Robust Trading for Ambiguity-averse Insiders

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ABSTRACT

In an asset market with explicit trading rules we characterize the trading activity of an ambiguity-averse insider who faces Knightian uncertain over other market participants' beliefs and implements a *robust* trading strategy. Such insider employs a *max-min* choice mechanism, so that in any round of trading she selects as her market order that which maximizes her expected profits against those market beliefs which penalize her most. Her trading strategy is equivalent to that of a risk-averse insider who does not face any Knightian uncertain and possesses risk-sensitive recursive preferences. As she finds it optimal to trade more aggressively and reveal her private information at a faster pace than her risk-neutral (expected-profit maximizer) counterpart, we find that ambiguity-aversion is beneficial to the efficiency of the market.

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Introduction

Trading activity in securities markets depends on investors' assessment of securities' fundamentals *and* of other agents' behavior. In particular, when trading securities an investor needs to take into account the beliefs on securities' fundamentals of other market participants, as these affect their investment decisions, the securities prices and individual profit opportunities. Then, since many investors find it difficult to discern the prevailing opinion in the market place over securities' fundamentals, it is interesting to investigate the interplay which exists between a trader's uncertainty about other investors' beliefs, her trading decisions and the characteristics of securities markets.

We propose an analysis of this interplay in an asset market regulated by explicit trading rules. We formulate a model in which a strategic trader is endowed with some private information on the fundamentals of a risky asset, but is uncertain about the beliefs of the market maker which sets the corresponding transaction price. Since in this formulation she cannot calculate the exact probability distribution of her profit opportunities this insider faces Knightian uncertainty. Aversion to such uncertainty, namely *ambiguity-aversion*, results in the insider adopting a *robust* trading strategy identified via a *max-min* choice mechanism, according to which she selects as her market orders those which maximize her expected profits against the worst market beliefs, ie. against those beliefs which penalize her profits most.

This robust trading strategy is found to be equivalent to that of a risk-averse insider who does not face any Knightian uncertainty over the market maker's beliefs but possesses recursive risk-sensitive preferences. This results in an equilibrium when the insider is risk-averse which is observationally equivalent to that which prevails when she is ambiguity-averse.

As in equilibrium it is found that such an insider trades more aggressively, revealing a larger proportion of her informational advantage and increasing the efficiency of the market, we conclude that ambiguity-aversion improves market quality. Such conclusion is particularly striking, as in the existing literature on ambiguity-aversion in asset markets it is typically found that ambiguity-averse agents display *portfolio-inertia* and trade slowly on their information. Consequently ambiguity aversion results in less efficient markets (see, among others, Dow and Ribeiro da Costa Werlang (1992); Caskey (2009); Condie and Ganguli (2011, 2014); Ozsoylev and Werner (2011); Easley, O'Hara, and Yang (2014); Mele and Sangiorgi (2015)).

Our novel conclusions originate from the particular form of Knightian uncertainty we introduce, which pertains to agents' beliefs rather than assets' fundamentals, and from the specific protocol of trading we consider, which differently from the existing literature prescribes that the insider acts strategically and solves a dynamic optimization exercise.

This paper is organized as follows. In the next Section we introduce Chau and Vayanos' model of an asset market in which an insider possesses some private information on the fundamentals of a risky asset. In this Section: i) we describe the protocol of trading which regulates how this risky asset is traded, the dynamics of its fundamentals and the characteristics of the market participants which trade it; ii) we introduce the insider's Knightian uncertainty about the beliefs of the market maker who sets the transaction price for the risky-asset; and iii) we define the robust choice mechanism she adopts to select her trading strategy. In Section 2 we derive the insider's robust trading strategy and characterize the stationary linear equilibria of the market for the risky asset.

In Section 3 we introduce the risk-sensitive recursive preferences for a risk-averse insider who does not face any Knightian uncertainty and show how her optimal trading strategy is equivalent to the robust one of the ambiguity-averse insider studied in Sections 1 and 2. In Section 4 we investigate the impact of ambiguity-aversion on the trading strategy of the insider and on market quality when trading approaches a continuous auction and when it takes place at equally-spaced-in-time call auctions. In both cases, ambiguity-aversion forces the insider to trade more aggressively than her risk-neutral counterpart, revealing a larger proportion of her informational advantage, so that the market for the risky asset becomes more efficient. In this Section we also discuss the theoretical underpinnings of our main results and the empirical implications of our analysis. A final Section summarizes our findings.

