which determine the optimal solution. The green wave provides, moreover, an effective graphical representation of the solution, which allows an immediate visual interpretation of the goodness of the solution found.

## 1.4 Maximal bandwidth problem

The problem of maximal bandwidth is adopted to solve the synchronization issue of real urban artery. This approach can be applied both in the case of nodes having a different green both in the case the nodes have a different spacing. In the first condition the bandwidth will be equal to the smaller green of artery, while in the second occurrence the bandwidth, generally, will be less than the duration of the green. The purpose of this problem is to search the vector of the phase shift that maximizes the sum of the bands in the both directions, in respect to constraints on the life cycle, the speed and the value of the variables. From mathematical point of view the problem can be recapped in the following way:

$$\max f = (b + b') \quad (1.1)$$

It is possible to formulate two different problem. The first (A) is an almost concave problem, that is it admits infinite optimum solutions, while the second problem (B) is strictly concave and therefore has a unique solution.

Constrains problem A

$$b > 0; b' > 0$$

$$0 \leq \vartheta < C$$

$$v_{min} \leq v \leq v_{max}$$

$$\max_i(C_{min,i}) \leq C \leq C_{max}$$

$$C_{min,i} = \frac{(L_i)}{(1 - \max_h(y_{i,h}) - \max_k(y_{i,k}))}$$

Constrains problem B

$$b = b' > 0$$

$$0 \leq \vartheta < C$$

$$\begin{aligned}
v_{min} &\leq v \leq v_{max} \\
max_i(C_{min,i}) &\leq C \leq C_{max} \\
C_{min,i} &= \frac{(L_i)}{(1 - max_h(y_{i,h}) - max_k(y_{i,k}))}
\end{aligned}$$

In particular, the second case admits only two possible value of the phase shift, 0 or  $C/2$ . In this study it is considered only the second problem, which presents a strict analogy with the ideal synchronization.

### Ideal synchronization

The ideal synchronization is an important instrument to solve the problem of the synchronization of the real urban artery. This type of synchronization is based on the following hypothesis:

- all junctions are equally spaced:  $l_{ij=A}, \quad \forall i \neq j, \quad i=1,2,\dots,n$
- no entering or exiting traffic along the artery, this condition implies also the cycle and the green is equal in all nodes:  $g_i = g_j = b; \quad C_i = C \forall i, j$

In this case, it is possible to realize a progression of vehicles which travel at constant speed without to stop at the nodes, imposing that in each node the green just starts when the first vehicle of the platoon arrives and ends immediately after the passage of the last vehicle. This condition, shown in the figure 1.2, requires the interval between the green start in two nodes  $i, j$  also not contiguous is equal to the time of travel, that is:

$$t_{ij} + t_{ji} = \frac{A}{v_1} + \frac{A}{v_2} = mC \quad m \text{ integer} \quad (1.2)$$

The equation 1.2 defines an important relationship between the distance of the nodes and the speed of vehicles (called synchronization speed) and the cycle. Assuming that the speeds are the same in both ways  $v_1 = v_2 = v$  and considering two contiguous nodes, the 1.2 becomes:

$$t_{ij} + t_{ji} = \frac{A}{v} + \frac{A}{v} = C) \quad \Rightarrow C = \frac{2A}{v} \quad (1.3)$$

to this hypothesis follows that the phase shift between two contiguous nodes is half cycle, that is:

$$\vartheta_{i,i+1} = 0.5C \quad (i = 1, 2, \dots, n) \quad (1.4)$$

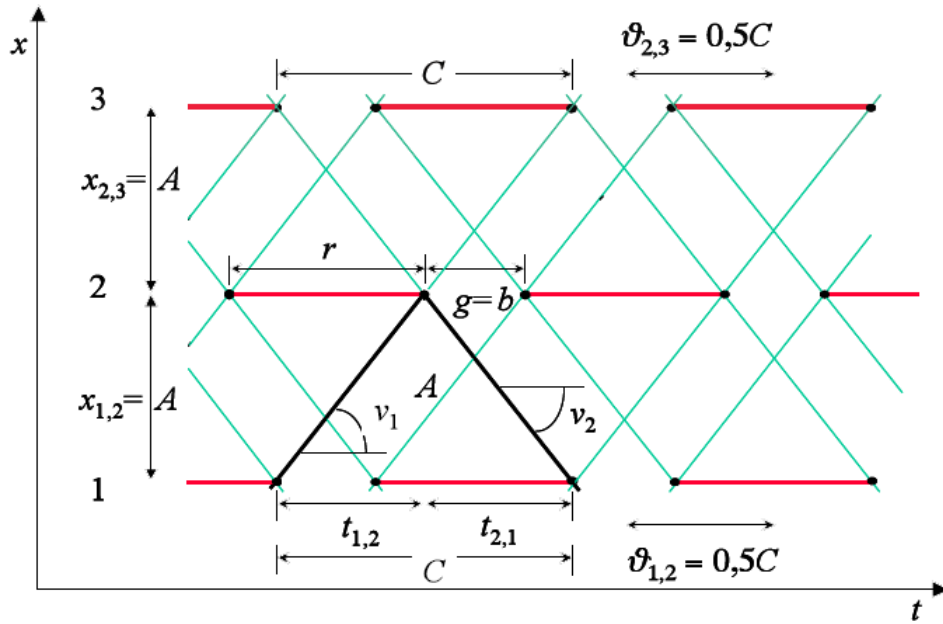


Figure 1.2: Scheme of the ideal synchronization in the place-time plan

In the formulation of the problem it is necessary to introduce constraints on the variables:

$$v_{min} \leq v \leq v_{max}$$

$$C_{min} \leq C \leq C_{max}$$

The constraint of the speed defines that the lower bound must guarantee a level of minimum service, value under which the synchronization would not be cheaper, while the upper bound must ensure safe driving conditions. The lower limit of the cycle, differently, must guarantee the capacity condition along the several approaches of the road artery, moreover, the upper constraint must ensure the length of the red time.

### 1.4.1 Periodicity and symmetry properties of the synchronization scheme

The cycles of the several nodes are all equal to each other, the synchronization scheme is, therefore, periodic in time, and at the same time, is periodic in the space, because the distance between the nodes is constant. In general, the contiguous nodes can be in phase or in opposite phase, so as described to the following relations:

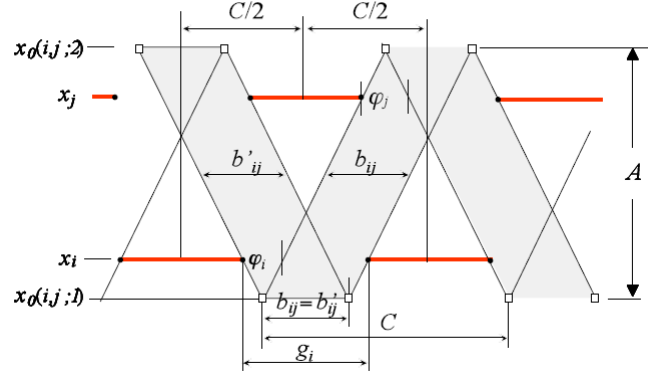


Figure 1.3: Maximal bandwidth of 2 nodes placed to the distance lower of  $\frac{A}{2}$

$$\vartheta_i = 0 \quad \text{if} \quad \frac{x_i - x_0}{2A} = m, \quad m = 0, 1, 2, \dots$$

$$\vartheta_i = \frac{C}{2} \quad \text{if} \quad \frac{x_i - x_0}{2A} = \frac{2m + 1}{2}, \quad m = 0, 1, 2, \dots$$

The relations above mentioned represent the solution of the ideal synchronization problem with amplitude of ascending  $b$  and descending  $b'$  equal the common length of the green. The synchronization scheme is symmetrical with respect to  $A$  and periodic in the space with a period of  $2A$ . The distance  $A$  between the nodes is therefore the module of the ideal synchronization.

## 1.5 Synchronization of two nodes

The synchronization of two nodes with the maximum symmetric band admits two possible value of offset:  $\vartheta = 0$  or  $\vartheta = 0,5$ , considering a cycle length  $C = 1$ . The type of offset can be choose in relation to the value of the distance existing between two nodes. In particular, it is necessary to be worth the following relations:

$$\vartheta_i = 0 \quad \text{if} \quad 0 \leq \frac{x_i - x_0}{2A} < 0,25 \quad \cup \quad 0,75 \leq \frac{x_i - x_0}{2A} < 2A$$

$$\vartheta_i = 0,5 \quad \text{if} \quad 0,25 \leq \frac{x_i - x_0}{2A} < 0,75$$

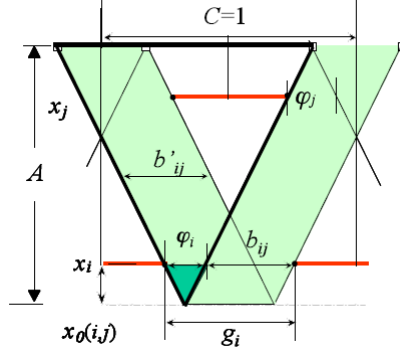


Figure 1.4: Ideal synchronization scheme in space-time plan: in each node the green starts when the first platoon arrives and ends immediately after the passage of the last vehicle.

### 1.5.1 Ideal equivalent system

The value of the maximum bandwidth of two nodes  $i$  and  $j$  can be obtained using the equivalence between 2 real nodes and 2 ideal nodes, having length of green time equal to the maximum bandwidth between the nodes  $i$  and  $j$  and the distance  $A$  equal to the ratio  $vC/2$ , so as displaced in the figure 1.4.

The two real and the ideal nodes of the system obtained to the extension of the bands in both directions produce the same solution, that is the same bandwidth. Therefore, they can be considered equivalent. Considering the similarity between the two green small triangles shown 1.4 and the triangle with the thickest line it is possible to write the following relationships:

$$x_i - x_0(i, j) = A\varphi_i \quad (1.5)$$

$$b_{ij} = b'_{ij} = g_i - \varphi_i \quad (1.6)$$

These relations permit to know many elements of the synchronization, in particular, the relationships above mentioned allow to obtain the ideal equivalent system to the first pair of nodes.

### Synchronization of 2 nodes

1. Case  $x_i - x_0 \leq \frac{A}{2}$ ;  $\vartheta = 0$

In the case the 2 junctions are distance to each other of quantity lower of  $A = 0.5$ , these nodes must be put in phase, that is with  $\vartheta = 0$ . In order to solve this problem it is necessary to adopt the ideal equivalent

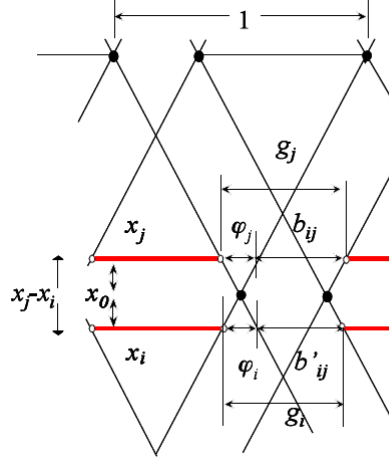


Figure 1.5: Synchronization of 2 nodes at a distance less than half the module.

system, exactly, the 2 nodes are one above and other node under the ideal node, this is shown in the figure 1.5.

Applying the relationship 1.7 and 1.8 to the node  $i$  it obtains to the triangle of base  $\varphi_i$  the following relations:

$$x_i - x_0(i, j) = A\varphi_i \quad (1.7)$$

$$b'_{ij} = g_i - \varphi_i \quad (1.8)$$

Explicating  $\varphi$  from the 1.7 and substituting this value in the relation 1.9 it obtains the following equation, related to the value of the bandwidth:

$$b'_{ij} = g_i - \frac{x_i - x_0}{A} \quad (1.9)$$

Adopting the same procedure for the node  $j$  it obtains as objective function, sum of two bands, the following relationship:

$$f = b'_{ij} + b_{ij} = g_i + g_j - \frac{x_i - x_0}{A} \quad (1.10)$$

The maximum bandwidth of 2 nodes, having a distance less than  $A/2$  and in phase  $\vartheta = 0$  is equal to the following equation:

$$b_{ij} = b'_{ij} = \frac{1}{2}(g_i + g_j - \frac{x_i - x_0}{A}) \quad \text{if } \varphi = 0 \quad (1.11)$$

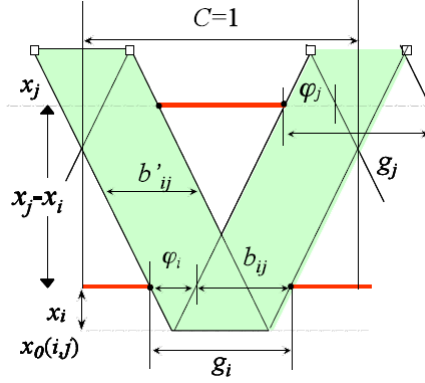


Figure 1.6: Synchronization of 2 nodes at a distance more than the half of the module.

2. case  $x_i - x_0 > \frac{A}{2}$ ;  $\vartheta = 0.5$

In the case the 2 nodes are distance to each other of quantity more than  $A = 0.5$ , these nodes must be put in opposite phase, that is with  $\vartheta = 0$ . In this case the ideal node is not among the 2 real nodes, as shown in the figure 1.6.

In this case the node  $i$  is above the ideal node, so it is possible to consider the following relations:

$$x_i - x_0(i, j) = A\varphi_i \quad (1.12)$$

$$b_{ij} = g_i - \varphi_i \quad (1.13)$$

from which

$$b_{ij} = g_i - \frac{x_i - x_0}{A} \quad (1.14)$$

The node  $j$  is placed respect the ideal node of a value equal  $x_0 + A$ . For this reason applying the law corresponding to the similarity of the triangles, it obtains:

$$x_0 + A - x_j = A\varphi_i \quad (1.15)$$

$$b'_{ij} = g_j - \varphi_j \quad (1.16)$$

$$b'_{ij} = g_j - \frac{x_i - x_0}{A} \quad (1.17)$$



Therefore, the sum of the 2 band is:

$$f = b_{ij} + b'_{ij} = g_i + g_j - 1 + \frac{x_j - x_i}{A} \quad (1.18)$$

The maximum bandwidth of 2 nodes, having a distance more than  $A/2$  and in opposite phase  $\vartheta = 1/2$  is equal to the following equation:

$$b_{ij} = b'_{ij} = \frac{1}{2}(g_i + g_j - 1 + \frac{x_j - x_i}{A}) \quad \text{if } \varphi = \frac{1}{2} \quad (1.19)$$

### Identification of the ideal equivalent system to a pair of nodes

In order to identify the ideal equivalent system between two real nodes it is necessary to consider that if the nodes are distance less than  $1/2$  the ideal node is between two real nodes, while, in the case the distance is more than  $1/2$  the ideal node is under the node  $i$  and above the node  $j$ , it is possible to choose indiscriminately one of two ideal nodes. According to these hypothesis the ideal node can be calculated in the following ways:

$$x_0 = x_i + (g_i - b'_i A) \quad \text{if } \frac{x_j - x_i}{A} \leq 0,5 \quad (1.20)$$

$$x_0 = x_i + (g_i - b'_i A) \quad \text{if } \frac{x_j - x_i}{A} > 0,5 \quad (1.21)$$

In the case the nodes are a distance more than  $A$ , it is necessary to substitute the ratio  $\frac{x_j - x_i}{A}$  with the value  $\text{mantissa}(\frac{x_j - x_i}{A})$ , operation which allows to remain within the interval  $[0, 1)$

### 1.5.2 Synchronization of urban artery

The relationships and the several case above mentioned are very important to synchronize the urban artery made up of several nodes. The synchronization of artery is equivalent to the the study of a pair of nodes. For example, the urban artery is made up of 3 nodes, the third signalized junction, that is equal to a node of the artery, has been synchronize through the ideal node. Therefore, the ideal equivalent system is an important tool through which it is possible to solve the synchronization problem with the different nodes of the urban artery, because this system avoids to consider all feasible combinations between the nodes. From mathematical point of view the width of the band after the synchronization to the third nodes is given to the following relationships:

$$b_{ij,r} = b'_{ij,r} = \frac{1}{2}(g_r + b_{ij} - \text{mant}\left[\frac{x_r - x_0(ij)}{A}\right]) \quad (1.22)$$

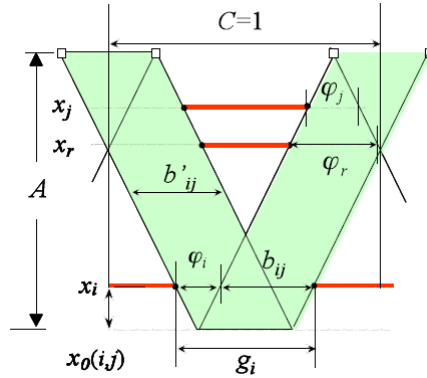


Figure 1.7: Synchronizing a third node  $r$  with the pair of nodes  $i$  and  $j$ .

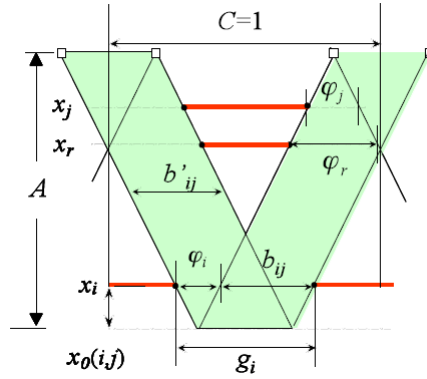


Figure 1.8: Reduction of bandwidth and traslation of the ideal system after after the synchronization of a third node  $r$  with the pair of nodes  $i$  and  $j$ .

$$if \quad 0 \leq mant \left[ \frac{x_r - x_0(ij)}{A} \right] < 0,5$$

$$b_{ij,r} = b'_{ij,r} = \frac{1}{2}(g_r + b_{ij} - 1 + mant \left[ \frac{x_r - x_0(ij)}{A} \right]) \quad (1.23)$$

$$if \quad 0,5 \leq mant \left[ \frac{x_r - x_0(ij)}{A} \right] < 1$$

The figure 1.7 displaces that the third node is in phase with the ideal node. The synchronization of node  $r$  reduces the width of the band and therefore a movement of the ideal grid, so as shown in the figure 1.8.

In general, each nodes is put in phase with the ideal nodes near, its synchronization decreases the distance between the real node and the ideal node.

The follow steps, note the new position of the ideal node, is to synchronize to this point to the fourth node of the artery. These are the steps necessary to synchronize all nodes of the urban artery.

The procedure of resolution of synchronization of the urban artery requests the following several steps:

- coordination of a first pair of nodes;
- identification the ideal system, equivalent to the first pair of nodes;
- coordination of the third node in the ideal system;
- identification of the new ideal equivalent system to the succession of the coordinate nodes;
- extension of the procedure to the all nodes.

It can be noted that the solution depends on the order of the node synchronization. The optimum solution must be found considering all possible combinations of first two nodes and the  $(n - 2)!$  permutations of the followings.

## 1.6 Car following model

### 1.7 Overview

Car following is a microscopic model which analyzes the behavior of the single vehicle, taking into account its kinematic characteristics and its interaction with other vehicles. The model above mentioned is completely different to the macroscopic model, which studies the aggregate behaviors of the vehicular stream. Car following, exactly, is a model describes as a driver reacts to the changes of the driving state of the vehicle ahead, considering, in particular, the longitudinal conditioning between the vehicles in a dense traffic condition, which is the origin of the congestion problem [3]. Various models were formulated to define the interaction between the vehicles, therefore, these models can be classified in:

- safety distance model: is a model based on the hypothesis that the follower vehicle is maintained a safety distance from the vehicle ahead. The first formulation is attributed to Kometani and Sasaki (1959), while the most famous evolution of this model is introduced by Gipps (1981), which provides two different functions for the free-flow and car-following regime.