1 Sequential Auctions, Insider Trading and Ambiguity-Aversion

In our analysis we introduce Knightian uncertainty into Chau and Vayanos' model of a market for a risky asset governed by an explicit protocol of trading (Chau and Vayanos, 2008). In their model a risk-neutral monopolistic insider trades a risky asset with a competitive market maker over an infinite sequence of call auctions. In any round of trading the insider observes a private signal on the rate at which the risky asset's dividends grow overtime and submits a market order for the risky asset which maximizes the expected value of her discounted future profits, while the competitive market maker breaks even by fixing the transaction price for the risky asset equal to the expected discounted value of all future dividends it pays out.

We depart from Chau and Vayanos' formulation in that we assume that the insider is uncertain on the pricing rule applied by the market maker. However, before we can explain how this translates into Knightian uncertainty, let us briefly describe the key elements of Chau and Vayanos' model.

1.1 The Analytical Framework

In Chau and Vayanos' set up a risk-neutral market maker trades a risky asset over an infinite sequence of call auctions with a population of clients. These clients comprise a group of unsophisticated agents who trade for liquidity reasons and an informed agent who possesses some private information on the risky asset's fundamentals and trades to gain speculative profits.

The risky asset pays at time $t = n \cdot \Delta$, where Δ is a time interval and n is an integer representing a trading period, a dividend equal to $d_n\Delta$. The dividend yield, d_n, is subject to stochastic shocks and reverts to a time-varying mean, g_n, according to the following Markovian specification

$$d_n \quad = \quad d_{n-1} \;+\; \nu\Delta(g_{n-1} - d_{n-1}) \;+\; \epsilon_n^d\,, \tag{1.1}$$

with $\nu > 0$ and $\nu\Delta \in (0, 1)$. The shocks $\{\epsilon_n^d\}$ are i.i.d.. They are normally distributed with mean zero and variance $\sigma_d^2\Delta$. The time-varying mean, g_n, represents the underlying profitability of the risky asset. This is also subject to stochastic shocks, while reverting to a long-run mean value $\bar{g}$, according to another Markovian specification

$$g_n \quad = \quad g_{n-1} \;+\; \kappa\Delta(\bar{g} - g_{n-1}) \;+\; \epsilon_n^g\,, \tag{1.2}$$

with $\kappa > 0$ and $\kappa\Delta \in (0, 1)$. The shocks $\{\epsilon_n^g\}$ are i.i.d. and independent of the shocks $\{\epsilon_n^d\}$. They are normally distributed with mean zero and variance $\sigma_g^2\Delta$.

In period n the market maker runs call auction n. In this auction, firstly, his clients select their market orders (x_n for the insider and ϵ_n^l for the liquidity traders) which are batched together and passed to the market maker; secondly, the dividend yield, d_n, is publicly observed and the insider *privately observes* the risky asset's underlying profitability, g_n; thirdly, the market maker selects the transaction price for the risky asset, p_n, at which all orders are executed.

In n the market order of the liquidity traders, ϵ_n^l, is normally distributed, with mean 0 and variance $\sigma_l^2\Delta$, while the insider's market order, x_n, is chosen strategically, taking into account its impact on the transaction price, in order to maximize the expected value of her discounted profits. These discounted profits are $\Pi_n = \sum_{j=n}^{\infty} e^{-r(j-n)\Delta}\,\pi_j$ where π_j is her trading profit in auction j and r is the continuously compounding interest rate paid by a safe bond. The insider's trading profit in period j is equal to $x_j(v_j - p_j)$ where v_j is her assessment of the present value of the risky asset's dividends, conditional on the information she possesses at the end of auction j, while p_j is the corresponding transaction price set by the market maker.

1.2 Knightian Uncertainty and Ambiguity-aversion

We believe that the insider can be subject to Knightian uncertainty as we envisage situations in which she is unsure about the price formation process followed by the market maker. Consider, in fact, that according to Chau and Vayanos Bertrand competition with other dealers forces the market maker to set the transaction price on the basis of a semi-strong form efficiency condition, so that p_n is the expected present value of all future dividends the risky asset pays out conditional on the information he possesses in period n. This implies that

$$p_n \quad = \quad A_0 d_n \;+\; A_1 \hat{g}_n \;+\; A_2 \bar{g}\,, \tag{1.3}$$

where $\hat{g}_n$ is the market maker's conditional expectation of the underlying profitability in n, while $A_0 = \frac{\Delta}{1-e^{-(r+\mu)\Delta}}$, $A_1 = A_0\frac{(1-e^{-\mu\Delta})e^{-r\Delta}}{1-e^{-(r+\kappa)\Delta}}$ and $A_2 = A_0\frac{(1-e^{-\mu\Delta})(1-e^{-\kappa\Delta})e^{-2r\Delta}}{(1-e^{-r\Delta})(1-e^{-(r+\kappa)\Delta})}$, with μ such that $e^{-\mu\Delta} = 1-\nu\Delta$.

Equation (1.3) illustrates how the transaction price set by the market maker crucially depends on his beliefs about the underlying profitability g_n. Then, the insider can be uncertain about the price formation process followed by the maker maker because she is unsure about the mechanisms which regulate how he forms his beliefs about this underlying profitability.

In order to see how this is possible notice that if the market maker is rational and if he conjectures that in any auction n the insider's market order is linear in the excess profitability of the risky asset ($x_n = \beta(g_n - \hat{g}_n)$ with $\beta > 0$), his expectation of the underlying profitably is formulated in n on the basis of the information he extracts from the dividend yield d_n and the overall market order $x_n + \epsilon_n^l$ according to the following expression

$$\hat{g}_n \quad = \quad \bar{g} + (1-\kappa\Delta)(\hat{g}_{n-1} - \bar{g}) + \lambda_d(d_n - (1-\nu\Delta)d_{n-1} - \nu\Delta\hat{g}_{n-1}) + \lambda_x\left(x_n + \epsilon_n^l\right), \tag{1.4}$$

with λ_d and λ_x two positive coefficients.

Given equation (1.3), on observing the transaction price at the end of period n the insider is able to determine the value $\hat{g}_n$. However, since she does not observe the market order of the liquidity traders, ϵ_n^l, she cannot establish with certainty whether a particular large value of $\hat{g}_n$ is the consequence of a large value of ϵ_n^l or of some deviation from equation (1.4).

While Chau and Vayanos assume that the insider is certain about the mechanisms which regulate how the market maker forms his beliefs, we believe that she may suspect that they actually deviate from those described above. The insider may in fact fear that the market maker's expectations of the underlying profitability are set incorrectly, possibly because the market orders of the liquidity traders contain predictable components he ignores. Alternatively she may suspect that these expectations are deliberately twisted by the market maker to manage his inventory of the risky asset or that they are simply biased for some unspecified reason.

In general, we consider a scenario in which the insider is uncertain about the maker maker's beliefs. In this scenario she assumes that the market maker correctly estimates the underlying profitability of the risky asset, as indicated in equation (1.4). Hence, equation (1.4) represents the *approximating* specification the insider presumes for the dynamics of the market maker's expectation of the underlying profitability. Although, she suspects that her conjecture is incorrect and that he forms his expectation according to the following alternative specification

$$\hat{g}_n = \bar{g} + (1 - \kappa\Delta)(\hat{g}_{n-1} - \bar{g}) + \lambda_d(d_n - (1 - \nu\Delta)d_{n-1} - \nu\Delta\hat{g}_{n-1}) + \lambda_x\,(x_n + \epsilon_n^l) + \sigma_{\hat{g}}\eta_n\,,$$

with η_n an undetermined value and $\sigma_{\hat{g}}$ the standard deviation of the expected profitability. This alternative specification, in which for some undefined reason the market maker sets his expectation incorrectly, is termed the *distorted* specification.

Three important features of this scenario ought to be emphasized. Firstly, as already mentioned, the insider's uncertainty on the validity of the approximating specification persists overtime because she does not directly observe the market orders of the liquidity traders, $\{\epsilon_n^l\}$. Thus, the insider cannot easily establish whether a large value of $\hat{g}_n$ is the consequence of a large market order by the liquidity traders (a large ϵ_n^l) or of a large error in the market maker's expectation of the underlying profitability (a large η_n).[1]

[1]It might be argued that the insider could learn overtime the mechanisms which regulate how the market maker forms his beliefs and it may be argued that learning could mitigate and in the long-run eliminate the insider's uncertainty over the pricing process. However, if this uncertainty is limited it is plausible to assume that the insider finds it impossible to learn the correct pricing process.