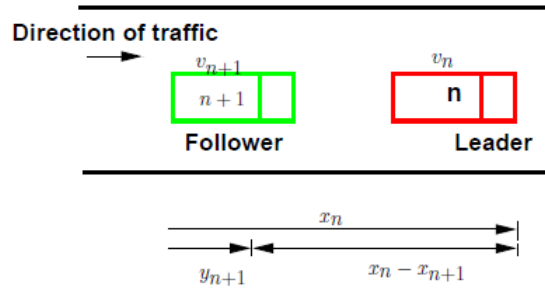


Figure 1.9: Notation for car following model

- response-stimulus model: is a model based on the hypothesis that the response or the reaction of follower vehicle is directly proportional to the stimulus of leader vehicle. The first model is attributed to Chandler et al. (1958). The first systematic experimentation on driver behavior, promoted by General Motors in the early 1960, is the GHR model, from the name of Gazis, Herman and Rotery (1959), and is also known as General Car Following Model. This model was then applied in the *Transmodeler* micro simulation software.
- psycho-physical model: is a model based on modeling of the human decision process during the phase of the guide, taking into account the driver's perception and some behavioral rules. Michaels is the first researcher to implement this model (1963); then Wiedemann (1974) presented a way to calculate the thresholds and then make a simulation. The model proposed to Wiedemann is at the base of the VISSIM micro simulation software.

### 1.7.1 Response-stimulus model

Response-stimulus model is a simple linear model, in which the response of vehicle is directly proportional to the stimulus of the vehicle ahead. In particular the reaction is proportional to the relative speed between the follower and leader vehicle. This car following model is the product of a series of experiments carried out on the vehicles. The most significant test is executed in the 1960s by General Motors, which allowed them to calibrate their parameters and validate the results. In other words, the basic philosophy of car-following theories can be summarized by the following equation:

$$Response \propto Stimulus \lambda \quad (1.24)$$

where:

The response of the follower vehicle represents the acceleration  $dv_{n+1}/dt$  which driver controls acting on accelerator or brake. The sensitivity  $\lambda$  is a factor that defines direct proportionally between the stimulus function and the control function. The stimulus is the relative speed between follower and leader vehicle  $[v_n(t) - v_{n+1}(t)]$ , that is the variation of displacement. In general, the car following model supposes that the driver during its movement carries out the following assignments:

- follows the leader vehicle, in this way its relative speed is  $v \approx 0$ ;
- avoids the collision which implies a big collision time  $t_c$ .

Driver's reaction is not immediately respect the stimulus of leader vehicle, therefore, it is necessary to consider the reaction time in the base equation, so as suggested by Chandler:

$$\frac{dv_{n+1}(t+T)}{dt} = \lambda[v_n(t) - v_{n+1}(t)] \quad (1.25)$$

where:

- $\frac{dv_{n+1}(t+T)}{dt}$  acceleration of the follower vehicle at time  $(t+T)$ ;
- $[v_n(t) - v_{n+1}(t)]$  relative speed of a couple of vehicles;
- $\lambda$  driver sensitivity coefficient is constant.

In particular, the driver sensitivity coefficient  $\lambda$  takes into account the drivers awareness and intensity of reaction to stimulus from the vehicle in front.

The relationship 1.25 represents an adaptation of the generic driver to the change of the distance respect to the leader vehicle. In the first car following model driver will attempt to keep relative speed as small as possible between its vehicle and the one immediately ahead. The stimulus changes the speed of the vehicle ahead and the lagged response modifies the velocity of the following vehicle, whereas the process is constrained to minimize the differences in terms of relative velocity between the two vehicles. This approach, therefore, allows to study the dynamic driving of a couple of vehicles, known the law of the first vehicle, and in general, to analyze the dynamic of the vehicular stream.

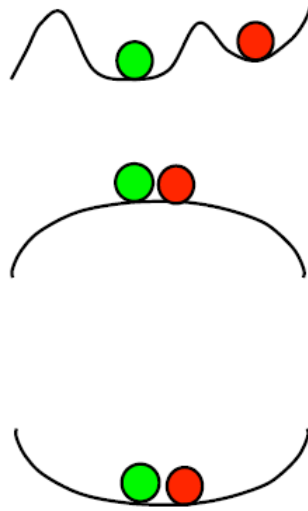


Figure 1.10: Analysis of the system stability: stable system, unstable system and asymptotically stable system

### 1.7.2 System stability

From theoretical point of view, it is very interesting to determine the conditions which causes the system instability (hiccup march and collision between vehicles) and the laws regulate the system to the stationary state. The last aspect is a meaningful factor useful to study the system performance in normal condition, that is the vehicles travel all at the same speed, without to consider the human behavior. Differently, the first aspect causes a worsening of the system performance.

In other words, from engineering point of view, the determination of the stationary state is relevant in the design and planning of the system components, instead, the instability is a fundamental element to the planning of the control system. In terms of stability needs to define, from physical point of view, the possible state of a system:

1. stable system: after a small perturbation, it comes back to its initial state;
2. unstable system: after a small perturbation, it gets away from its initial state indefinitely;
3. asymptotically stable system: after an infinite time all solutions of the system tend to the same value;

Table 1.3: Value of  $\lambda$

System	Value of $\lambda$
Stable system	$0 < \lambda T \leq 0,414$
Asymptotically stable system	$0,414 < \lambda T < 1$
Unstable system	$\lambda T > 1$

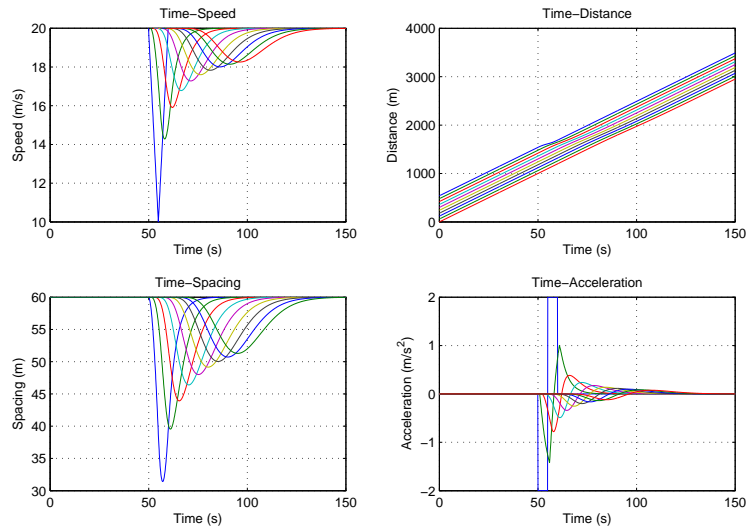


Figure 1.11: Example of stable system with  $0 < \lambda T \leq 0,414$ , considering 10 vehicles and a small perturbation connected to driving behavior of the first vehicle.

as shown in the figure 1.10.

In the response-stimulus model the stability of system is determined the value of the constant  $\lambda$ . Therefore, it is possible to recap the several behavior of system in the table 1.3.

Modifying the value of  $\lambda$  in the range defined in the table 1.3, it is possible to analyze the state of the considered system and the driving behavior according to the laws that regulate the response-stimulus model(see figure 1.11 and 1.12).

Therefore, the only factor that influences the state of the system is the driver sensitivity coefficient, which describes driver's capacity as a function of his reaction or response caused to driving behavior of the vehicle in front.

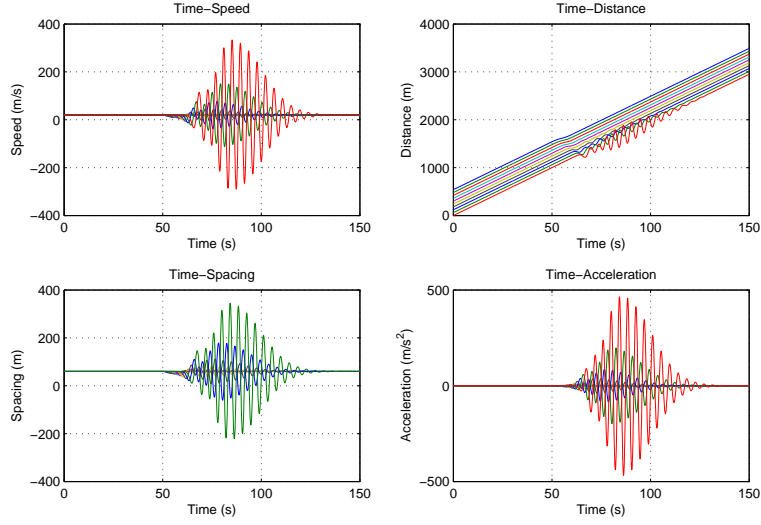


Figure 1.12: Example of unstable system with  $\lambda T > 1$ , considering 10 vehicles and a small perturbation connected to driving behavior of the first vehicle.

### 1.7.3 Safety-distance model

Safety-distance model assumes that follower vehicle keeps a safety distance respect to leader vehicle. This distance  $\Delta x$  is a function of the speed of vehicle  $v_n$  and  $v_{n+1}$  and of the reaction time  $T$  of driver. A feasible relation of distance according Kometani and Sasaki is:

$$\Delta x(t - T) = \alpha v_n^2(t - T) + \beta_1 v_{n+1}^2(t) + \beta v_{n+1}(t) + b_0 \quad (1.26)$$

where  $\alpha$ ,  $\beta$  and  $\beta_1$  are the constant to calibrate the model. The safety distance, instead, is connected to the braking distance, that is:

$$s_f = \frac{v^2}{2\gamma} + v\tau \quad (1.27)$$

The most important evolution of this model is introduced by Gipps in the year 1981. In this model there are two different functions related to 2 different driving regimes:

- free flow regime: driver speed tends to the desired one so that the rate of acceleration decreases until it is canceled once it is the desired speed is reached;
- car following regime: the follower tries to modify its speed in order to reach a safe distance from the vehicle ahead. This distance is considered



safe if it is possible to driver to respond to each action of the vehicle ahead, avoiding the collision.

#### 1.7.4 General motors model

The General Motors model, as described in [21], is the most popular of the car-following theories for the successive reasons:

- agreement with field data; the developed simulation model based on General motors' shows good correlation to the field data;
- mathematical relation to the macroscopic model; Greenberg's model for speed-density relationship can be derived from General motors car following model;

The car following model are various, but the model, that is often considered, is associated with the work of Herman et al. at the research laboratories of General Motors Corporation. The model proposed by the General Motors is founded on follow-the leader concept. This model, in particular, is based on two assumptions:

- higher the speed of the vehicle, higher will be the spacing between the vehicles;
- to avoid collision, driver must maintain a safe distance with the vehicle ahead.

Therefore, the General Motors model is made up: of term  $\Delta x_{n+1}^t$  is the gap available for  $(n+1)^t$  vehicle, of factor  $\Delta x_{safe}$  is the safe distance and of terms  $v_{n+1}^t$  and  $v_n^t$  are the velocities, consequently the gap required is given by:

$$\Delta x_{n+1}^t = \Delta x_{safe} + \tau(v_{n+1}^t) \quad (1.28)$$

where  $\tau$  is a sensitivity coefficient, which can not be a constant, because the behavior of the drivers changes during its movement. In particular, when vehicles are very close to each other, drivers pay more attention and hence their awareness and sensitivity to the actions increase. In other words, this factor explains the sensitivity coefficient variation. The above equation can be written as follows:

$$x_n - x_{n+1}^t = \Delta x_{safe} + \tau v_{n+1}^t \quad (1.29)$$

Differentiating the above equation with respect to time, it obtains:

$$v_n - v_{n+1}^t = \tau a_{n+1}^t \quad (1.30)$$

$$a_{n+1}^t = \frac{1}{\tau[v_n^t - v_{n+1}^t]} \quad (1.31)$$

General Motors has proposed various forms of sensitivity coefficient term resulting in five generations of models. The most general model has the form:

$$a_{n+1}^t = \left[ \frac{\alpha_{l,m}(v_{n+1}^t)^m}{(x_n^t - x_{n+1}^t)^l} \right] [v_n^t - v_{n+1}^t] \quad (1.32)$$

where

- $l$  is a distance headway exponent and can take values from  $+4$  to  $-1$ ;
- $m$  is a speed exponent and can take values from  $-2$  to  $+2$ ;
- $\alpha$  is a sensitivity coefficient;

These parameters are to be calibrated using field data. This equation, exactly, is the core of traffic simulation models. In the implementation of the car following models, three things need to be remembered:

- a driver will react to the change in speed of the front vehicle after a time gap called the reaction time during which the follower perceives the change in speed and react to it;
- the vehicle position, speed and acceleration will be updated at certain time intervals depending on the accuracy required. Lower the time interval, higher the accuracy;
- vehicle position and speed is governed by Newtons laws of motion, and the acceleration is governed by the car following model;

Therefore, the equations of a traffic flow, considering  $\Delta T$ , as the reaction time, and  $\Delta t$ , as the updating time, can be written as follows:

$$v_n = v_n^{t-\Delta t} + a_n^{t-\Delta t} \Delta t \quad (1.33)$$

$$x_n^t = x_n^{t-\Delta t} + v_n^{t-\Delta t} \Delta t + \frac{1}{2} a_n^{t-\Delta t} \Delta t^2 \quad (1.34)$$

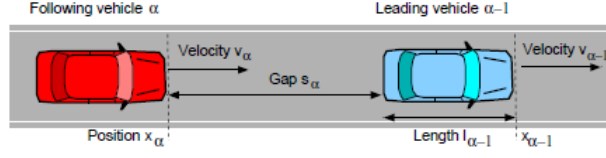


Figure 1.13: Vehicle dynamics

$$a_{n+1}^t = \left[ \frac{\alpha_{l,m} (v_{n+1}^t)^m}{(x_n^t - x_{n+1}^t)^l} \right] [v_n^{t-\Delta t} - v_{n+1}^{t-\Delta t}] \quad (1.35)$$

The relationship 1.33 has similar to Newtons law of motion  $v = u + at$  and the same consideration it can be made for the equation 1.34 which has an equivalent form to the relation  $s = ut + \frac{1}{2}at^2$ . The acceleration of follower vehicle depends on the difference between the speed of leader and follower vehicle, sensitivity coefficient and the gap between the vehicles. In this method it is not able to control the distance a priori because it depends on initial conditions hypothesized.

### 1.7.5 IDM

Intelligent driver model (IDM) is a continuous-time single-lane car following model, through which it is possible to simulate freeway and urban traffic. It is developed by Treiber, Hennecke and Helbing in 2000 to improve the results provided with other intelligent driver models such as Gipps' model, which lose realistic properties in the deterministic limit. The IDM, as a car-following model, describes the dynamics of the positions and speed of single vehicle, in particular it studies the interaction between vehicles, considering driving behavior is influenced to the vehicle ahead, called leader vehicle, so shown in the figure 1.13. The IDM is used as the basic car following model in this work because it reproduces realistically the observed phenomena of urban traffic. Moreover, the model parameters are all relevant and therefore they allow an intuitive description of different driving styles.

#### Equation

A microscopic traffic flow model describes the single vehicle dynamic, exactly the IDM considers the interaction between follower and leader vehicle influences the driving behavior [1]. This peculiarity is described with acceleration function  $a_n(t)$  of each vehicle, which is a continuous function of the actual

velocity  $v_n(t)$ , the net distance gap  $s_n(t)$  and the velocity difference  $\Delta v_n(t)$  to the leading vehicle:

$$a_n = a_0 \left[ 1 - \frac{v_n^\delta}{v_0} - \frac{s^*(v_n, \Delta v_n)^2}{s_n} \right] \quad (1.36)$$

where

- $a_0$  is the maximum acceleration of the vehicle  $n$ ;
- $v_0$  is the desired speed of the vehicle  $n$ ;
- $\delta$  is a parameter of model;
- $s_n$  is the net distance;
- $s^*$  is the minimum desired net distance.

The deceleration term depends on the ratio between the effective desired minimum gap:

$$s^*(v_n, \Delta v_n) = s_0 + T_n v_n - \frac{v_n \Delta v_n}{2\sqrt{a_n b_n}} \quad (1.37)$$

where

- $b_n$  is the desired deceleration of the vehicle  $n$  (comfortable deceleration);
- $s_0$  is a minimum desired net distance (safety distance);
- $T$  is the safety time distance of the vehicle  $n$ .

In particular, the minimum distance  $s_0$  is a significant element in the congested condition only for low velocities. The main contribution in stationary traffic situation is given to the term  $vT$  with  $T$  constant temporal distance between follower and leader vehicle. The last term is only active in non-stationary traffic with  $\Delta v_n \neq 0$  and it implements an intelligent driving behavior including a braking strategy that, in nearly all situations, limits braking decelerations to the comfortable deceleration  $b$ . Moreover, the IDM braking strategy guarantees collision free driving. The IDM is defined through the equation 1.36 which is composed to six parameters. For the sake of simplicity, the acceleration exponent  $\delta$  is fixed constant because this setting corresponds to the most realistic acceleration behavior. The remaining five parameters of Intelligent Driver Model, assuming typical values or for freeway traffic or for urban traffic, as shown in the table 1.4 and 1.5.

Table 1.4: Characteristic parameters of the Intelligent Driver Model for free-way.

IDM parameter	Value
Desired speed $v_0$ ( $m/s$ )	33.3
Headway $T$ ( $s$ )	1,5
Safety distance $s_0$ ( $m$ )	2,0
Maximum acceleration $a$ ( $m/s^2$ )	1,4
Comfortable deceleration $b$ ( $m/s^2$ )	2,0

Table 1.5: Characteristic parameters of the Intelligent Driver Model for urban traffic flow.

IDM parameter	Value
Desired speed $v_0$ ( $m/s$ )	10
Headway $T$ ( $s$ )	1,5
Safety distance $s_0$ ( $m$ )	2,0
Maximum acceleration $a$ ( $m/s^2$ )	1,0
Comfortable deceleration $b$ ( $m/s^2$ )	2,0

In contrast to the stationary behavior, the stability properties of the IDM are mainly determined by the maximum acceleration  $a$ , the desired deceleration  $b$  and by  $T$  (see figure 1.14 and figure 1.15). All these characteristics allow to consider this method a significant tool for the engineering simulation, because it is made up of a number of parameters which it is possible to select properly in order to describe the real phenomena of urban traffic. In other words, the feasibility to fix the value of desired speed, headway, safety distance, maximum acceleration and comfortable deceleration not only consents to manipulate the intrinsic parameters of the model but also to define the desired driving behavior; in this way it is possible to simulate the manners and wanted vehicle dynamic. In the figure 1.16 it can observe as the IDM is able to replicate the desired driving behavior considering the data shown in the table 1.6 considering a perturbation at an instant of time of 60 *sec* and with a deceleration of 0,5  $m/s^2$ .

Table 1.6: Parameters used to test the Intelligent Driver Model for urban traffic flow.