Secondly, as η_n is undetermined, under the distorted specification her uncertainty on how the market maker forms his beliefs amounts to *Knightian uncertainty*. In fact, since she cannot calculate the exact probability distribution of the profits any trading decision will generate, the insider cannot measure exactly the risk she is facing.

Thirdly, the approximating specification the insider presumes for the market maker's expectation of the underlying profitability coincides with the correct one. Therefore, under this specification the insider does not commit systematic errors in predicting the market maker's expectations and the transaction prices he charges. However, because she fears that her conjecture on the market maker's beliefs could be incorrect she considers as possible alternative specifications.

We investigate this scenario because we believe that in many real situations some investors may have privileged information on securities' fundamentals, but limited knowledge of the activity and beliefs of other investors and market participants. Indeed, in equity markets company stakeholders, such as main shareholders or senior managers, may have privileged access to companies' information and a better understanding of their fundamentals. However, these insiders are likely to possess only limited knowledge of the activity of other traders and of the mechanisms which dictate how these market participants form their beliefs about stocks' fundamentals. Therefore, it is likely that they are uncertain about the process dealers follow in setting transaction prices in equity markets. Consequently, within Chau and Vayanos' model, it is reasonable to assume that the insider is unsure about the pricing rule the market maker applies, as in the scenario described above.

This scenario is novel with respect to the existing literature on ambiguity-aversion in securities markets, in that it is typically assumed in this literature that investors are uncertain about assets' fundamentals, while we argue that they are more likely to be uncertain about the price formation process since they find it hard to divine what other market participants believe. Indeed, this may be considered a more realistic vision of ambiguity-aversion in securities markets, as investors are typically more uncertain about other market participants' beliefs than about securities' fundamentals.

The sort of Knightian uncertainty described above is more conveniently investigated if we reformulate such uncertainty in terms of the excess profitability, $g_n - \hat{g}_n$, as this represents the difference in opinions between the market maker and the insider and it is the state variable in the dynamic optimization the latter solves. Indeed, using equations (1.1) and (1.2), the approximating specification can be rewritten in terms of the excess profitability as

$$g_n - \hat{g}_n \;=\; \big(1 - (\kappa + \lambda_d \nu)\Delta\big)(g_{n-1} - \hat{g}_{n-1}) \;-\; \lambda_x x_n \;+\; \epsilon_n\,, \tag{1.5}$$

where $\epsilon_n = \epsilon_n^g - \lambda_x \epsilon_n^l - \lambda_d \epsilon_n^d$, while the distorted specification can be written as

$$g_n - \hat{g}_n \;=\; \big(1 - (\kappa + \lambda_d \nu)\Delta\big)(g_{n-1} - \hat{g}_{n-1}) \;-\; \lambda_x x_n \;+\; \epsilon_n \;+\; \sigma_\epsilon w_n\,, \tag{1.6}$$

with σ_ϵ the standard deviation of ϵ_n and w_n such that $\sigma_{\hat{g}} \eta_n = -\sigma_\epsilon w_n$.

In order to define the insider's degree of ambiguity-aversion we introduce a measure of probabilistic discrepancy between the approximating and distorted specifications known as *discounted conditional entropy* (see Hansen and Sargent, 2001, 2008). In particular, in period n denote with z_n the state variable $g_n - \hat{g}_n$ and with $f_a(z_n \mid z_{n-1})$ and $f_d(z_n \mid z_{n-1})$ the probability density function of z_n, conditional on z_{n-1}, respectively under the approximating and distorted specifications. Denote with $m(f_a(z_n \mid z_{n-1}))$ the log of the ratio between these density functions, $\log(f_a(z_n \mid z_{n-1})/f_d(z_n \mid z_{n-1}))$. The *conditional relative entropy* in n is then defined as the conditional expectation of the log-likelihood ratio for the approximating and the distorted specifications, calculated under the distorted one,

$$I(f_a, f_d)(z_{n-1}) \equiv \int m(f_a(z_n \mid z_{n-1}))\, f_d(z_n \mid z_{n-1}) dz_n.$$

In the Appendix we show that this conditional relative entropy is equal to $\frac{1}{2} w_n^2$. An intertemporal measure of probabilistic distance between the two specifications is then given by the expected discounted value of all conditional relative entropies, or discounted conditional entropy,

$$\mathcal{R}_n^w \;\equiv\; 2\, E_n \left[\sum_{h=0}^{\infty} e^{-rh\Delta}\, I(f_a, f_d)(z_{n+h-1}) \right] \;=\; E_n \left[\sum_{h=0}^{\infty} e^{-rh\Delta}\, w_{n+h}^2 \right] ,$$

where the expectation is taken in n under the distorted specification. This aggregate measure represents the probabilistic distance between the approximating specification conjectured by the insider and the distorted one she suspects is regulating the dynamics of $g_n - \hat{g}_n$. We assume that the insider considers as potential alternatives all the distorted specifications for which $\mathcal{R}_n^w \leq \phi$, where ϕ (with $\phi > 0$) defines the maximum probabilistic distance between the approximating and distorted specifications she deems feasible.[2]

[2]If the approximating and distorted specifications are defined in terms of the expected profitability $\hat{g}_n$ the

In this way we envision a situation in which the insider assumes that $g_n - \hat{g}_n$ is generated according to equation (1.5) and suspects that it is actually governed by equation (1.6). This distorted specification is assumed to be not too *far* from the approximating one, where in measuring their distance the insider refers to the discounted conditional entropy $\mathcal{R}_n^w$.

We assume that in n the insider selects a *robust* trading strategy which works for *all* distorted specifications for which $\mathcal{R}_n^w \leq \phi$. This selection criterion is particularly demanding, in that it requires that in any auction n she chooses the market order which maximizes her expected discounted profits in the *worst* distorted specification (among all admissible ones).

Since the insider observes the history of the risky asset's underlying profitability, alongside that of its dividends, her assessment of its fundamental value in n respects a formulation similar to equation (1.3), with the expected underlying profitability, $\hat{g}_n$, replaced by the actual one, g_n (so that $v_n = A_0 d_n + A_1 g_n + A_2 \bar{g}$). Then, given her normalized profits in j, $\pi_j^* = x_j(g_j - \hat{g}_j)$, she selects a robust trading strategy solving the following *constraint program*[3]

$$\max_{\{x_{n+h}\}_{h=0}^{\infty}} \min_{\{w_{n+h}\}_{h=0}^{\infty}} E_n \left[\sum_{h=0}^{\infty} e^{-rh\Delta} \, \pi_{n+h}^* \right] \tag{1.7}$$

$$\text{s.t.} \qquad g_n - \hat{g}_n \text{ respects equation (1.6) and } \mathcal{R}_n^w \leq \phi \,.$$

According to this program in n the insider firstly isolates among all alternative specifications the worst one, i.e. the one which minimizes the present value of her expected profits, and secondly she selects the market order which maximizes such profits within this worst-case specification. She applies this particularly restrictive selection criterion because it allows her to deal with her inability to calculate the probability of the outcomes of her trading activity.

Solving program (1.7) may be difficult. However, Hansen and Sargent (2001) prove a very useful result, in that they show that the constraint program (1.7) is equivalent to the following

conditional relative entropy in n is $\frac{1}{2}\eta_n^2$, while, since $\eta_n = \frac{\sigma_\epsilon}{\sigma_{\hat{g}}} w_n$, the corresponding discounted conditional entropy is $\mathcal{R}_n^\eta = \frac{\sigma_\epsilon^2}{\sigma_{\hat{g}}^2} \mathcal{R}_n^w$. Then, the set of alternative distorted specifications the insider consider as possible can equivalently be defined in terms of $\hat{g}_n$ as those such that $\mathcal{R}_n^\eta \leq \frac{\sigma_\epsilon^2}{\sigma_{\hat{g}}^2} \phi$.

[3]The normalized profits, π_j^*, are employed here rather than the actual ones, π_j, to simplify the comparison with Chau and Vayanos' results. However, the two formulations, with normalized and actual profits, are economically equivalent in that $v_j - p_j = A_1(g_j - \hat{g}_j)$ and $\pi_j = A_1 \pi_j^*$.

multiplier program

$$\max_{\{x_{n+h}\}_{h=0}^{\infty}} \min_{\{w_{n+h}\}_{h=0}^{\infty}} E_n \left[\sum_{h=0}^{\infty} e^{-rh\Delta} \left(\pi_{n+h}^* + \vartheta w_{n+h}^2 \right) \right], \tag{1.8}$$

s.t. $\quad g_n - \hat{g}_n$ respects equation (1.6) and ϑ is some positive constant.