IDM parameter	Value
Desired speed $v_0$ (m/s)	3,3
Headway $T$ (s)	1,6
Safety distance $s_0$ (m)	2,0
Maximum acceleration $a$ (m/s <sup>2</sup> )	0,73
Comfortable deceleration $b$ (m/s <sup>2</sup> )	1,67

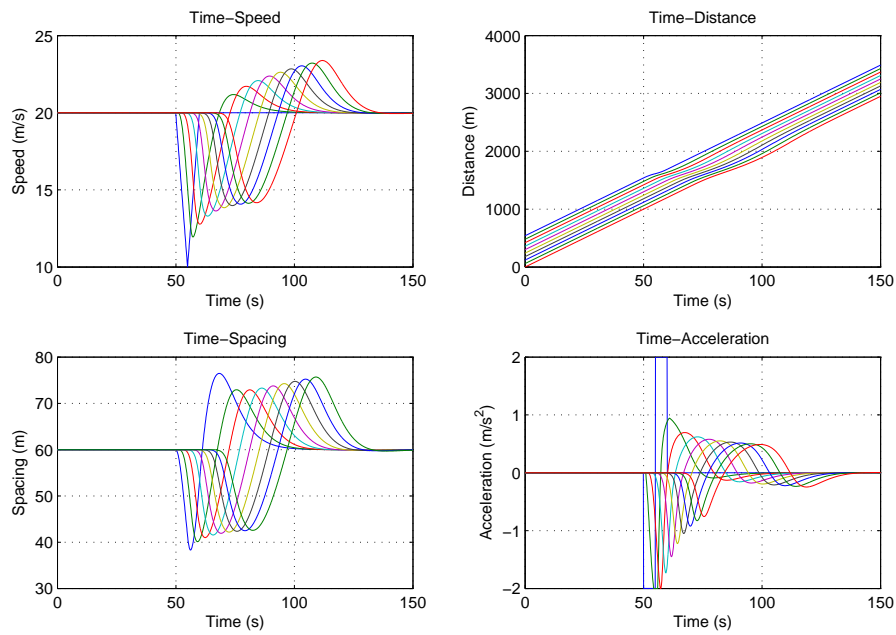


Figure 1.14: Example of stable system with 10 vehicles.

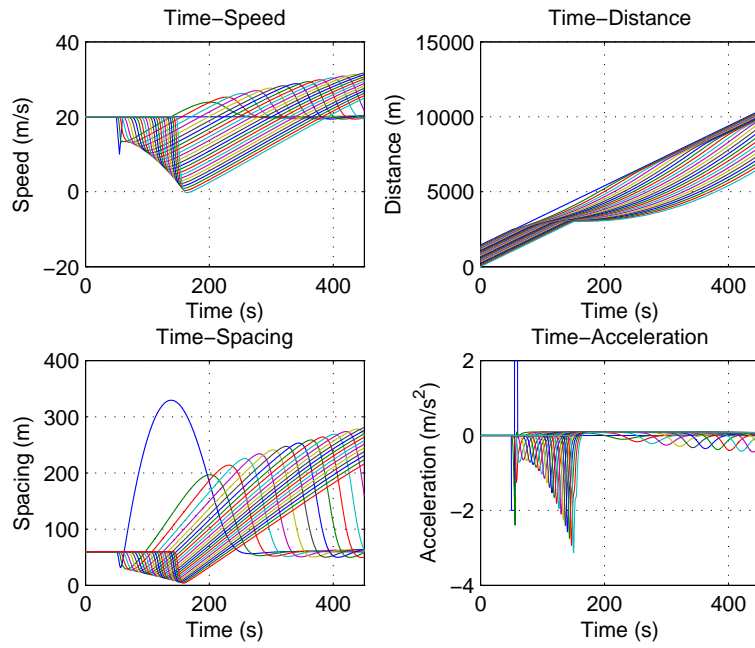


Figure 1.15: Example of unstable system with 10 vehicles.

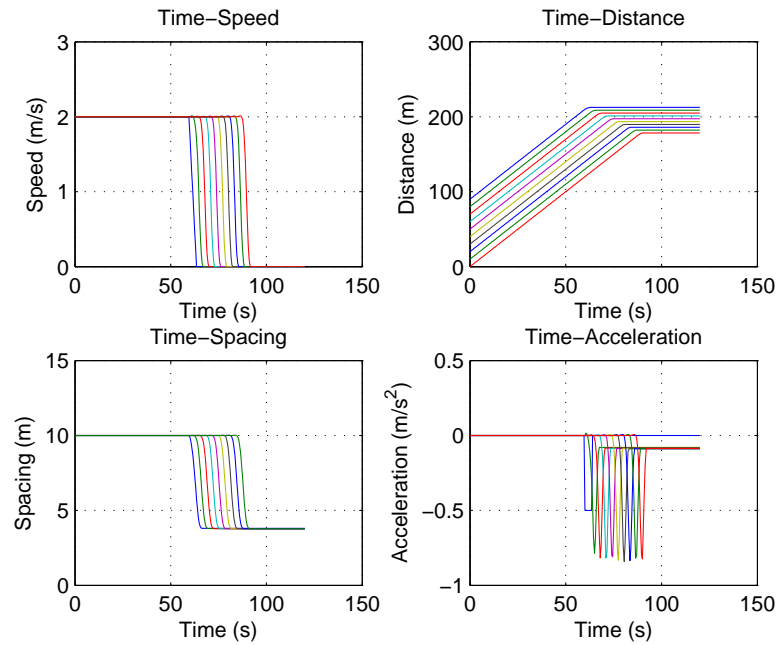


Figure 1.16: Respect to safety distance during evolution of vehicle

## 1.8 Meta-heuristic algorithms

### 1.8.1 Overview

In this thesis the optimization techniques used in order to reduce the delay accumulated to the each vehicles, during the movement along the urban artery, are genetic and firefly algorithms. Before to analyze the implementation of the platform, it is necessary to introduce the method used in general terms, without to examine in depth the application that is the subject of this text, but providing basic information for understanding the reasons bring about the choice of these strategy.

### 1.8.2 Optimization technique

The optimization technique, generally, is a procedure that is able to return as a result the optimum of a function. It is possible to consider three different methods: numeric, randomized and enumerated search.

Numerical methods are based on mathematical considerations and for this reason they need to know the function to optimize and its dependence on each parameter; in particular, it is possible to divide the numerical methods into two subgroups:

- direct method;
- indirect method.

The first, considering a system of nonlinear equations, finds the minimum of a function putting equal zero its gradient, while the second method bases on an iterative algorithm that, step by step, under certain conditions, approaches more and more to the absolute minimum: better representative method of this category are the Newton-Raphson one. The numerical technique is analytical method and, if verified the initial hypotheses, it ensures the truthfulness of the obtained result. The problem, however, is to define these initial hypotheses, because all numerical methods, in particular indirect ones, are based on the search for the minimum using the direction of the first derivative of the function or more generally the direction of the gradient. In the case of multi-modal functions, so as shown in the figure 1.17, characterized by many relative highs and minimums, classical numerical methods would have remain in a point that does not correspond to the optimum solution of the problem. Analytical methods, therefore, return true results only when it is sure that they are a parabolic function or they have only a minimum. Another optimization technique is to calculate the value of function



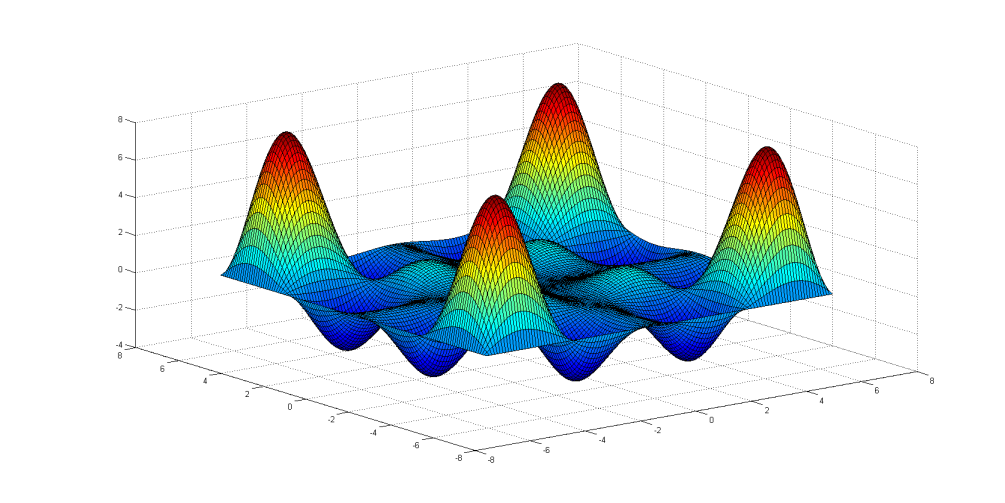


Figure 1.17: Trend of multimodal function

to optimize in whole domain, obviously, it is not able to consider all points of the domain, but only a part of ones, for this reason, needs to carry out a partition of the solution field. This technique called enumerated methods, borns thanks to the evolution of electronic computers and to their computational capacity.

The third and final optimization technique is random guided research, which also includes evolutionary algorithms, as genetic algorithms. It is a particular enumerated technique which, rather than to analyze the whole domain, used complementary information useful in the search for the best solution; this procedure allows to reduce the time and computing cost needed to achieve the optimum. Example of evolutionary algorithms are: the Simulated Annealing which is able to simulate thermodynamic processes, during which a body previously heated, it is cooled slowly until to the elimination of any retinal defects, the genetic algorithm which is based on the Charles Darwin's theory [19] and the Firefly algorithm [10] which simulate the natural behavior of this specific insect.

### 1.8.3 Structure of the genetic algorithm

The peculiarity determine the use of genetic algorithms GA in respect to other optimization technique are three:

- GA can process temporarily a certain number of individuals, avoiding so to find in a local minimum;

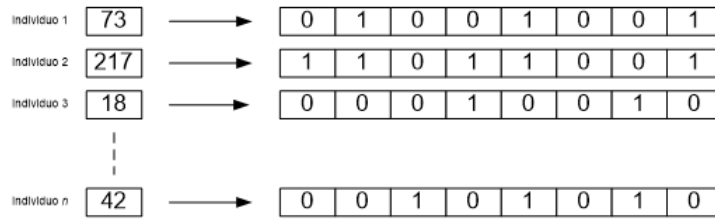


Figure 1.18: Binary conversion of the individuals of the population

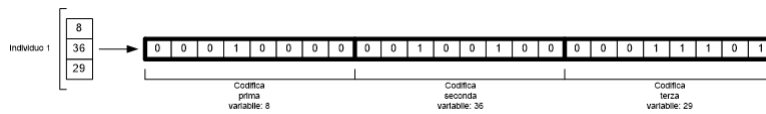


Figure 1.19: Example of multiparameter concatenation

- GA is a probabilistic method, differently to classic methods which are deterministic;
- GA can be used also without the knowledge of  $n$ th derivative, for this reason is only necessary to write the objective function.

It is defined the basic concept useful to implement the genetic algorithm. The first operation consists to generate a number of points belonging to the domain of the objective function randomly; each point represents an individual, that is a possible solution to the problem. All these points represent the first population of the algorithm. In the search of the minimum of function with a single variable (obviously for this type of applications, rarely, it is used genetic algorithms), after the selection of all random  $n$  individuals of the first population, it is necessary to codify these points in a structure similar to the chromosomal form of DNA; one of the most used configuration is the binary structure. Therefore starting from  $n$  points or individuals belonging to the domain of interest,  $n$  binary strings are obtained, as displaced in the figure 1.18.

When the number of variables is more than 1 it needs to carry out a multi-parameter concatenation: according to this procedure each individual is represented to a vector made up of a number of elements equal to the number of the variables. Each element is converted in a binary structure and finally every information is put near to each other in order to create an unique individual, which will represent to binary structure, as that shown in the figure 1.19

### 1.8.4 Operators and selection technique

The essence of the Genetic Algorithm is based on the way through which individuals of new generation are produced starting from the ones of the first generation. The new generation is made up of a set of individuals or solutions that can be better than the previous population. According to Charles Darwin's theory, the individuals of the future generation are made up of only ones which have greater probability to survive and to hand down our genetic patrimony to the successive generation, avoiding to the survival of the weakest individuals. Therefore, from mathematical point of view, only the solutions that are near to the minimum of the function have more probability to survive and to generate the solutions able to solve the problem in the best way. In order to simulate the behavior of Mother Nature and to apply this specific manner to Genetic Algorithm, three operators are used:

1. reproduction;
2. cross-over;
3. mutation.

The first operator aims at the selection of a certain number of individuals and at the copy of one in the successive generation. This operator works in a similar way to nature. In fact, according to the natural laws, the strongest individuals are able to survive and to hand down our genetic structure to the next generation. This factor doesn't imply which only the best  $k$  individuals are selected, rather it is applied a function which selects, based on the characteristics of each solution,  $k$  individuals through a random method. The selection can be carried out through the functions: tournament selection, proportional roulette wheel selection, rank-based roulette wheel selection. Another operator used in the GA is the elitism which is a technique consists of the copy, automatically, the best element from a generation to the successive one, so this operator selects an individual, without the use of selection function above described. This technique, therefore, has not any parallelism to the evolutionary theory, because selected individual even if is the best it could not hand down his genetic heritage to the next generation.

The second operator mixes up the genetic patrimony of two individuals in order to generate two different persons but with similar characteristics; this operation is able to evolve the algorithm toward a better solution than the previous generation. From the practical point of view, this operator allows to combine two sub-strings, made up a certain number of bits of random length, belonging to two different strings, in order to generate a new individual able to have the positive peculiarity of the both parent. In the figure 1.20 it is

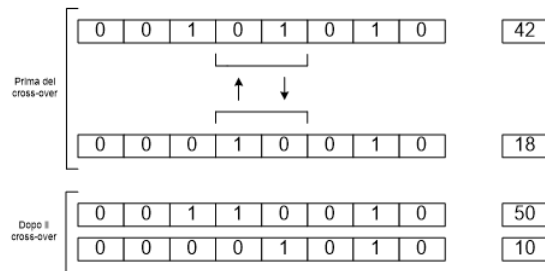


Figure 1.20: Example of cross-over

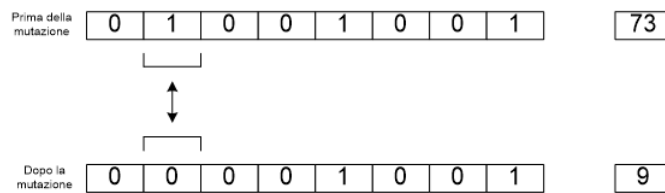


Figure 1.21: Example of mutation

possible to observe two individuals are made up of 8 bits one, on which it is carried out the cross-over operation. This operator allows to select a pair of bits, one for each, and the cross-over operation is done to generate a new pair of elements.

The mutation is the last operator through which it is possible to commute a single individual's bits in order to have an individual is made up of completely different characteristics than the initial ones. This operator allows solving the problem to find a minimum local, analyzing the part of the unexplored domain, within which there are better solutions than those that were followed before. This operator is applied to a certain number of individuals according to stochastic criteria. In the figure 1.21 it is shown the way through which carries out the selection and switching of individual's bits.

### 1.8.5 Fitness function and objective function

In the genetic algorithm, there are two different functions: fitness and objective functions, which have a different conceptual meaning. The first function measures the individual's goodness, exactly, it assigns a higher value to the individual which has more probability to generate individuals are able to return the minimum of the objective function, instead the second one is the

function to minimize. In particular, the goodness of the individuals allows to apply the operators above mentioned.

### 1.8.6 Stop criteria

In the genetic algorithm is possible to use many stop criteria. The first and more simple stop criteria is to fix a number of maximum iterations or generations, in other words the algorithm will return the element of the last population which minimizes in better way the objective function. Another possibility it is to fix a maximum time in the search of the result, beyond which the algorithm is interrupted. Another technique is to supervise the element to the highest fitness and the average value of the fitness of all individuals of a population: when these two indicators are sufficiently close to one another, it means that most of the individuals are converging towards the absolute optimum. Finally, it can impose the algorithm stops when you get the same result for a certain number of generations.

### 1.8.7 Seeding

The first population is usually generated randomly, but in the case of the initial information, it is possible to use these ones in order to bring the algorithm to convergence in a less time than necessary one to the first generation completely random. This technique allows a convergence of the algorithm more quickly, because it has an element with a good fitness and, therefore, an element that is able to evolve the individuals towards the right direction. In other words, introducing a seed in the search of the optimum, it involves the analysis of a certain part of domains that are more promising than others. This technique, obviously as well as an elitism criteria, modifies intensely the random nature of the algorithm, exactly, both techniques leave to Charles Darwin's theory.

### 1.8.8 Flowchart diagram of the Genetic Algorithm

In general, it is possible to recap the several steps characterized the Genetic Algorithm in a flowchart, as shown in the figure 1.22. This diagram explains the simple scheme of the principle operation of GA. In relation to the purposes of the present study, the Genetic Algorithms are only a tool through which to achieve the results. However, before to think as to improve the method, it is necessary to understand as to apply it to the problem.

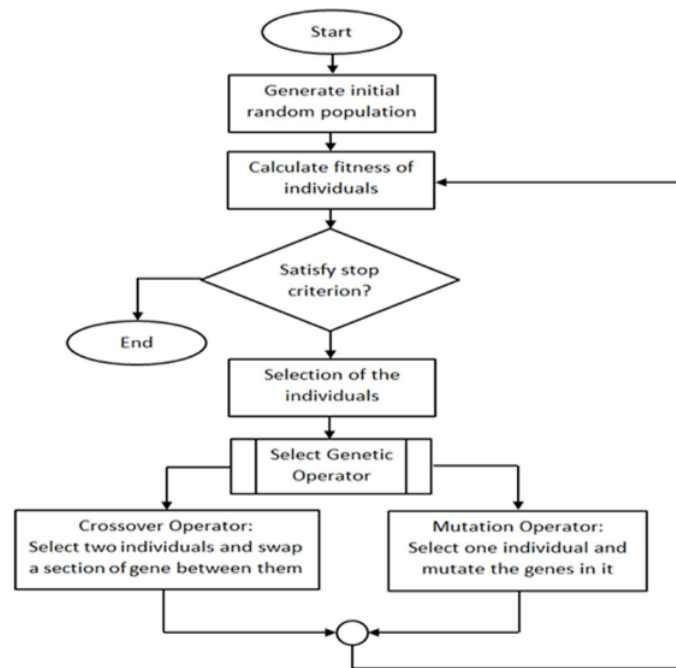


Figure 1.22: Flowchart diagram of a generic genetic algorithm

## 1.9 Firefly algorithm

### 1.9.1 Overview

The Firefly Algorithm (FA), proposed by Xin-She Yang at Cambridge University, is a meta-heuristic optimization method, inspired by the behavior of fireflies [20]. This method together with many other meta-heuristic ones is based on existing mechanisms of nature. The natural systems, in fact, represent a starting point in the planning of new techniques dedicated to solve many optimization problems. For example, the ant and bee algorithms are two methods inspired by the natural behavior of the insects and their interactions. The structure of the FA method is very interesting because the algorithm is inspired by the natural behavior in the search for the optimum, trying to reproduce a similar manner in mathematical terms.

### 1.9.2 Firefly behavior

The flashing light is produced by a process of bioluminescence. The produced light and its flashing has two fundamental functions:

1. to attract partners (communication);

2. to attract potential prey.

Furthermore, flashing light is a tool which the fireflies can be used as a protective warning mechanism. The flashing light and the rate of one are intrinsic features of both firefly's sexes. The communication system between this type of insects takes place through the radiated light and its intensity, which can observe only at a particular distance  $r$ . The brightness  $I$  is inversely proportional to the square of  $r$  ( $I \propto \frac{1}{r^2}$ ). Moreover, the air absorbs light which becomes weaker when the distance increases. Both factors make most fireflies visible only to a limited distance, usually several hundred meters at night, which is considered a good distance between the fireflies to communicate. Therefore, the light intensity can be, in general, associated to the objective function to be optimized, so as described in [10].

### 1.9.3 Characteristics of FA

The use of the FA as an optimization method involves the following three idealized rules:

- all fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex;
- Attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less brighter one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly;
- The brightness of a firefly is affected or determined by the landscape of the objective function.

These three rules together the firefly behavior can be summarized from mathematical point of view through the pseudo code shown in the figure 1.23. In FA, the attractiveness is connected to objective function and to distance according to a monotonous decreasing behavior.

## 1.10 Attractiveness

In the firefly algorithm, there are two important issues: the variation of brightness and formulation of the attractiveness. For the sake simplicity, it can assume that the attractiveness of a firefly is determined by its brightness which, moreover is associated with the objective function. In this study,

**Firefly Algorithm**


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*Objective function*  $f(\mathbf{x})$ ,  $\mathbf{x} = (x_1, \dots, x_d)^T$   
*Generate initial population of fireflies*  $\mathbf{x}_i$  ( $i = 1, 2, \dots, n$ )  
*Light intensity*  $I_i$  at  $\mathbf{x}_i$  is determined by  $f(\mathbf{x}_i)$   
*Define light absorption coefficient*  $\gamma$   
**while** ( $t < \text{MaxGeneration}$ )  
**for**  $i = 1 : n$  all  $n$  fireflies  
    **for**  $j = 1 : i$  all  $n$  fireflies  
        **if** ( $I_j > I_i$ ), Move firefly  $i$  towards  $j$  in  $d$ -dimension; **end if**  
        *Attractiveness varies with distance*  $r$  via  $\exp[-\gamma r]$   
        *Evaluate new solutions and update light intensity*  
    **end for**  $j$   
**end for**  $i$   
*Rank the fireflies and find the current best*  
**end while**  
*Postprocess results and visualization*

---

Figure 1.23: Firefly pseudo firefly algorithm

the analyzed problem is a case of minimum optimization problems, for this reason the brightness  $I$  of a firefly in a specific point of the domain  $x$  can be chosen as  $I(x) \propto 1/f(x)$ . The attractiveness  $\beta$ , therefore, is proportional to the brightness, both factors vary to increase the distance  $r_{ij}$  between firefly  $i$  and firefly  $j$ . In particular, light intensity decreases with the distance from its source, and light is also absorbed in the media, so the attractiveness varies with the degree of absorption. In the simplest form, the light intensity  $I(r)$  varies according to the inverse square law, that is:

$$I(r) = \frac{I_s}{r^2} I_s / r^2$$

where  $I_s$  is the intensity at the source. For a given means with a fixed light absorption coefficient  $\gamma$ , the light intensity  $I$  varies with the distance  $r$ . Precisely,

$$I = I_0 e^{-\gamma r}$$

where  $I_0$  is the original light intensity. In order to avoid the singularity at  $r = 0$  in the expression  $I_s/r^2$ , the combined effect of both the inverse square law and absorption can be approximated using the following Gaussian form:

$$I(r) = I_0 e^{-\gamma r} \quad (1.38)$$

Sometimes, it needs to have a function which decreases monotonically at a slower rate. In this case, it can use the following relation:

$$I(r) = \frac{I_0}{1 + \gamma r^2} \quad (1.39)$$



At a shorter distance, the above two forms are essentially the same. This is because the series expansions about  $r = 0$ .

$$e^{-\gamma r^2} \approx 1 - \gamma r^2 + \frac{1}{2}\gamma^2 r^4 + \dots \quad \frac{1}{1 + \gamma r^2} \approx 1 - \gamma r^2 + \gamma^2 r^4 + \dots, \quad (1.40)$$

are equivalent to each other up to the order of  $O(r^3)$ . As a firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, therefore it can define the attractiveness  $\beta$  of a firefly through the successive relationship:

$$\beta(r) = \beta_0 e^{-\gamma r^2} \quad (1.41)$$

where  $\beta_0$  is the attractiveness at  $r = 0$ . As it is often faster to calculate  $1/(1 + r^2)$  than an exponential function, the above function, if necessary, can conveniently be replaced by  $\beta = \beta_0/(1 + \gamma r^2)$ . Equation 1.41 defines a characteristic distance  $\Gamma = 1/\sqrt{\gamma}$  over which the attractiveness changes significantly from  $\beta_0$  to  $\beta_0 e^{-1}$ . In the implementation, the actual form of attractiveness function  $\beta(r)$  can be any monotonically decreasing functions such as the following generalized form:

$$\beta(r) = \beta_0 e^{-\gamma r^m}, \quad (m \geq 1) \quad (1.42)$$

For a fixed  $\gamma$ , the characteristic length becomes  $\Gamma = \gamma^{-\frac{1}{m}}$  as  $m \rightarrow \infty$ . Conversely, for a given length scale  $\Gamma$  in an optimization problem, the parameter  $\gamma$  can be used as a typical initial value. That is  $\gamma = \frac{1}{\Gamma^m}$  [10].

### 1.10.1 Distance and Movement

The distance between any two fireflies  $i$  and  $j$  at  $x_i$  and  $x_j$ , respectively, is the Cartesian distance:

$$r_{ij} = \|(x_i - x_j)\| = \sqrt{\sum_{k=1}^d (x_{i,k} - x_{j,k})^2} \quad (1.43)$$

where  $x_{i,k}$  is the  $k$ th component of the spatial coordinate  $x_i$  of  $i$ th firefly. In 2-D case, we have  $r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ . The movement of a firefly  $i$  is attracted to another more attractive (brighter) firefly  $j$  is determined by:

$$r_{ij} = x_i + \beta_0 e^{-\gamma r_{ij}^2} (x_j - x_i) + \alpha \left( \text{rand} - \frac{1}{2} \right) \quad (1.44)$$

where the second term is caused the attraction, while the third term is randomization with  $\alpha$  being the randomization parameter.  $Rand$  is a random number generator uniformly distributed in  $[0, 1]$ . For most cases in this implementation, it can take  $\alpha = 1$  and  $\alpha \in [0, 1]$ . Furthermore, the randomization term can easily be extended to a normal distribution  $N(0, 1)$  or other distributions. The parameter  $\gamma$  characterizes the variation of the attractiveness, and its value is crucially important in determining the speed of the convergence and as the FA algorithm behaves [10].

### 1.10.2 Flowchart diagram of the Firefly Algorithm

In general, it is possible to summarize the several steps characterized the FA in a flowchart, as shown in the figure 1.24. This diagram explains the simple scheme of the principle operation of FA. In relation to the purposes of the present study, the Firefly Algorithms are only a tool through which to achieve the results as in the case of GA.

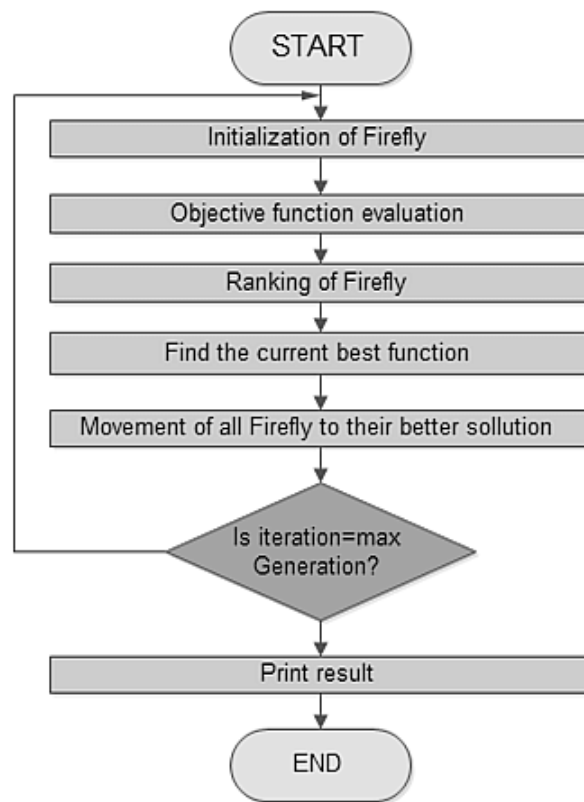


Figure 1.24: Flowchart diagram of a generic Firefly Algorithm

# Chapter 2

## Model

### 2.1 Overview

In this chapter the carried out choices are reported for the writing of a mathematical model, so that it is easily usable to any optimization algorithm. The developed traffic model is a model through which it is possible to represent a whole urban area, made up of several arteries, exactly, it represents an area composed of a main artery and several secondary arteries. This work aims at the fulfillment of a model that is able to contain the main concepts mentioned in the previous chapter, in order to return the total delay of the vehicles during their movement along the urban artery. The model can be imagined as a black box, including the relationships regulate the behavior of the single road user and the interaction between the drivers during the movement from the origin to the destination node of the area of interest, but also all characteristics information corresponding to the urban artery. In general, this box or this model is born as a tool through which to simplify the structure of the problem and to divide the model to the optimization methods, this division allows to change the optimization algorithm easily. The black box must be able to return, starting from some input data, the parameters through which to carry out a proper sizing of the signalized intersection and to change the surrounding conditions, as shown in the figure 2.1. Therefore, the aim is double: from side the goal is to determine the set of parameters that best fit to the traffic control and the traffic conditions presented in the study area, and to the other side, it is necessary to understand which is the best configuration, in respect of the problem constraints. This procedure represents better way to reduce the accumulated delay to the driver along its movement along the artery. After a brief analysis of the fundamental aspects associated with the optimization problem of a urban artery: analysis of the

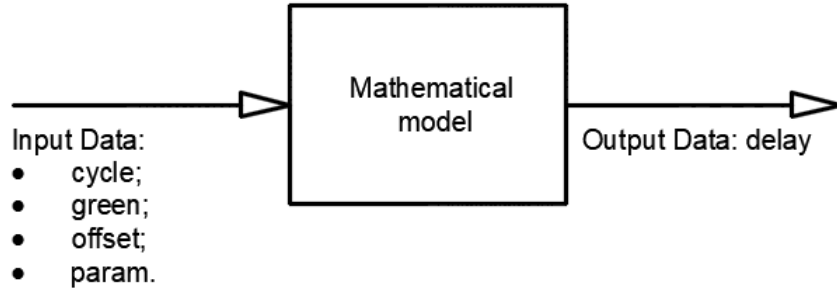


Figure 2.1: Mathematical model

current state of the artery, synchronization problem and the analysis of the behavior of driver and his interaction with the other drivers, it is able to motivate coherently the carried out choices to elaborate the mathematical model of the present thesis.

## 2.2 Constraints

The analyzed problem, from the analytical point of view, is a non-convex multi-variable and constrained function. In the definition of constraints it needs to define, first of all, the several intrinsic elements of the problem, exactly, the essential factors are made up of the area of study: the regulation of the traffic signal in terms of length of the cycle, length of the green time and the offset. The constraints are defined on the basis of the problem characteristics, these limits must be respected in the adjustment of the traffic signals. The constraints of the problem can be so summarized:

$$C_i \geq C_{min} \quad \text{with} \quad C_{min} = \frac{1}{1 - \sum y^*} \quad (2.1)$$

$$C_i > G_i + L \quad (2.2)$$

$$G_i > y^* C \quad (2.3)$$

The first constraint avoids the cycle assuming unrealistic value during the optimization process, exactly, it defines the minimum value of cycle, that is the inferior limit of the one. Another cycle limit is defined in the equation 2.2, in which the cycle is evaluated as the sum of the green length and the total lost time. The last constraint, instead, reviews the value of the green time

which must have a value connected to the cycle length and the saturation degree. All these limits are defined as capacity constraints. The restraints above mentioned allow to verify, every time, if each individual of GA or each firefly of FA respect the constraints capacity of the problem because if these limit are violated, the objective function is not evaluated for the incompatible individuals. Whenever incompatible individuals are evaluated, it needs to use a technique which penalizes the objective functions of unacceptable solutions. In other words, this individuals are rejected through a relationship which modifies the objective function, so it is evaluated a fictitious delay, as the product between a value of penalty parameter and the value assumed to constraints of incompatible individual or firefly. Therefore, it is necessary to introduce a penalty factor, which must bear in mind that the problem is not solved because the constraints are not satisfied. The penalty factor is so a measure of the violation restraints of the analyzed individual.

## 2.3 Delay

### 2.3.1 Definition

The purpose of this work is to calculate the value of the accumulated delay of drivers and to reduce this value through the implementation of the optimization algorithms. In the evaluation of this factor it needs, first of all, to take into account the feature of the urban artery and, at the same time, to consider the adopted car-following model.

In the analyzed problem, initially, is evaluated the evolution of the vehicular stream to the stationary state, that is in the absence of perturbation. Therefore, the vehicles move undisturbed along the urban artery according to the laws of uniform straight motion.

In unperturbed conditions (see figure 2.2) it is calculated the travel time or ideal time of the single vehicles, as a function of the desired speed and of the length of the artery, data equal for each vehicle of the traffic flow related to the area of interest. The travel time in a stationary system is:

$$time_{ideal} = \frac{LA}{v_{desired}}$$

In this study it is not evaluated only the stationary state, but also a perturbed system due to the presence of the signalized intersections. The perturbation represents an event that modifies the system stability and, consequently, the normal evolution of the vehicles along the urban artery, so as shown in the figure 2.3.

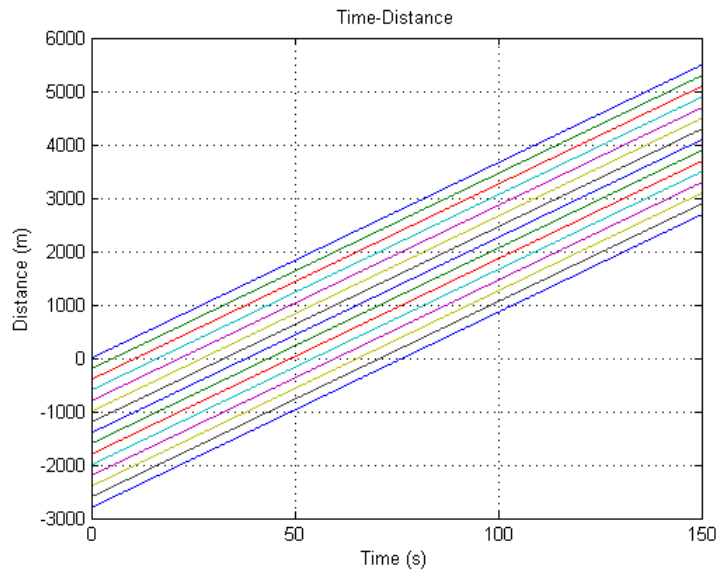


Figure 2.2: Undisturbed vehicular stream

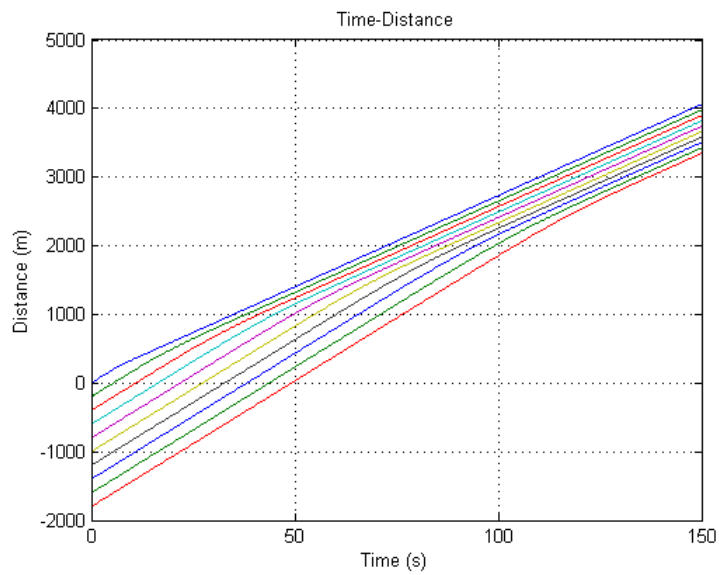


Figure 2.3: Vehicular stream with a perturbation

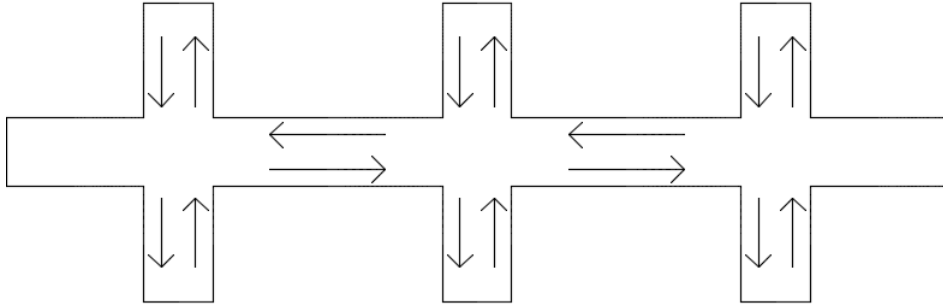


Figure 2.4: Urban artery

In the exam case, the evolution of vehicles is perturbed to the presence of the traffic signal, these perturbed elements can cause the possible training of the queue and, consequently, an increasing of the travel time, which can be considered as the origin of the delay of a vehicle. In the computation of the requested travel time to cross the artery with several signalized junctions, differently, to the previous estimation of the travel time, it is necessary to take into account the initial instant in which the single-vehicle goes in the artery and the instant in which the vehicle passes the end of artery. This information is used to calculate the travel time in the presence of the elements that perturb the vehicular evolution. The travel time, known as the real time, is equal to the following relationship:

$$time_{real} = t_{final} - t_{initial}$$

Therefore, from theoretical point of view the delay of i-vehicle or the total travel time is calculated as:

$$R_i = time_{real} - time_{ideal}$$

Ultimately it needs to consider the features of the urban artery, exactly, the used approach to evaluate the delay of vehicles. The artery of interest is made up of one main artery and tree secondary artery crosses in both directions, as shown in the figure 2.4. The delay is evaluated for each vehicle, for each direction and for each artery, therefore the delay function is called a number of times equal to the number of vehicles. The total delay (value of the objective function) of each individual is equal to the sum the delay of each vehicles.



## 2.4 Time-discrete simulation

The carried out study requests the definition of the simulation time, that is the time within which is simulated the evolution of vehicles along the urban artery. The analysis time is equal to 8000 s which is consider a wider interval time than the necessary interval because extension of this gap allows to evaluate the movement of all vehicles of the traffic flow during their movement along the artery. Therefore, this value consents to define the dimension of the matrix which are used to solve the optimization problem. In reality, the simulation of the traffic flow along the road stops when the last vehicle crossed the end of the artery. The analyzed problem, from the theoretical point of view, is a very complex issue which requests, according to its intricacy, a simplification through a discretization time. The mathematical model above described is created in order to take into consideration this time division. The time discretization is based on the division of time in steps equal to 1s, this subdivision allows to know:

1. the position of the vehicles in the artery every second;
2. the instant of the crossing of the artery by the vehicle;
3. the initial and final instant of the crossing of artery;
4. the initial and final instant of the red, the green and the orange time;

The four and last point above mentioned explains as the time discretization allows to know the state of the traffic signal in every instant, as for example in the figure 2.5. For the sake of simplicity inside of the model it is created a matrix in which the x-axis is the time-axis with the time discretization, while the y-axis is the axis of the intersections. This matrix is made up of a serious elements equal 0, 1 or 0,5: the first term indicates the red time, the second one indicates the green time and finally the third term defines the orange phase. This simple matrix, therefore, allows to know the state of the traffic light in every second and in each signalized intersection (see figure 2.6 and, consequently, to valuate the distance of vehicle to the traffic signal and, in relation to state of one, to carry out the braking process.