Not only this equivalence is useful because program (1.8) is much easier to deal with, but it is also particularly important in that it entails that the insider's preferences we consider correspond to a specific form of ambiguity-aversion. In fact, Maccheroni, Marinacci, and Rustichini (2006) define a general class of preferences which subsumes the *multiple priors preferences* of Gilboa and Schmeidler (1989), commonly employed in the ambiguity-aversion literature, and the *multiplier preferences* of Hansen and Sargent, according to which an agent facing Knightain uncertainty seeks to follow a robust strategy by solving program (1.8).[4]

Under the class of preferences considered by Maccheroni and his coauthors, the smaller ϑ the greater the agent's degree of ambiguity-aversion.[5] This implies that we can measure the insider's degree of ambiguity-aversion through the inverse of ϑ, $\theta \equiv \vartheta^{-1}$.[6] Importantly, for $\theta \downarrow 0$ ($\vartheta \uparrow \infty$) the minimum wrt to $\{w_{n+h}\}_{h=0}^{\infty}$ in program (1.8) is reached for $w_{n+h} = 0 \; \forall h$. In this case the trading strategy of the insider coincides with that of the informed trader considered by Chau and Vayanos and hence a straightforward comparison between their formulation and ours ensues.

The parameter θ (or equivalently the parameter ϕ), measuring the insider's degree of ambiguity-aversion, is constant overtime in the formulation we investigate. One could argue that the degree of ambiguity-aversion evolves overtime, as agents are usually more ambiguity-averse during periods of economic downturns. Investigating an extension with a time-varying degree of ambiguity-aversion would be interesting and potentially very fruitful. However, it would entail non-stationary equilibria, which would be very difficult to characterize, and it would require some formalization of time-varying ambiguity-aversion which so far has not been proposed in the literature.

We now study stationary linear equilibria in which in any auction the insider solves the

[4]Alternative, closely related, axiomatizations of ambiguity-aversion have been put forward by Schmeidler (1989), Epstein (1999) and Ghirardato and Marinacci (2002). For a general presentation of the literature on ambiguity-aversion see Machina and Siniscalchi (2013).

[5]See Proposition 8 in Maccheroni, Marinacci, and Rustichini (2006).

[6]The constants ϑ and ϕ are also inversely related, in that $\phi = \phi(\vartheta)$ with $\phi(\vartheta)$ decreasing in ϑ as proved in Lemma 8.5.1 in Hansen and Sargent (2008). This implies that θ and ϕ can equivalently be treated as measures of the insider's degree of ambiguity-aversion.

constraint program in (1.7), or equivalently the multiplier program (1.8), while the market maker sets the transaction price according to the efficiency condition (1.3).

2 Stationary Linear Equilibria

We start the characterization of stationary linear equilibria by studying the insider's robust trading strategy. As she selects her market order according to the constraint program (1.7), or equivalently according to the multiplier program (1.8), we define the value function

$$\boldsymbol{\mathcal{V}}_n \equiv \max_{\{x_{n+j}\}_{j=0}^{\infty}} \min_{\{w_{n+j}\}_{j=0}^{\infty}} E_n\left[\sum_{j=0}^{\infty} e^{-rj\Delta}\left(\pi^*_{n+j} + \theta^{-1} w^2_{n+j}\right)\right].$$

Because the insider's per-period profits are linear in the excess profitability, we can conjecture that in a stationary equilibrium the value function is $\boldsymbol{\mathcal{V}}_n = B(g_{n-1} - \hat{g}_{n-1})^2 + C$, for B and C two positive constants. Then, the multiplier program (1.8) admits a (modified) Bellman equation which allows to determine the value function and the robust market order selected by the insider in n. This Bellman equation is as follows

$$B(g_{n-1} - \hat{g}_{n-1}) + C = \max_{x_n} \min_{w_n} E_n[\pi^*_n + \theta^{-1} w_n^2 + e^{-r\Delta} B(g_n - \hat{g}_n)^2 + e^{-r\Delta} C],$$

where the state variable, $g_n - \hat{g}_n$, respects equation (1.6) and the expectation is taken with respect to the distribution of ϵ_n. Crucially, this Bellman equation yields the same modified Riccati equation and the same robust market order of the non-stochastic version in which $\epsilon_n \equiv 0$. In the non-stochastic version of the Bellman equation C disappears and the robust market order in n is identified solving the double recursion

$$\begin{aligned} B(g_{n-1} - \hat{g}_{n-1})^2 &= \max_{x_n} \min_{w_n}[\pi^*_n + \theta^{-1} w_n^2 + e^{-r\Delta} B(g_n - \hat{g}_n)^2], \quad \text{with} \\ g_n - \hat{g}_n &= \big(1 - (\kappa + \lambda_d \nu)\Delta\big)(g_{n-1} - \hat{g}_{n-1}) - \lambda_x x_n + \sigma_\epsilon w_n. \end{aligned} \tag{2.1}$$