## 2.5 Mathematical model

The Intelligent driver model (IDM) is the starting point of this thesis. This model, as previously described, simulates the evolution of vehicles in the urban artery, exactly, it is able to reproduce all intrinsic traffic dynamic

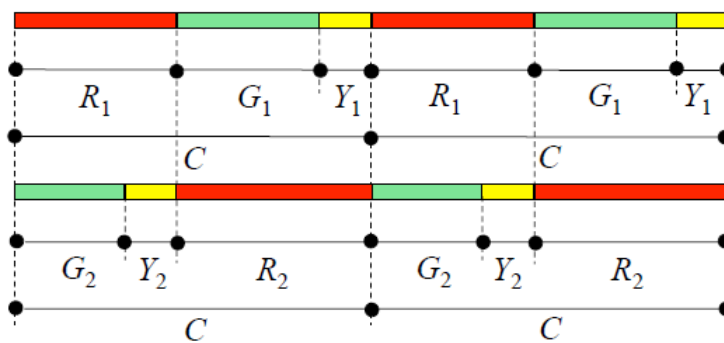


Figure 2.5: Example of traffic signal



Figure 2.6: Traffic signal: succession of green, red and orange time

phenomena observed on road. The IDM is a model with which the driver can regulate his desired speed and the safety distance. It, as a car following model, describes the dynamics of the positions and velocities of single vehicles and it is characterized by the following relationships:

$$a_n = a_0 \left[ 1 - \frac{v_n}{v_0} - \frac{s^*(v_n, \Delta v_n)^2}{s_n} \right] \quad (2.4)$$

$$s^*(v_n, \Delta v_n) = s_0 + T_n v_n - \frac{v_n \Delta v_n}{2\sqrt{a_n b_n}} \quad (2.5)$$

Both equations represent the foundation of Intelligent Driver model, whose essential properties can be summarized as follows [1]:

1. the IDM acceleration is a continuous function incorporating different driving modes for all speed in freeway and in urban context. Besides IDM also takes into account speed differences, which play an essential stabilizing role in real traffic, especially when approaching traffic jams;
2. the IDM contains only a few parameters which are known to be relevant and are empirically measurable;
3. the IDM has different regimes for braking and accelerating and it shows a plausible microscopic acceleration and deceleration behavior of single driver-vehicle;

### 2.5.1 Selected parameters of the Intelligent Driver Model

The characteristic parameters of IDM, above introduced, can be assumed several values in relation to the initial hypothesis of the analyzed problem. Before, to define the parameter values, it needs to specify the index used to determine the follower and the leader vehicle. In this thesis, in fact, it is assumed that the index of the leader vehicle is  $n$ , while the index of the follower one is  $n - 1$ . Therefore, defined index, it needs to describe the importance of each term characterizing the IDM and, consequently, the values assigned to each element. The IDM is defined through the relation 2.4 in which are presented six parameters.

The first analyzed term is  $\Delta v_n$  which is given to the speed difference between the follower and leader vehicle, exactly  $\Delta v_n = v_n - v_{n-1}$ . The speed  $v_n$  considered in the simulations is equal to  $10 \text{ m/s}$ , it is fixed this value in order to take into account the evolution of traffic condition characteristics of the road network. Another parameter is the minimum distance  $s_0$ , which is a relevant element in the congested traffic condition in which the speed

Table 2.1: Characteristic parameters of the Intelligent Driver Model

IDM parameter	Value
Desired speed $v_0$ ( $m/s$ )	10
Headway $T$ ( $s$ )	1
Safety distance $s_0$ ( $m$ )	6
Maximum acceleration $a$ ( $m/s^2$ )	1
Comfortable deceleration $b$ ( $m/s^2$ )	2

is very low, as the case analyzed in this thesis. The value assigned to the safety distance or the minimum distance  $s_0$  between the vehicles is equal  $6m$ . It is chosen this value because the vehicle is considered as a point. This choice, in fact, allows to evaluate its dimension and the spatial distance from the vehicle ahead. The term  $vT$  is the main contribution in stationary traffic condition, because this factor defines the temporal distance between the follower and leader vehicle, in particular the headway  $T$  is equal 1. The last term  $(v_n \Delta v_n) / (2\sqrt{a_n b_n})$ , instead, is only active in non stationary traffic condition with  $\Delta v \neq 0$  and it describes an intelligent driving behavior and a braking strategy that, in nearly every case, limits braking decelerations to the comfortable deceleration  $b$ , that it is assumed equal  $2m/s^2$ . Moreover, the IDM braking strategy avoids the collision between drivers. The parameter  $\delta$ , for the sake of simplicity, is considered as a constant term, exactly  $\delta = 4$ , because this setting corresponds to the most realistic acceleration behavior. All parameters and their value are summarized in the table 2.1.

All these parameters allow a description of the heterogeneous driving behavior. In this thesis project the IDM is used in order to analyze the single-vehicle dynamics, as shown in the figure 2.7. In relation to the assigned parameters and to the features of the urban artery it is necessary to inhibit the negative speed, which would present in the braking process, chancing the real behavior of the driver, also in the case of an instability of the system.

## 2.6 Initial conditions

### 2.6.1 Used parameters to initialize the model

In the section time-discrete simulation, it talks about the simulation time. This time allows simulating the vehicle dynamics in the urban artery. In the initial phase of simulation, exactly to the instant  $t = 0$  it observes the fluctuations (see figure 2.8, especially the first sub-figure to right), which only after an interval time is dumped and, consequently, it allows the system to reach

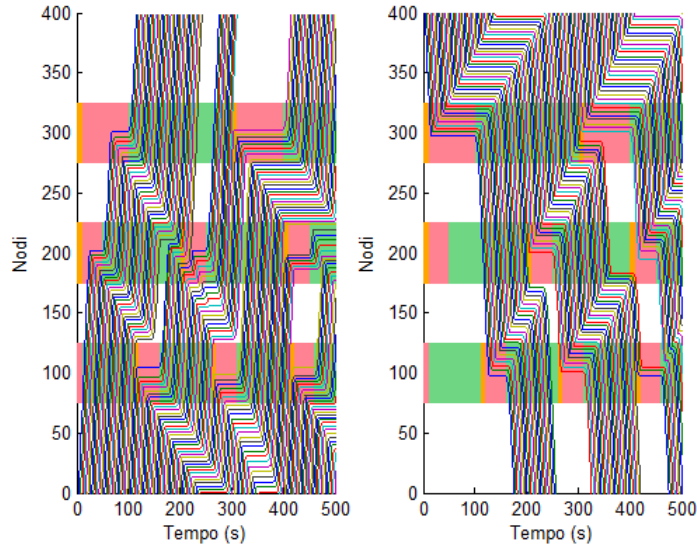


Figure 2.7: Example of queue

the asymptotic stability condition. For this reason, it needs to initialize the several parameters, that is it imposes equal 0 the relation 2.4. This process implies all parameters must respect the condition  $a_n = 0$ . Therefore, it is necessary to evaluate a parameter (unknown parameter is the headway) in order to avoid an initial instability of the system, understating as a beginning oscillations, which dump slowly. This process allows having a stable system even the initial instant, as shown in the figure 2.9, in which doesn't need to wait the reaching a transient beyond which the system stabilizes.

### 2.6.2 Fictitious artery

The vehicle dynamics represents a significant point in this work, because it simulates the progressive evolution of the vehicles in the urban artery. In order to reproduce the driving behavior of the fixed traffic flow needs to consider a fictitious artery along which are distributed all vehicles. This expedient allows a distribution of vehicles in an artery which begins in the initial point of the real artery and it extends backwards until a length obtained through the following relationships:

$$distance_{vehicle} = \frac{v_{initial}}{flow}$$

$$Length_{fictitiousartery} = (n - 1)distance_{vehicle} \quad \text{with } n = flow$$

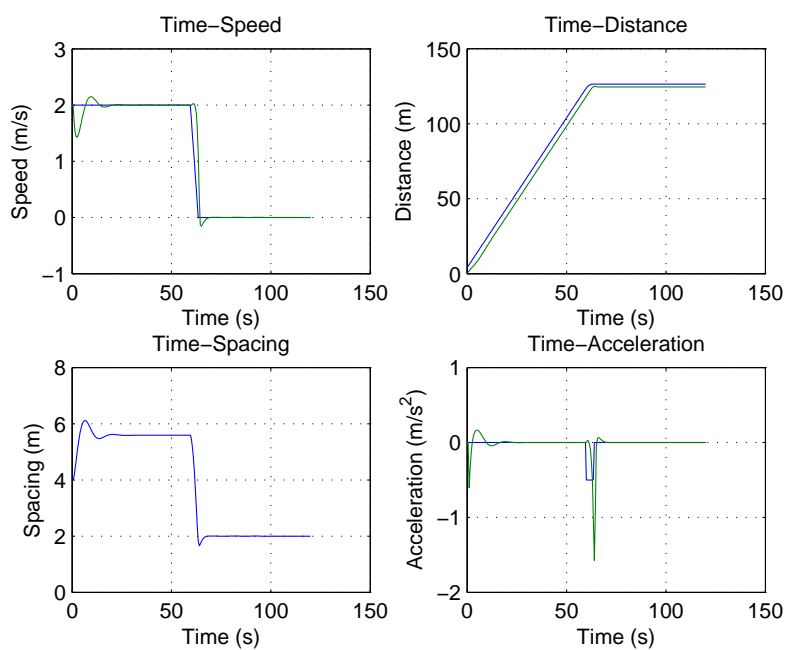


Figure 2.8: Not initialization

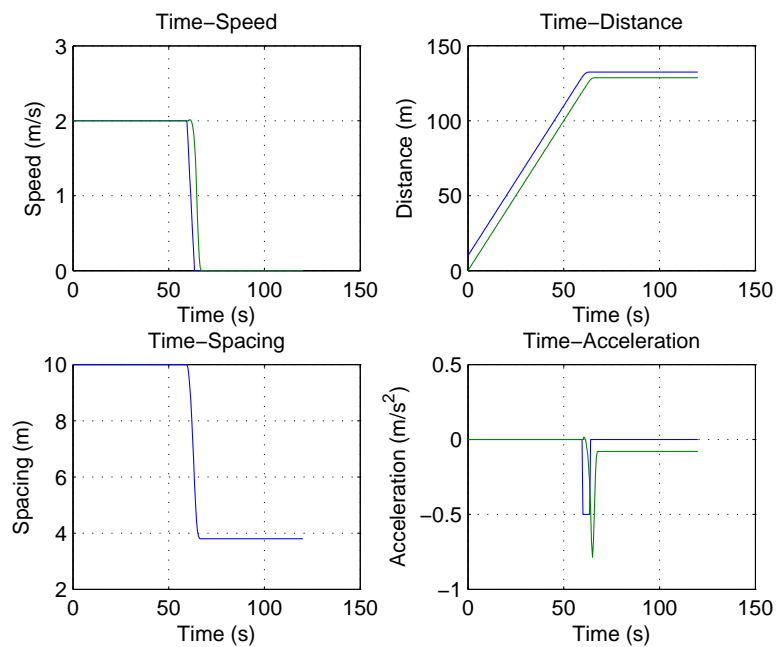


Figure 2.9: Initialization

The fictitious artery, so built, makes sure that in one hour a number of vehicles equal the flow crosses urban road, evolving with a desired speed. Therefore, this expedient guarantees the evolution of all vehicles along the fixed real artery.

### 2.6.3 Fictitious vehicle

Similar thinking is made to describe the vehicle dynamics according to the microscopic car following model. The movement of the several vehicles from the origin to the destination node can take place fixing a fictitious vehicle, which it moves in undisturbed way in the urban artery. In particular, the fictitious vehicle is as ghost vehicle which allows to the other vehicles to move according to the laws characterized the car following model. Therefore, this vehicle can be consider as an input element to the evolution of the remaining vehicles.

## 2.7 Braking to the traffic signal

### 2.7.1 Philosophy

In the stationary condition, the vehicles move undisturbed along the urban artery according to the laws of the car following model. In reality, the vehicular stream is perturbed to many factors: bottlenecks, work in progress, accidents and slowing downs. In this study, the main disturbing factor is represented by the signalized intersection. The presence of traffic signal causes the braking of the vehicles and, consequently, a perturbation of the driving behavior and the running state. The braking process takes place when of one the perturbation factors, previously defined, arise. In the analyzed problem, the braking process occurs when the leader vehicle runs into the red or orange. Therefore, the state of the traffic signal imposes initially the slowdown and, after the vehicle stop. The behavior of the leader vehicle influences the driving reaction of the other vehicles, which follow the leader according to the microscopic car following model, except the first or leader vehicle, which follows the laws of the motion uniformly decelerated. Now, the main problem is to convert this information or real driving behavior in mathematical terms compatible with the characteristics code of the environment used to simulate the motion of the vehicles and their interaction. In order to convert this process in mathematical terms needs, as previously defined, to know the state of the traffic signal in each time instant. This knowing allows to have all information corresponding to the traffic lights, that is the instant time in

which the one is red, green or orange. The problem, as built, represents a snapshot of the traffic signals and their configurations in the analyzed time. The snap-shot of the traffic lights represents, from the mathematical point of view a matrix, called *MatCicli*, which together with the characteristic ones of the single vehicle (distance, velocity and spacing matrices) allows to convert all data in MATLAB code. Exactly, each vehicle is identified by a code which allows knowing the vehicle position in a certain instant time and, consequently, to understand if the vehicle must carry out the braking or not. The matrices that come into play in this study are able to take into account all information of vehicles, therefore, they consent to know if the vehicle is near to the traffic light or if it must break or if can go undisturbed in the crossing of the urban artery.

### 2.7.2 Calculation of braking distance and driving behavior

The braking process takes place when the driver is a certain distance to the traffic signal. This distance is called as a braking distance, which is evaluated through the characteristic parameters of the Intelligent Driver Model, exactly using the desired velocity and the comfortable deceleration. The obtained distance from these parameters allows vehicle to carry out in a safety condition the braking process. From behavioral point of view, the vehicle when is near to the signalized intersection verifies its distance to the traffic signal; if this distance is equal to braking one, it is necessary that driver checks the traffic light configuration. Therefore, if is the red or orange phase the vehicle starting its braking, if instead the traffic signal is green the vehicle can cross the intersection in undisturbed way. In other words, the braking process interests the first vehicle of the traffic stream, which represents the leader vehicle, while the remaining vehicles represent the followers, that is the drivers move according to the laws of the car following model. Therefore, for each signalized intersection there is only braking vehicle followed to other drivers.

### 2.7.3 Management of braking vehicles

The vehicle dynamic and the braking process are two significant elements in this study, because both describe the driver behavior during his movement. The first factor is regulated to the laws of the car following model, previously defined from both theoretical both practical point of view, instead, the second point defines all several steps to follow in the break phase, aspect described only from theoretical point of view. In the practical terms it needs to convert



the braking process of the vehicle and, consequently, his driver behavior in MATLAB code. In particular, when the break phase takes place is increased a counter, which takes into consideration the number of the braking vehicles. Every vehicle is identified to an index, which allows to know the position of the vehicles respect the traffic lights, in correspondence to which they must stop. Therefore, noticed the braking vehicles it starts the stop phase regulated to the scheme above described. After the break phase it need to verify the configuration of the traffic signal, if the one is green it takes place:

1. initially the restart of the vehicles;
2. finally an updating of braking vehicles.

Both phases is a significant commands because they convert the human and driving behavior in MATLAB code, allowing a simulation of vehicles dynamics.

# Chapter 3

## Applications

### 3.1 Overview

In the field of study of this thesis project, it is possible to identify two distinct phases described below briefly. The first phase, supported by adequate theoretical and analytical procedure, aims to develop and to test a model of traffic of an urban artery. The second phase consists of the analysis and of the comparison of methods found in the literature for the solution of the optimization problem and in the implementation of innovative and intelligent algorithms, such as genetic and firefly algorithms. In other words, the purpose of this work is to solve an optimization problem, taking into account not only the characteristics of the traffic flow of the artery of interest, but also the interaction between the vehicles. The optimum of the system is searched in the optimization of several feature parameters of the traffic signals and in a proper planning of ones.

### 3.2 Reference artery

The reference artery is made up of a main artery and 3 secondary arteries, all cross in both direction, as shown in the figure 3.1.

For the sake of simplicity, it is decided to analyze a simple urban artery characterized to the parameters shown in the table 3.1.

This configuration is chosen, first of all, to test the mathematical model and to verify if the developed model can simulate properly the vehicle dynamics and driving behavior but this choice is connected strictly to the computation time.

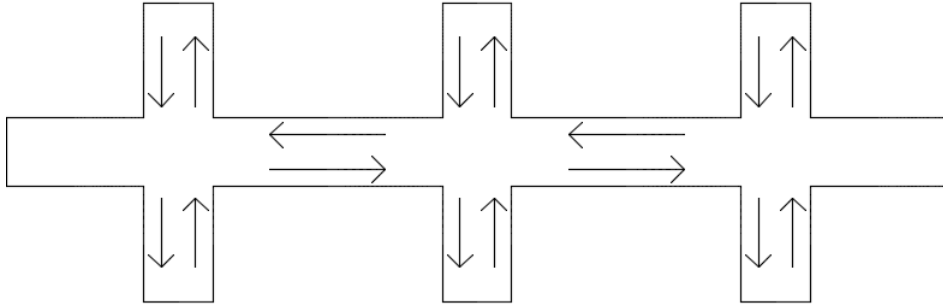


Figure 3.1: Urban artery

Table 3.1: Characteristic parameters of the reference artery

IDM parameter	Value
Length real artery $LA$ (m)	400
Flows $f$ (veh/h)	100
Saturation flows $s$ (veh/h)	1000
Total lost time $L$ (s)	10

### 3.3 Objective function

The aim of this work is to create a mathematical model within which are contained the main concept mentioned in the chapter 1: the analysis of the current state of artery, the synchronization, the laws of the car following model, the analysis of the driving behavior and the interaction between the drivers. The model, so built, allows to simulate the vehicle dynamics during the movement from the origin to the destination node of the artery of interest and also to evaluate the value of the accumulated delay. Therefore, this black box is considered as a tool through which simplify the structure of the problem and to divide the model to the optimization methods. In other words, the mathematical model can be applied to any optimization algorithm, because the model is independence to the objective function  $f_{ob}$ , defined to the successive relationship:

$$R = f_{obiettivo}(C, G, O, param) \quad (3.1)$$

The structure of objective function, shown in the relation 3.1, highlights the independence of the  $f_{ob}$  to the mathematical model. The model, in particular, can be considered as a black box within which can be imagined various sections, each of which has a significant rule in the study; one of this sections is occupied to the objective function, another to the optimization

algorithm, elements which if opportunely called return the delay.

## 3.4 Tested algorithms

The structure of the mathematical model is designed as a box made up of  $n$  compartments. This subdivision allows to extrapolate and to modify the section correspondence to the optimization algorithm and, consequently, to apply several method to minimize the objective function. The use of several optimization algorithm allows a comparison between the obtained results through each method. The comparison is, furthermore, useful to understand which is the best optimization algorithm to adopt in the resolution of this problem. The analyzed problem is a complex case and its intricacy is strictly connected to the structure of the objective function. The  $f_{ob}$  to minimize is a multi-variable and constrained function, therefore it can not always to be solved through analytical methods. Another reasonable optimization method is the enumerative one, which is not suitable to solve a multi-variable problem with a wide domain. In other words, the best methods can be adopt to solve a problem so complex are so-called 'intelligent' methods, which can guarantee the maximum effectiveness. A method extensively adopted is the Genetic Algorithm already widely used in the literature for the resolution of similar problems, another method, more recent, is the Firefly algorithm considered an efficient method to find the optimum of the problem. Wherefore the adopted methods in this project are:

1. Genetic Algorithm;
2. Firefly Algorithm.

### 3.4.1 GA

The Genetic Algorithm is a meta-heuristic optimization method, inspired to the process of natural selection and based on Charles Darwin's theory. The working principle and the features of the GA was defined in the chapter of the state of the art, in which the algorithm was defined in general terms. Therefore, it needs to define the intrinsic characteristics of this optimization algorithm which can not fixed a priori: the number of individuals and of generations are the feature elements which must be selected properly in order to allow the convergence of the algorithm to the optimum. A correct selection of the parameters of the genetic algorithm allows to manage the computation time which often is unacceptable time to obtain the results. Beyond all, it

Table 3.2: Obtained results to 3 equal simulation with 200 generations.

Output	Symbol	Unit	Run 1	Run 2	Run 3
Cycle Int. 1	$C_1$	s	101	72	95
Cycle Int. 2	$C_2$	s	101	132	95
Cycle Int. 3	$C_3$	s	102	132	108
Green Int. 1	$G_1$	s	38	27	40
Green Int. 2	$G_2$	s	33	46	24
Green Int. 3	$G_3$	s	40	48	52
Offset Int. 1	$O_1$	s	27	103	57
Offset Int. 2	$O_2$	s	25	8	79
Offset Int. 3	$O_3$	s	3	18	78
Total delay	$R$	h	6,2	5,8	6,6

needs to find the proper compromise between the feature elements of the GA and the parameters of the analyzed problem.

### Individuals and generation number

In general, the number of individuals must be sufficiently high so that the algorithm can converge to the solution of the problem in a limited number of generations, but this number can not so much great to increase significantly the computational load. Differently, the number of generations must be able to converge the method to the absolute minimum. In order to define these indicators, it is fixed, initially, a number of 100 individuals and 200 generations and consequently 3 equal simulations are lunched, trough which it is possible to understand if this selection of indicators it is efficient or not to the resolution of the analyzed problem. Therefore, the effectiveness of method is strictly connected to the results of 3 simulations which must be, to validate the method, similar or equal between them. In the opposite case, in order to understand the proper number of the generations necessary to reach the solution, it needs, first of all, to analyze the trend of the best individuals of the each algorithm and, furthermore, to verify the generation in which the 3 simulations deal in a neighborhood sufficiently small than the final value. This first configuration shown as 200 generations is not able to converge the objective function to the solution because the 3 carried out simulations produced 3 different results, as shown in the table 3.2.

In other words, the set of used data is not valid in this study, for this reason it needs to modify the number of generations, that is the stop criteria. In particular, these criteria can not be more based on the maximum num-

ber of generations but on the number of generations in which the objective function assumed the same value.

### Stop criteria

The stop criteria is a significant element in every optimization algorithm, because it defines the condition through which can be stopped the simulation. In the genetic algorithm is possible to use many stop criteria. The first and more simple stop criteria is to fix a number of maximum generations; in the end the algorithm will return the element of the last population, which best minimizes the objective function. This criteria, as above described, is not an efficient method, because it doesn't allow to converge to the solution of the problem. Therefore, it is necessary to define a new criteria through which it must be reached the best solution. The stop criteria, in other words, must be based on the stall number, that is when the objective function returns for at least 50 generations the same value of the function to minimize.

### Bits precision

A simple method to determine the variable precision is to assign all the ones a certain number of bits. This number is chosen considering the configuration which is able to give a certain percentage accuracy. In the present case, it is supposed to want to encode the interval from 0 to 255 with 8 bits in order to obtain a precision of 1 s, in other words, it means that the algorithm works with integer variables.

### Reproduction and Selection

The reproduction aims at the selection of a certain number of individuals and at the copy of one in the successive generation, working according to the natural laws. Therefore, a relevant aspect in the GA is the choice of the proper function to select the individuals on which is applied the operators. In general, the considered functions are stochastic and they are able to select with more probability those individuals which have a high fitness value.

Some of known techniques are already mentioned in the chapter 1. In this section, differently, it describes the principle of operation and the main features. The most common technique is *tournament selection*, which is a selection criteria based on competition between a certain number of individuals. First of all, chromosomes are selected among all those of population, later it is evaluated the physicality of each element and, finally, it is extracted the best element of the group, that is the one with greater fitness. The procedure is repeated a number of times equal to the number of individuals which

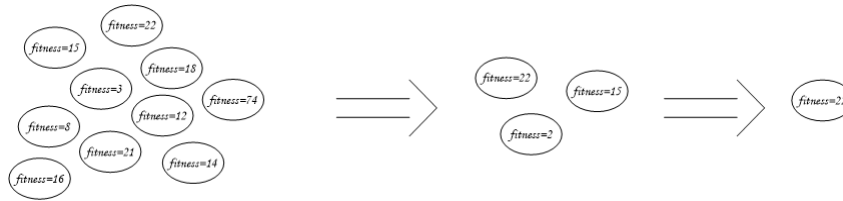


Figure 3.2: Example of Tournament Selection

it desires to select. The figure 3.2 shows an example of *tournament selection*, a selection criteria which selects from the initial population, composed of 10 individuals, 3 of ones, according to a random criteria.

This criteria implies weak element have a minor likelihood to pass to the successive generation, because it will confront to stronger individuals. This intrinsic characteristic of the *tournament selection* influences in a negative way the genetic diversity of chromosomes and may lead to convergence at a local minimum.

Another technique often used is the *Proportional Roulette Wheel Selection*, which simulates the behavior of a roulette wheel; each individual is associated to a given sector of a wheel and its size is proportional to its physicality. In particular, this selection criteria turns the circumference a number of times equal to the number of individuals which must be selected. At each turns the indicator will randomly stop on a date slice, correspondence to the selected individual.

This selection implies, although the best individuals occupy a segment of wheel proportional to their physicality, a greater probability to be selected then weak ones, also an individual with a low physicality has possibility to pass to the next generation. The intrinsic feature of this selection method allows a genetic diversity in the population, which will allow to the algorithm to exit from a possible local minimum. In other words, the significant characteristic of this method is to preserve the diversity. The figure 3.3 shows a possible configuration of the roulette wheel.

The main disadvantage of this selection method is connected to possible premature convergence, caused of highly gifted individual which occupies a high percentage of circumference and therefore it is chosen several times.

A possible solution related to the *Proportional Roulette Wheel Selection* might be to use the *Rank based Roulette Wheel Selection*. This method allows, differently to the previous selection one, to divide the circumference in relation to the rank assigned to each individual. The rank is a number associated to all individual which follows an ascending order (from worst to best), depending on their physicality. In this way, even if a super-doted

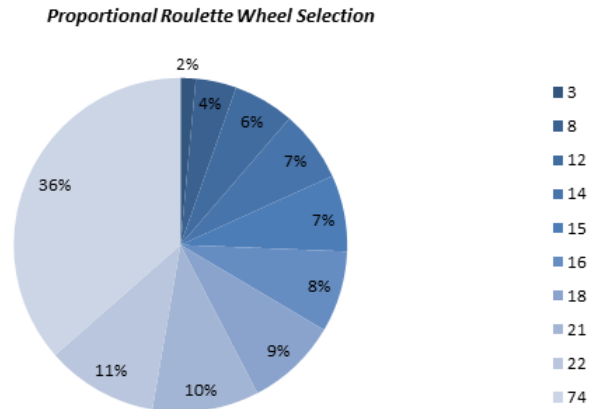


Figure 3.3: Proportional subdivision of roulette wheel selection

individual appears in the first generation, continuing to be more likely to move to the next generation, he does not have absolute domain in roulette wheel. The worst aspect of this selection method is the long time needed to the convergence, caused by the lesser role of fitness function and, therefore, to the presence of many fitted out individuals in each generation. The figure 3.4 shows the roulette subdivision in the *Rank based Roulette Wheel Selection*, while the table 3.3 shown the difference in the way in which the segments of the wheel are attributed to different methods.

In this study is not used any of the selection method above described but the *Stochastic Universal Sampling (SUS)*.

The *Stochastic Universal Sampling* is an evolution of the *Proportional Roulette Wheel Selection*: if an individual has a physicality of 4.5% respect to the sum of the fitness of all other individuals, it verifies that on 100 chromosomes this individual will appear between 4 and 5 times in the next generation, as shown in the figure 3.5. The SUS, in other words, enures this happens, while in the traditional roulette may happen that the same individual appears one time, 10 times, 50 times, or even one.

The relevant difference between two methods is connected to the way in which the roulette wheel is turned, in particular:

- the *Proportional Roulette Wheel Selection* turns a number of times equal to the number of individual which it desires to select;
- the SUS carries out only turn during which are take all individuals.

For example supposing to select 30 individuals, the roulette wheel is divided in 30 parts each with a  $360/30$  size, with an element selected at each



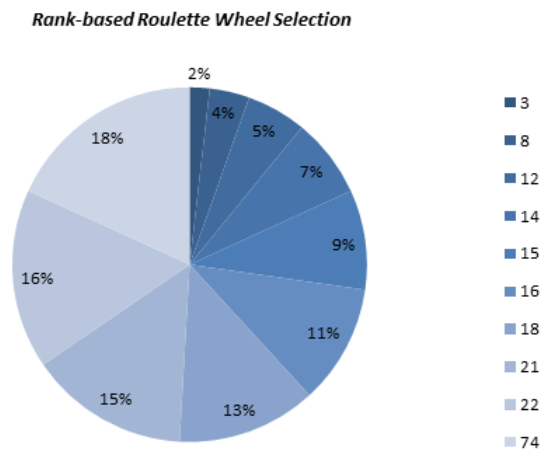


Figure 3.4: Proportional subdivision of roulette wheel selection

Table 3.3: Comparison between the Proportional Roulette Wheel Selection and the Rank based Roulette Wheel Selection

Individual physically	Roulette	Individual physically	Rank	Roulette
3	1,48%	3	1	1,82%
8	3,94%	8	2	3,64%
12	5,91%	12	3	5,45%
14	6,90%	14	4	7,27%
15	7,39%	15	5	9,09%
16	7,88%	16	6	10,91%
18	8,87%	18	7	12,73%
21	10,34%	21	8	14,55%
22	10,84%	22	9	16,36%
74	36,45%	74	10	18,18%

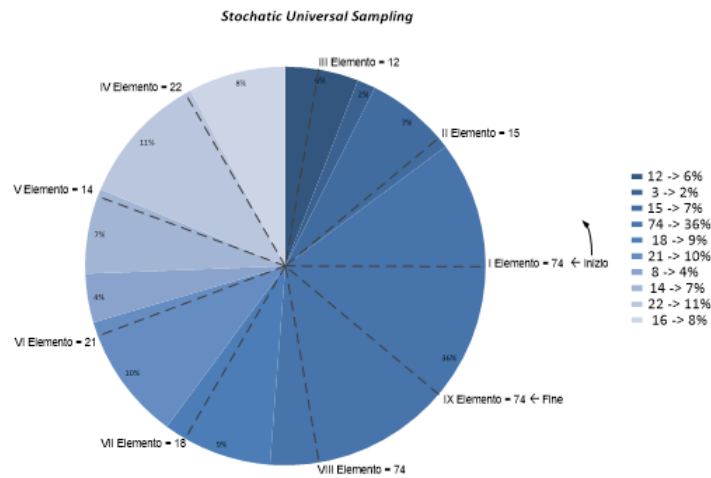


Figure 3.5: Stochastic universal sampling

step. In this way, it ensures to pick up every element a number of times proportional to its physicality, but also to reject all those individuals having a small physicality.

### Cross-over

The crossover is an operator combines the genetic patrimony of two individuals in order to generate two different entities, but with similar characteristics; this operation is able to evolve the algorithm toward a better solution than the previous generation. From the practical point of view, this operator allows to combine two sub-strings, made up a certain number of bits of random length, belonging to two different strings, in order to generate a new individual able to have the positive peculiarity of the both parent. This operation takes place through two different methods:

1. single point;
2. two point,
3.  $n$  point.

In this study is adopted the first method based on division in a random point of the characteristic vector of individual. In this way the chromosome of the new individual is made up of the first  $k$  elements of the father and the remaining  $n - k$  elements of the son, as shown in the figure 3.6. The other methods forecast: the two point method uses the same technique of the first



Figure 3.6: Single-point method

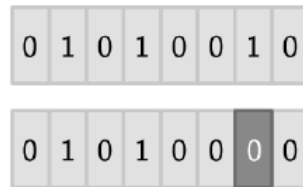


Figure 3.7: Example of mutation

one even if parents are divided into three parts, while the  $n$  point crossover is obtained by building a chromosome where the  $i$ th element is taken with the same probability from the first or second father.

### Mutation

The mutation operator allows to modify genetic heritage to produce new chromosomes. Differently to the crossover operator, it introduces genetic aspects, modifying a part of a single chromosome, as displayed in the figure 3.7. The effect of this operator is to modify deeply the chromosome, so that the mutated individual explores a part of domain not yet observed. This instrument, therefore, is introduced to avoid convergence to local minimum, thus favoring exploration of space and global research.

### 3.4.2 Results of 4 runs

In order to verify the reliability of the optimization method adopted it is implemented an experimental platform, used to optimize the urban artery and to reduce the delay accumulated to drivers. First of all, it needs to determine if the method is repeatable, that is if the results correspondence to

Table 3.4: Obtained results to 4 equal simulation with 50 stalls.

Output	Symbol	Unit	Run 1	Run 2	Run 3	Run 4
Cycle Int. 1	$C_1$	s	72	90	72	72
Cycle Int. 2	$C_2$	s	72	90	108	72
Cycle Int. 3	$C_3$	s	72	54	72	36
Green Int. 1	$G_1$	s	17	28	28	23
Green Int. 2	$G_2$	s	35	38	44	35
Green Int. 3	$G_3$	s	27	15	29	10
Offset Int. 1	$O_1$	s	29	18	19	23
Offset Int. 2	$O_2$	s	22	19	97	21
Offset Int. 3	$O_3$	s	55	20	55	18
Total delay	$R$	h	3,0	4,7	3,9	2,9
Individuals	$NIND$	-	100	100	100	100
Generations	$GEN$	-	370	192	239	263
Elapsed time	$T$	h	21	31,5	39	40,5

a given operating condition, while repeating the sizing several times, remain in any case same. This test, obviously, allows to determine if the algorithm converges to the same result, starting from different initial generations and quite casual. Therefore is carried out a set of simulations through which it is possible to verify the reliability of algorithm. The results are summarized through the table 3.4 which shows the outcomes of 4 runs of the Genetic Algorithm and the figure 3.8 shows the behavior of the best individual for each generations in each run. In the figures 3.9, 3.10, 3.11 and 3.12 it is possible to see the optimized configuration of the signalized intersections for each run. From the results shown in the table 3.4 it is possible to see that 2 runs (50%) return a very close outcomes (3,0 hours and 2,9 hours) and the same conclusion can be made for the values of the variables. The other 2 runs return a different results; the reason of this behavior of the Genetic Algorithm is connected to the high dimension of the domain of the objective function. In order to understand the dimension of domain it is possible to consider that it is defined in  $N^9$  and each variable can assume all integer values from 0 to 255; it means that the number of all possible combination of variables is  $256^9$  (a number with 21 zeros). The implemented Genetic Algorithm, through the evaluation of only 263 generations and consequently only 26300 individuals it is able to find the global optimum with a probability of 50%.

The analysis of the figures 3.9, 3.10, 3.11 and 3.12 generates the following

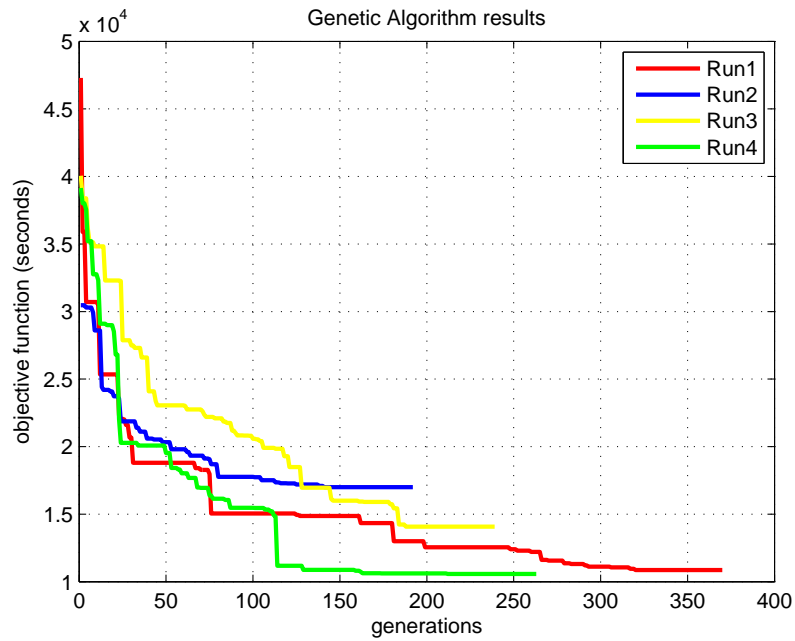


Figure 3.8: Trend Genetic Algorithm

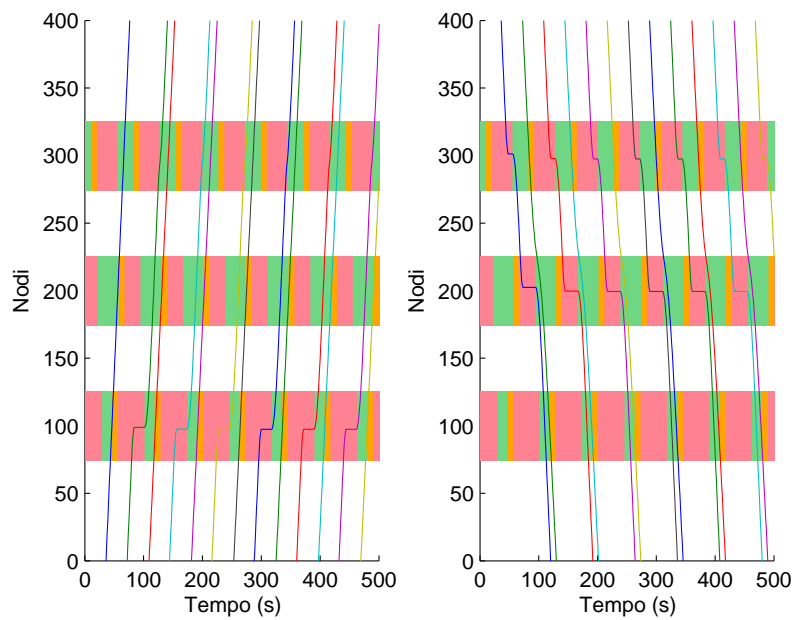


Figure 3.9: Vehicle dynamics after optimization (Run 1)

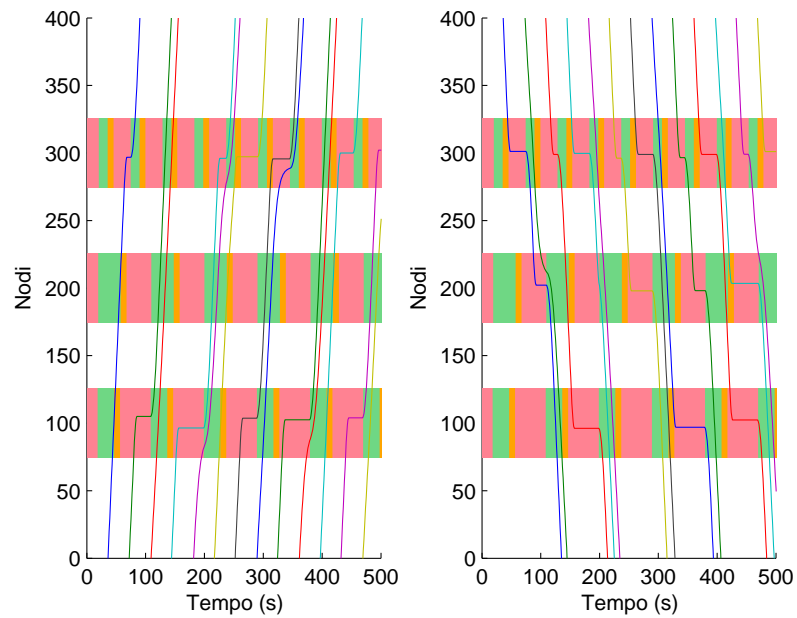


Figure 3.10: Vehicle dynamics after optimization (Run 2)