We can then prove the following Lemma, which describes the exact specification of the insider's market order in any auction n according to her robust strategy.

Lemma 1 *In auction n, according to the insider's robust trading strategy, her market order is*

$$\begin{aligned}
x_n &= \beta(g_{n-1} - \hat{g}_{n-1})\,, \quad \textit{with} &(2.2)\\
\beta &= \frac{\big(1-(\kappa+\nu\lambda_d)\Delta\big)(1-2e^{-r\Delta}\lambda_x B)}{2\lambda_x(1-e^{-r\Delta}\lambda_x B) \,+\, \frac{1}{2}\theta\sigma_\epsilon^2} \quad \textit{and} &(2.3)\\
B &= \frac{e^{r\Delta}}{2\lambda_x}\left(1+\frac{1}{4\lambda_x}\theta\sigma_\epsilon^2 - \left[\left(1+\frac{1}{4\lambda_x}\theta\sigma_\epsilon^2\right)^2 - e^{-r\Delta}\big(1-(\kappa+\nu\lambda_d)\Delta\big)^2\right]^{1/2}\right). &(2.4)
\end{aligned}$$

Proof. See the Appendix.

Unsurprising corollary of Lemma 1 is the following result which confirms that our formulation subsumes that of Chau and Vayanos.

Corollary 1 *For $\theta \downarrow 0$ the insider's trading strategy converges to that of the risk-neutral (profit-maximizer) informed trader considered by Chau and Vayanos.*

Then, using the projection Theorem for Normal random variables, we find that if the insider chooses her market order according to equation (2.2), with β some positive constant, the market maker applies equation (1.4) in period n to formulate his expectation of the underlying profitably, with the coefficients λ_d and λ_x as follows

$$\lambda_d = \frac{(1-\kappa\Delta)\Sigma_g\nu\sigma_l^2\Delta}{\Sigma_g(\beta^2\sigma_d^2+\nu^2\sigma_l^2\Delta^2)+\sigma_d^2\sigma_l^2\Delta}\,, \tag{2.5}$$

$$\lambda_x = \frac{(1-\kappa\Delta)\beta\Sigma_g\sigma_d^2}{\Sigma_g(\beta^2\sigma_d^2+\nu^2\sigma_l^2\Delta^2)+\sigma_d^2\sigma_l^2\Delta} \tag{2.6}$$

and Σ_g, the conditional variance of g_n given the market maker's information at the end of period n, equal to

$$\Sigma_g = \frac{(1-\kappa\Delta)^2\sigma_d^2\,\sigma_l^2\Delta}{\Sigma_g(\beta^2\sigma_d^2+\nu^2\sigma_l^2\Delta^2) \,+\, \sigma_d^2\,\sigma_l^2\Delta}\,\Sigma_g \,+\, \sigma_g^2\Delta\,. \tag{2.7}$$

Combining all results presented in this Section we see that in a stationary linear equilibrium the transaction price for the risky asset set by the market maker in n is a linear function of the dividend yield, d_n, the market maker's conditional expectation of the underlying profitability, $\hat{g}_n$, and its long-run mean value, $\bar{g}$ (equation (1.3)). As in n the market maker receives informative signals on the underlying profitability from the dividend yield, d_n, and his clients' market orders, $x_n + \epsilon_n^l$, his conditional expectation of g_n is a linear function of such variables (equation (1.4)). At the same time, the insider exploits her informational advantage by submitting a market order which is a linear function of the perceived mis-pricing of the risky asset, measured by the difference between the actual profitability and the corresponding market maker's conditional expectation, $g_{n-1} - \hat{g}_{n-1}$ (equation (2.2)).

The following Proposition sums up this characterization of a stationary linear equilibrium.