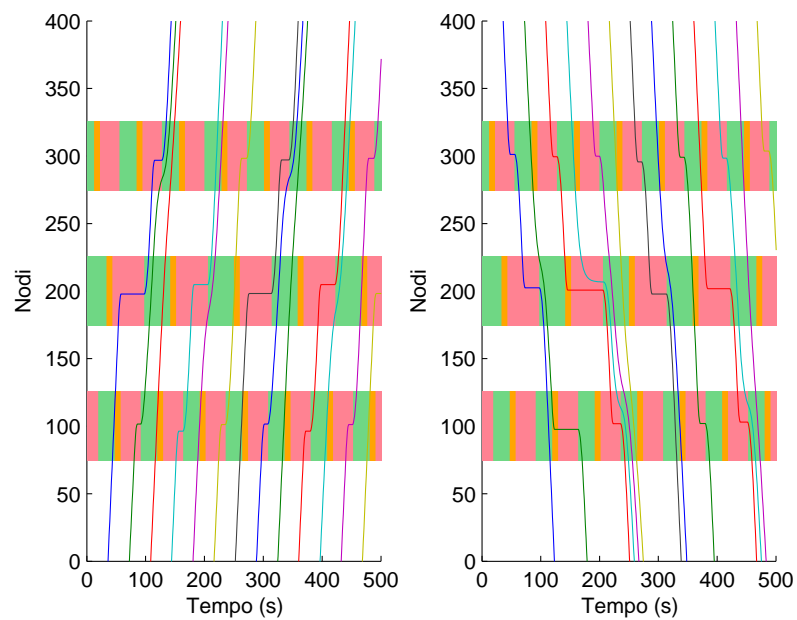


Figure 3.11: Vehicle dynamics after optimization (Run 3)

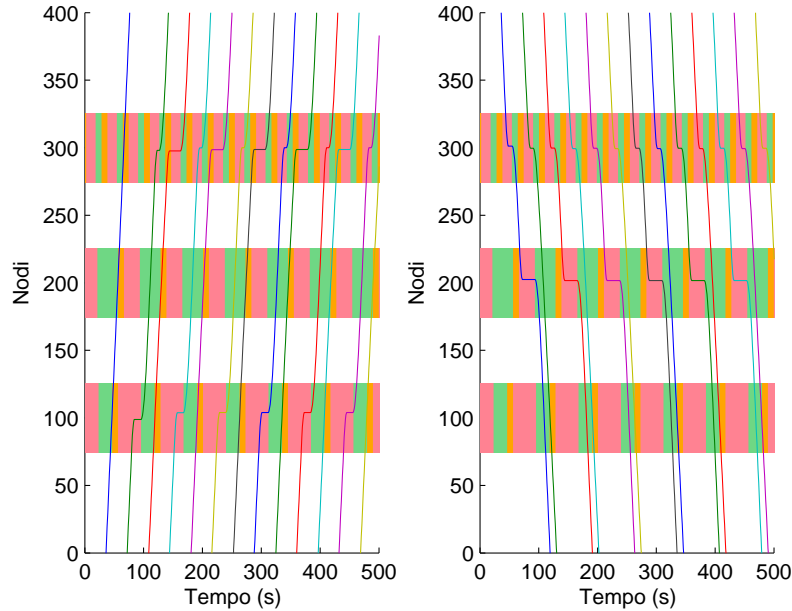


Figure 3.12: Vehicle dynamics after optimization (Run 4)

conclusions:

- the visual inspection allows to conclude that the minimum found with runs 1 and 4 is the global minimum: thanks to the simplicity of the reference artery used for the test it is possible to see that the found solution synchronizes the intersection in the best possible way;
- the results confirm that the choice of a very simple artery was a good choice to test the optimization algorithms;

### 3.4.3 FA

The Firefly Algorithm is a meta-heuristic optimization method proposed by Xin-She Yang and inspired by the flashing behavior of fireflies. In the FA there are two important issues: the variation of light intensity and formulation of the attractiveness. In other words, in this section is not describe the working principle of the FA, but the characteristic parameters of this optimization algorithm are defined, especially the elements which can not fixed a priori: the number of individuals and of generations are the feature elements which must be selected properly, in order to allow the convergence of the algorithm to the optimum. A correct selection of the parameters, in fact,

Table 3.5: Characteristic parameters of FA

Parameter	Value
Maximum iterations	500
Number of individuals	10
Light absorption coefficient $\gamma$	1
Attraction coefficient base value $\beta$	0.1
Mutation coefficient $\alpha$	0.2
Mutation coefficient damping ratio $\alpha_{damp}$	0.999
Exponent m	2

allows to manage the computation time which often is unacceptable time to obtain the results. In other words, it needs to find the proper compromise between the feature elements of the FA and the parameters of the analyzed problem.

### Parameters

In order to use the Firefly algorithm to optimize the delay of urban artery are fixed specific value of the intrinsic parameters of this optimization algorithm, which are summarized in the table 3.5.

The number of firefly and of generation are two significant factors which influence deeply the working principle of FA in terms of computation time and of results. In the resolution of the analyzed problem, in fact, it takes into account different scenarios in terms of generations and individuals number and also two different optimization method:

1. a method without evaluation of new solution and updating of brightness;
2. a method with evaluation of new solution and updating of brightness.

### 3.4.4 Results

The first method and their outcomes, as shown in the table 3.7, are not able to converge the objective function to the optimum solution, even if considering an elevate number of fireflies (100) because the objective function is evaluated only n times. For this reason, it is adopted the second method which is more efficient than the first, because it is able to analyze part of domain not observed with the method without evaluation of new solution and updating of brightness. In particular, the second method than the first



Table 3.6: Obtained results to 4 simulations with 800, 3000 and 5000 generation with 10 individuals and one with 500 generation and 20 individuals.

Output	Symbol	Unit	Run 1	Run 2	Run 3	Run 4
Cycle Int. 1	$C_1$	s	154	154	154	90
Cycle Int. 2	$C_2$	s	216	216	216	72
Cycle Int. 3	$C_3$	s	72	72	72	108
Green Int. 1	$G_1$	s	69	69	69	45
Green Int. 2	$G_2$	s	122	122	122	35
Green Int. 3	$G_3$	s	27	27	27	60
Offset Int. 1	$O_1$	s	101	101	101	75
Offset Int. 2	$O_2$	s	126	126	126	57
Offset Int. 3	$O_3$	s	57	57	57	58
Total delay	$R$	h	8,4	8,4	8,4	5,2
Individuals	$NIND$	-	10	10	10	20
Generations	$GEN$	-	800	3000	5000	500
Elapsed time	$T$	h	8	10	12	14

is able to evaluate the  $f_{ob} n(n-1)/2$  times, its peculiarity and its structure show an improvement in terms of obtained results.

The first test is carried out on a number of fireflies equal 10 and a set of three different generations (800, 3000, 5000), this configuration, shown in the table 3.6, allows to observe which these set of parameters are not able to find the optimum solution because in all three analysis the objective function was trapped in the same local minimum. From the obtained results it is decided to modify the number of fireflies and the generation (20 fireflies and 500 generation). The last configuration returns the outcomes better than the previous results, as shown in the table 3.6. The figure 3.13, instead show the behavior of the best individual for each generations in each run. While the figures 3.15, 3.16, 3.17, 3.18 and 3.19 show the optimized configuration of the signalized intersections for each run, in particular the several figures above mentioned allow to realize that the results obtained through several runs doesn't reach the global optimum. In order to improve the results obtained by means of the use of the Firefly Algorithm with an evaluation of objective function  $n(n-1)/2$  times it is decided to add a swarm of random fireflies when the objective function assumed the same value for 10 generations. This expedient is a good tool through it is possible to analyze new part of the domain, but so that it is able to explore the definition interval of the objective function it need to chose conveniently the characteristic parameters of FA in

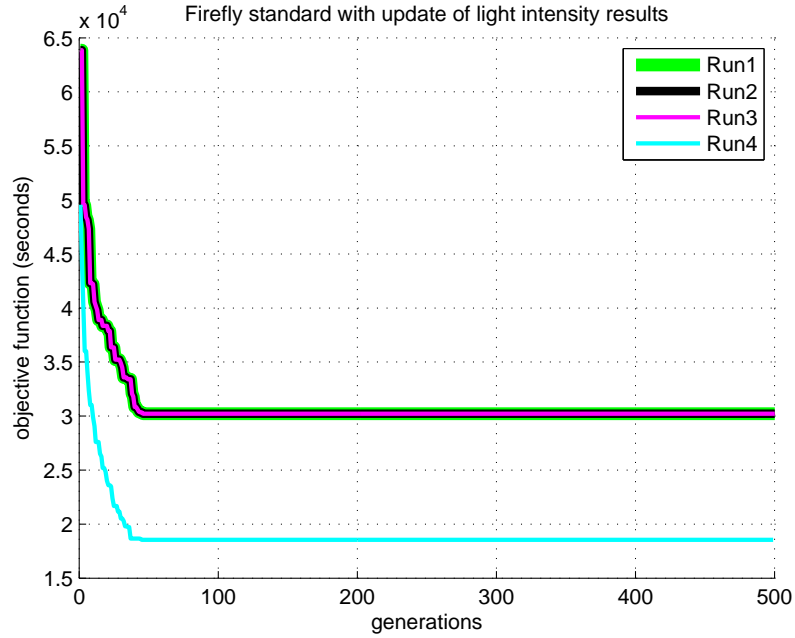


Figure 3.13: Trend Firefly Algorithm with evaluation of new solution and updating of brightness

Table 3.7: Obtained results to 3 equal simulation with 50 stalls.

Output	Symbol	Unit	Run 5	Run 6	Run 7
Cycle Int. 1	$C_1$	s	230	44	111
Cycle Int. 2	$C_2$	s	83	186	143
Cycle Int. 3	$C_3$	s	79	32	80
Green Int. 1	$G_1$	s	70	18	54
Green Int. 2	$G_2$	s	35	111	31
Green Int. 3	$G_3$	s	50	8	22
Offset Int. 1	$O_1$	s	132	28	1
Offset Int. 2	$O_2$	s	21	81	39
Offset Int. 3	$O_3$	s	71	25	39
Total delay	$R$	h	11,7	8,7	9,3
Individuals	$NIND$	-	100	100	100
Generations	$GEN$	-	64	69	96
Elapsed time	$T$	h	1	1,5	2

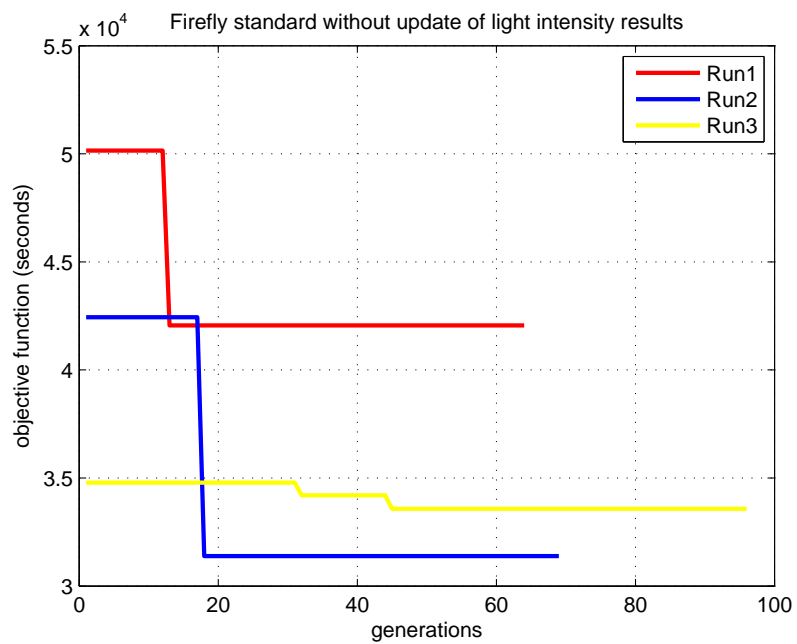


Figure 3.14: Trend Firefly Algorithm without evaluation of new solution and updating of brightness

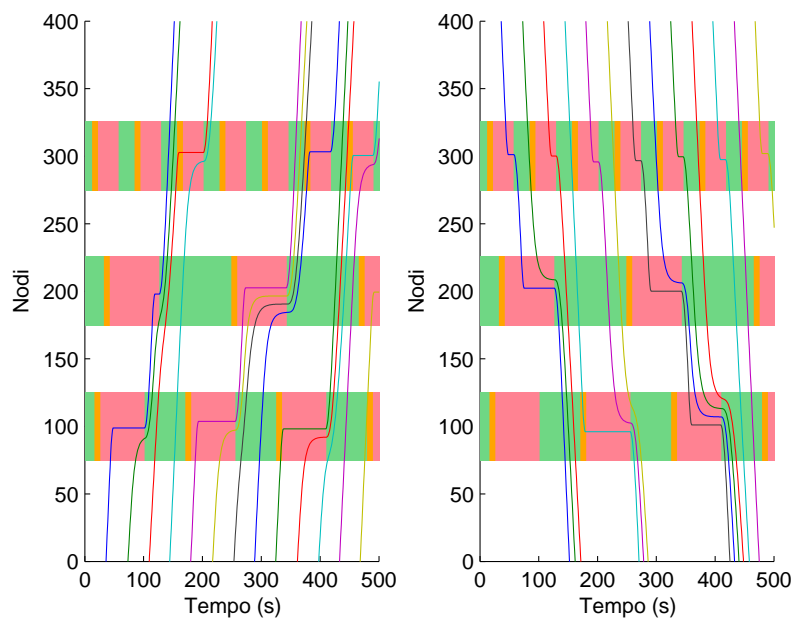


Figure 3.15: Vehicle dynamics after optimization (Run 1)

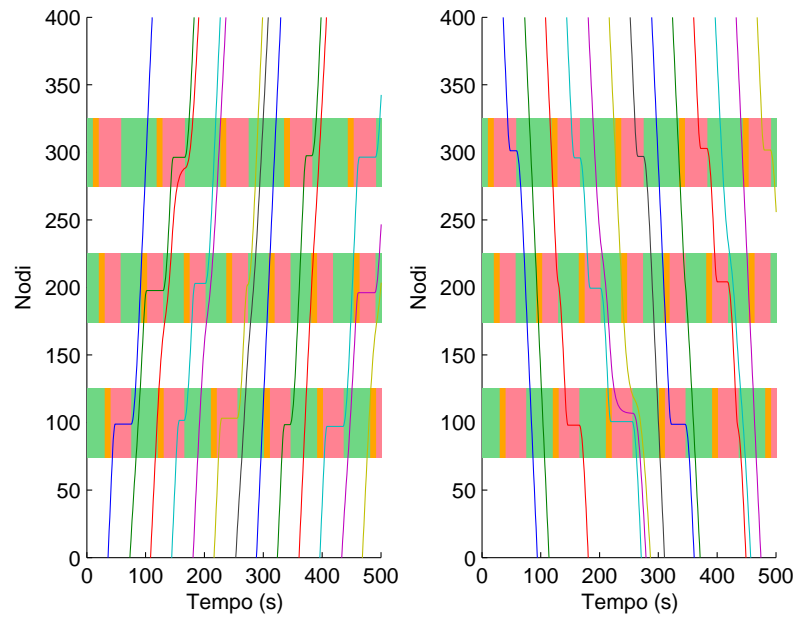


Figure 3.16: Vehicle dynamics after optimization (Run 4)

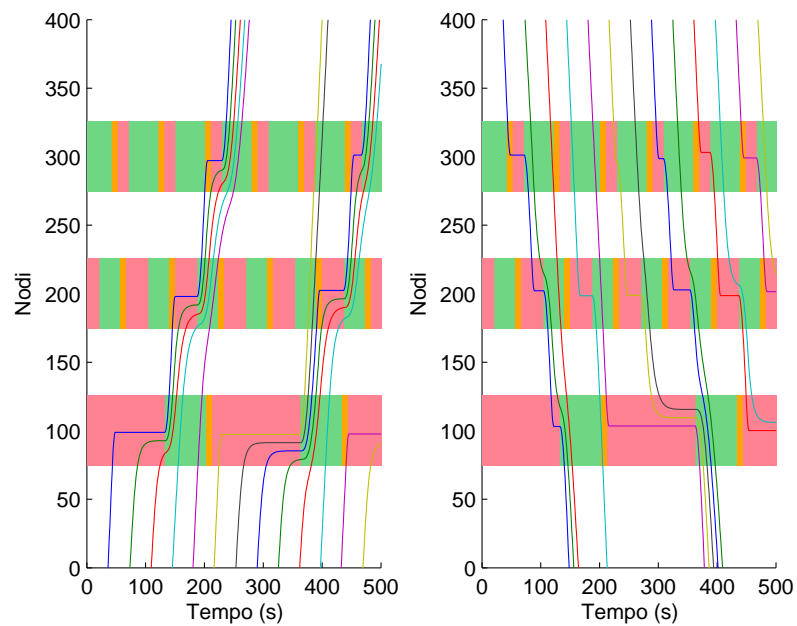


Figure 3.17: Vehicle dynamics after optimization (Run 5)

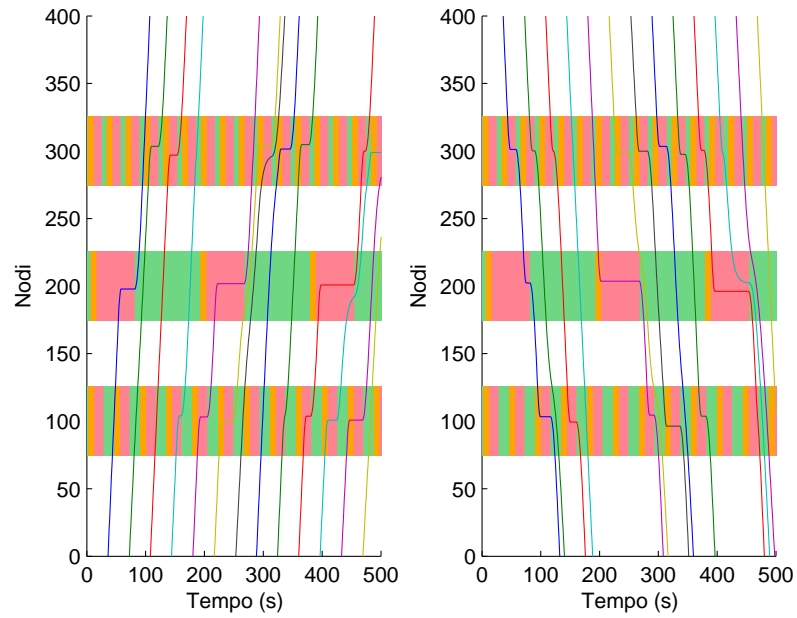


Figure 3.18: Vehicle dynamics after optimization (Run 6)

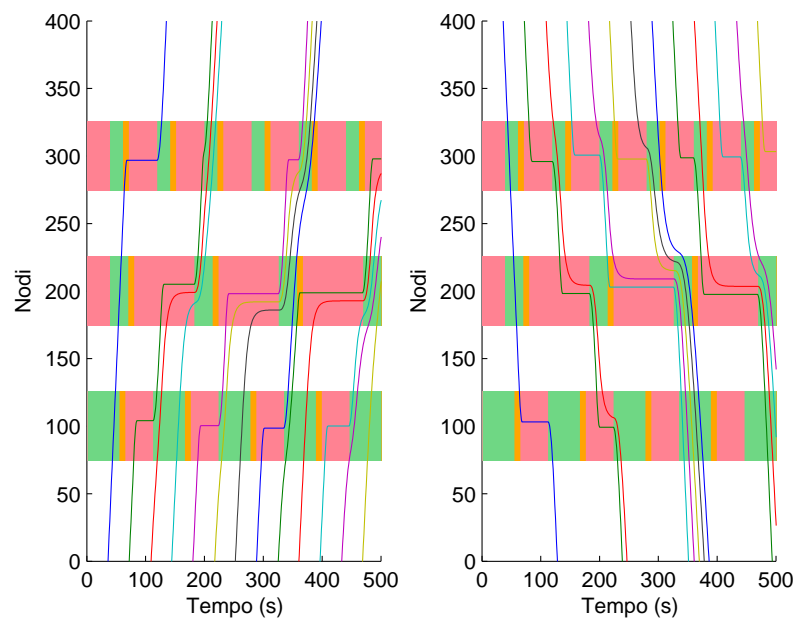


Figure 3.19: Vehicle dynamics after optimization (Run 7)

Table 3.8: Obtained results to 2 equal simulation with 50 stalls and a random swarm of fireflies.

Output	Symbol	Unit	Run 8	Run 9
Cycle Int. 1	$C_1$	s	71	108
Cycle Int. 2	$C_2$	s	202	173
Cycle Int. 3	$C_3$	s	130	52
Green Int. 1	$G_1$	s	74	23
Green Int. 2	$G_2$	s	144	65
Green Int. 3	$G_3$	s	0	70
Offset Int. 1	$O_1$	s	67	161
Offset Int. 2	$O_2$	s	114	255
Offset Int. 3	$O_3$	s	52	79
Total delay	$R$	h	6	4,5
Individuals	$NIND$	-	20	20
Generations	$GEN$	-	94	101
Elapsed time	$T$	h	2,4	2,7

terms of number of fireflies. In this case the number of selected fireflies is not sufficient to avoid to remain in a local minimum, so shown in the table 3.8.

### 3.5 Implementation of mathematical model on real artery

The analyzed artery is Via Tuscolana in the VII District, one of the most congested artery of Rome, which is made up of six signalized intersection (see the figure 3.20). These junctions that compose the real artery are summarized in the table 3.9 (see the figures 3.21).

The total length of the artery is 1970 *m* from the first junction to the last one, with the distances collected in the table 3.10. It is analyzed, first of all, the current configuration of the artery and after it is decided to optimize the real artery by means of the use the genetic algorithm. Via Tuscolana is one of the major artery of Rome, it is usually subject to heavy traffic, so it is decided to analyze one of the two more critic periods between the morning rush hour and the afternoon rush hour. The conducted analysis in the peak hour time of morning evaluates the most congested period with an evaluation of 15 minutes. In this period of analysis, the schools were still open and the roads are very congested. The traffic flows and their analysis, in fact, are very



Figure 3.20: The satellite view of the analyzed artery.

Table 3.9: Signalezed intersections in Via Tuscolana

Number	EB-WB	NB	SB
1	Via Tuscolana	Via delle Capannelle	Via di Torre Spaccata
2	Via Tuscolana	Circonvallazione-Tuscolana	Viale Palmiro Togliatti
3	Via Tuscolana	Viale Tito Labieno	Via Orazio Pulvillo
4	Via Tuscolana	Viale Giulio Agricola	Viale Marco Fulvio Nobiliore
5	Via Tuscolana	Via Valerio Publicola	Via Calpurnio Fiamma
6	Via Tuscolana	Via Lucio Sestio	Via Ponzio Cominio



(a) Junction 1



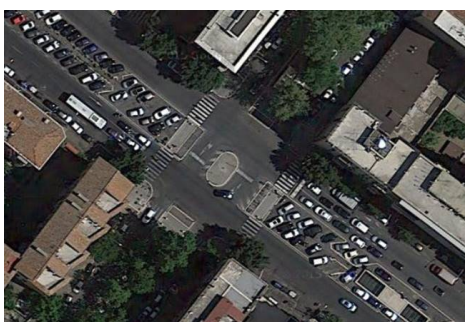
(b) Junction 2



(c) Junction 3



(d) Junction 4



(e) Junction 5



(f) Junction 6

Figure 3.21: Signalized Intersections.



Table 3.10: Flows in via Tuscolana

Number	Progressive distance $m$	Main artery		Secondary arteries	
		$\rightarrow$	$\leftarrow$	$\rightarrow$	$\leftarrow$
1	0	2000	1500	573	351
2	660	2000	1500	547	200
3	870	2000	1500	320	507
4	1390	2000	1500	156	138
5	1770	2000	1500	147	147
6	1970	2000	1500	196	0

interesting from engineering point of view because these characteristic flows are the cause of congestion and of significant delay accumulated to each road user (see the table 3.10). The mathematical model, so built, allows only the crossing of artery in both direction, differently along via Tuscolana are permitted both crossing both left and right turns. For this reason the crossing flows are considered in this study and, exactly in the simulations of the driving behavior.

The current state of the six signalized intersections is shown in the table 3.11, in which it is possible to see the several states of ones in terms of cycle, green time and offset, while in the figure 3.22 is shown single vehicle dynamic along via Tuscolana. Moreover, in order to simulate the real traffic flows it is necessary to calibrate properly the Intelligent Driver Model, in fact, it is decided to change the value of desired speed, desired time distance, minimum and maximum acceleration in order to avoid instability of the microscopic model. The used parameters are shown below:

- Desired speed  $22 m/s$ ;
- Initial speed  $11 m/s$ ;
- Desired time distance  $1,8 s$ ;
- Maximum acceleration  $4 m/s^2$ ;
- Comfortable deceleration  $3 m/s^2$ ;
- Delta  $4$ ;
- Safety distance  $6 m$ .

Table 3.11: Values of several parameters of six signalized intersections

Parameters	Intersections					
	1	2	3	4	5	6
C	131	165	96	96	96	96
G	74	90	50	51	52	53
O	0	80	0	48	0	48
Total delay	2,325 10 <sup>6</sup> s					
Average delay	6 min					

Table 3.12: Values of several parameters of six signalized intersections

Parameters	Intersections					
	1	2	3	4	5	6
C	157	157	157	157	157	157
G	79	94	79	94	79	94
O	31	94	94	47	47	47
Total delay	1,6036 10 <sup>6</sup> s					
Average delay	4 min					
Individuals	25					
Generations	117					

The optimization by means of the Genetic Algorithm it can observe a new signal control of different traffic lights as shown in the table 3.12 and a significant evolution of vehicles along the main artery as so shown in the figure 3.23. The optimization was able to optimize and to synchronize the six signalized intersections, but above all to simulate a green wave, that allowed to reduce the delay in a meaningful way (see the table 3.12), with an average delay for vehicle of 4 min respect of the initial configuration of Tuscolana with an average delay for vehicle of 6 min.

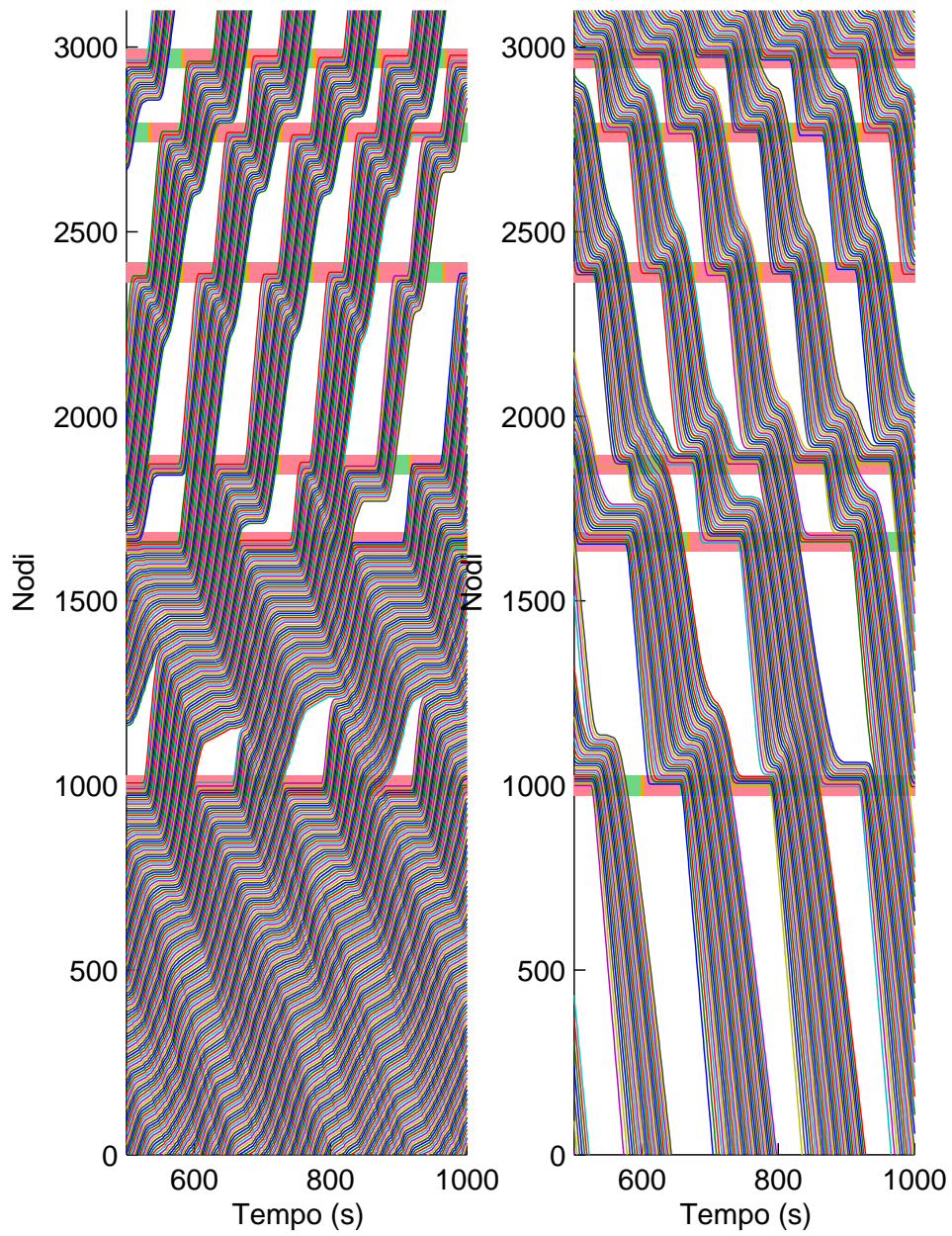


Figure 3.22: Via Tuscolana before optimization.

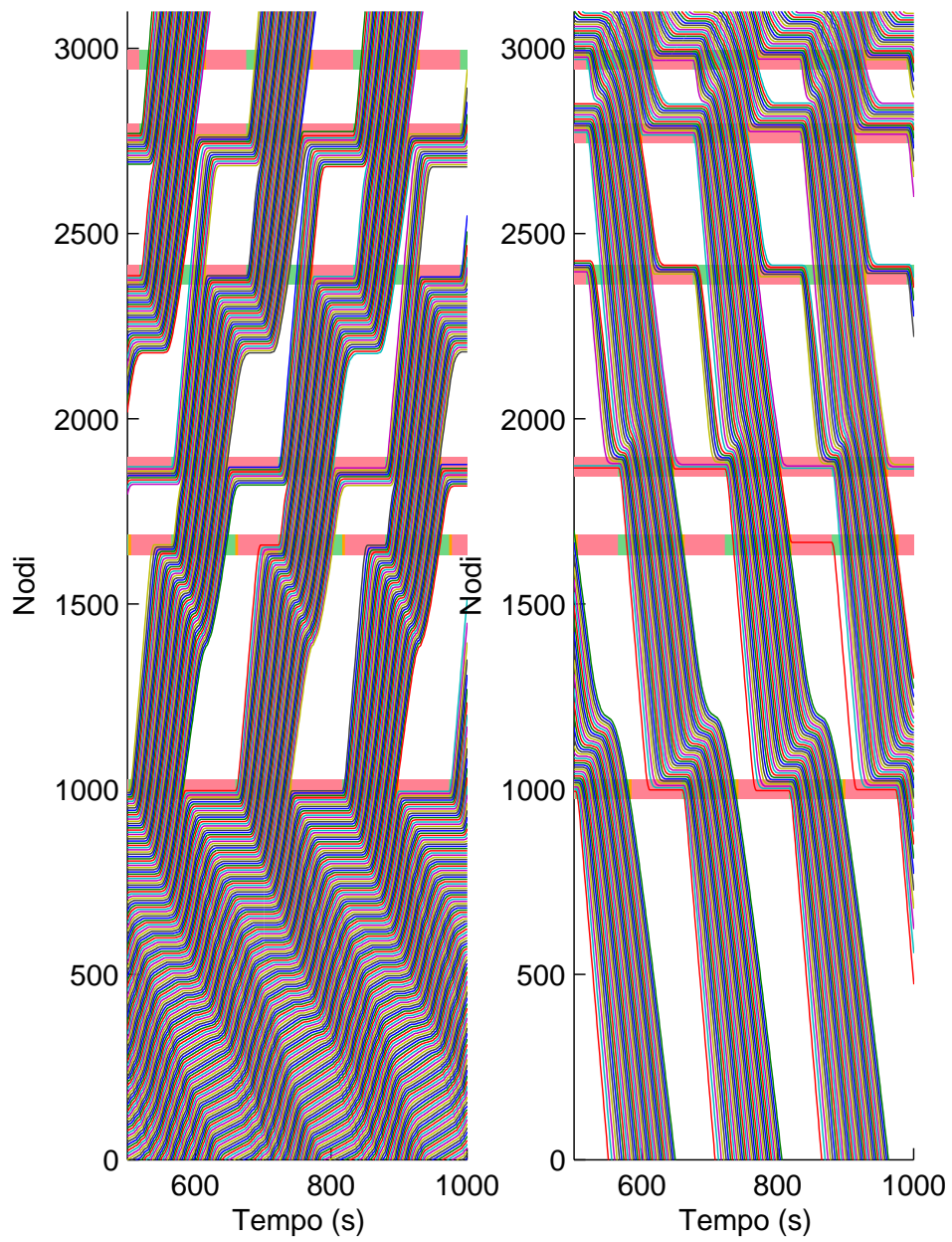


Figure 3.23: Via Tuscolana after optimization.

# Chapter 4

## Conclusions

In the previous section it is analyzed the several results obtained with the use of Genetic and Firefly algorithms. The results, so achieved, allow to compare the methods and to highlight the difference and the power of each single optimization algorithm.

The GA, in terms of results, is able to produce a significant solutions in only 4 simulations, considering a constant number of individuals equal 100, as shown in the table 3.4. In fact, the outcomes allow to show the Genetic Algorithm is a robust method able to conform to the features of the analyzed problem. Its strength and its efficiently can be consider a meaningful advantage if to compared to the FA, because it is to reach the global minimum of the problem. The limit of GA is the computation time, in fact, in order to achieve this important result was necessary 263 generation and approximately 48 hours of processing time.

Differently, the FA method is not able to reach the same or similar outcome to the GA; if  $n$  is equal to the number of fireflies, both in the first case in which was used an optimization method with  $n(n - 1)/2$  evaluation of objective function for each generation both in opposite case in which the  $f_{ob}$  is evaluated only  $n$  times for each generation, as shown in tables 3.6 and 3.7.

The very disadvantage of the Firefly Algorithm, although described in the literature as method able to optimize naturally and efficiently the multi-variable and constrained problem, is strictly connected to the computation time. In the chapter 3 two different methods are described in the application of FA:

1. evaluation of objective function  $n(n - 1)/2$  times;
2. evaluation of objective function  $n$  times.

The first method is more efficiently than the second because for each steps or iterations the brightness of fireflies is updated and, furthermore, it is

verified if the  $f_{ob}$  is evolved to new solution better than the previous outcome. The Firefly Algorithm, so structured, requests the processing time greater than one of GA, considering the same number of individuals, for this reason it is necessary to reach a compromise between the elapsed time and number of firefly. Therefore, the FA can be consider as an important optimization tool which must be modeled opportunely. In order to improve the results obtained by means of the use of the Firefly Algorithm with an evaluation of objective function  $n(n - 1)/2$  times it is decided to add a swarm of random fireflies when the objective function assumed the same value for 10 generations. This expedient is a good tool through it is possible to analyze new part of the domain, but so that it is able to explore the definition interval of the objective function it need to chose conveniently the characteristic parameters of FA in terms of number of fireflies. In this case the number of selected fireflies is not sufficient to avoid to remain in a local minimum, so shown in the table 3.8.

The congestion problem is an issue widely discussed in the literature and for which is proposed different strategies to its resolution. This thesis project, therefore, suggests another methodology to adopt in the resolution of congestion problem, intended mainly as the reduction of the travel time of each road users. Exactly, it is thought to adopt a technique fundamentally different from the traditional one; the traditional method forecasts an optimization of isolate signalized intersection and later a synchronization of urban artery, made up of several signalized junctions. In this work is submitted a methodology which is able to optimize and to synchronize contemporary urban artery, in order to reduce the accumulated delay of drivers during their movements and to synchronize n intersection between them. The implementation of this model forecasts to use of the microscopic model through which it is possible to reproduce the vehicle dynamics and the driving behavior. The choice of the microscopic model, exactly the car following one, is considered better than macroscopic model, because last model simulates the behavior of vehicular stream along the artery. In other words, the macroscopic model is not able to simulate driving behavior and the movement of each driver and, consequently, it doesn't allow to adopt intelligent tools introduced in the sector of transport engineering through which to improve the life in the urban congested area.

Beside the mathematical model, which simulates the vehicle dynamics, it needs to think a method to optimize the artery. In this work it is implemented two different methods, Genetic and Firefly algorithm, through which it is possible to validate the adopted mathematical model but, at the same time, to understand which method conforms to formulated problem.

From the obtained results to application of both methods can be deduced an interesting conclusions about the project methodology used. In

fact, thanks to the use of a technique deeply different from the traditional one, it was possible to determine the configuration that best conforms to operating principle in a particular operating conditions.

All this analysis are carried out a reference artery having relatively simplified features than the reality. This approach, therefore, allowed to validate the capability of mathematical model to reproduce in a simplifying case the real condition of artery and the best configuration of all parameters of the signalized intersection. The platform, so implemented, will can improve structure of urban artery and distribution of traffic flow. In other words, it needs to create an artery in which must be allow all possible maneuvers, all possible choosing of timing and all suitable traffic flow in order to reproduce the real artery. At the same time it is necessary to modify characteristics of optimization model in order to reduce the computation time than actual one.

In order to validate the model it is decided to optimize a real artery. The selected artery is via Tuscolana in Rome that is one of the most congested and complex artery in terms of flows and topology, as shown in the previous chapter it is made up of 6 signalized intersections with a total extension of 1970 *m*. Thanks to flexibility of the algorithm and the model it was very simple to implement the real configuration of the real artery. It is possible to conclude that, basing on the results shown in the chapter 3, the proposed method is able not only to study a very complex artery but also to simply conform to all possible real configurations. Taking into account the optimization results it is possible to conclude that:

- the algorithm is able to optimize a real complex artery, finding the signalized intersection configuration in terms of cycle, green time and offset that minimizes the total delay;
- the found solution (see table 3.12) is valid from engineering point of view;

The model, so thinking, can be consider as a starting point to the future study, through which it hopes to be able to propose a new and optimum strategies to reduce the congestion problem in urban network and to improve life quality of road users. *La ratio dicendi*, having identified all different or uncertain characters of this method, supported each single practical-experimental test, sealing, with a positive connotation, the desired results.

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