Proposition 1 *In a stationary linear equilibrium, in auction n the market maker sets the transaction price for the risky asset according to equation (1.3), where his conditional expectation of the underlying profitability, g_n, is a linear function of the dividend yield and of his clients' overall market order, as given in equation (1.4) with the coefficients λ_d and λ_x described in equations (2.5) and (2.6), and the corresponding conditional variance is given in equation (2.7). The insider's market order is a linear function of the market maker's mis-pricing of the risky asset as given in equation (2.2), with the coefficients β and B described in equations (2.3) and (2.4).*

Such equilibrium exists if there exist values for the coefficients B, β, λ_d, λ_x and Σ_g which simultaneously respect equations (2.4), (2.3), (2.5), (2.6) and (2.7).

While explicit formulae for the coefficients B , β, λ_d, λ_x and Σ_g are not available, a simple numerical procedure allows to solve the system of equations (2.4), (2.3), (2.5), (2.6) and (2.7). This is a numerical procedure which yields the trading intensity implicit in any initial guess β_0, $\beta = \boldsymbol{\mathcal{N}}(\beta_0)$. In particular, starting from an initial guess β_0, the conditional variance Σ_g is derived from equation (2.7); then, the coefficients λ_d and λ_x are obtained from equations (2.5) and (2.6); eventually, equation (2.4) yields the constant B, while equation (2.3) allows to obtain a final value for the trading intensity β.

A root of the numerical procedure $\boldsymbol{\mathcal{N}}(\cdot)$ yields a solution to the system of equations (2.4), (2.3), (2.5), (2.6) and (2.7). Finding such root is simplified by the fact that, as shown in Section 4.1, an explicit and unique solution for the system of equations (2.4), (2.3), (2.5), (2.6) and (2.7) always exists in the continuous-time limit. Then, one can start the procedure searching for the root of $\boldsymbol{\mathcal{N}}(\cdot)$ from the value of β consistent with the stationary linear equilibrium which prevails in the continuous-time limit.

Before we turn to the analysis of the properties of the equilibrium described in Proposition 1 we prove an important result which suggests that an equivalence holds between the formulation with an ambiguity-averse insider considered so far and an alternative one with an insider endowed with risk-sensitive recursive preferences.

3 An Insider with Risk-sensitive Recursive Preferences

In this Section we consider an insider who does not face any Knightian uncertainty over the price formation process. This means that as in Chau and Vayanos she is sure that in n the market maker applies equation (1.4) in formulating his expectation of the underlying profitability g_n. However, differently from what it is assumed by Chau and Vayanos, she is risk-averse, so that in n she chooses her market order, x_n, solving the following recursive optimization

$$\boldsymbol{\mathcal{C}}_n \quad = \quad \min_{x_n} \left\{ \frac{2}{\rho} \ln \left(E_n \left[\exp \left(\frac{\rho}{2} (c_n \, + \, e^{-r\Delta} \boldsymbol{\mathcal{C}}_{n+1}) \right) \right] \right) \right\} , \tag{3.1}$$

where ρ is a positive coefficient, c_n is a per-period cost function equal to the opposite of the the insider's per-auction profits, $c_n = -\pi_n$, and $\boldsymbol{\mathcal{C}}_n$ is the optimization criterion in n.[7]

The optimization criterion in (3.1) accommodates risk-aversion through the curvature of the exponential function.[8] As the convexity of $\ln(E[\exp(\frac{\rho}{2}\boldsymbol{\mathcal{X}})])$ increases with ρ, this coefficients determines the insider's degree of risk-aversion. In addition, for $\rho \downarrow 0$ the recursive optimization in (3.1) converges to $\boldsymbol{\mathcal{C}}_n = \min_{x_n} E_n[c_n + e^{-r\Delta}\boldsymbol{\mathcal{C}}_{n+1}]$. This corresponds to the Bellman equation which solves the insider's optimization exercise within Chau and Vayanos' formulation. Thus, for $\rho > 0$ an insider endowed with the recursive preferences described by the optimization criterion in (3.1) is more risk-averse than the risk-neutral one considered by Chau and Vayanos.

It is worth noticing that as $v_n - p_n = A_1(g_n - \hat{g}_n)$ the modified optimization criterion $\boldsymbol{\mathcal{W}}_n = \boldsymbol{\mathcal{C}}_n / A_1$ can equivalently be employed. In fact, it is immediate to see that the recursive optimization (3.1) corresponds to

$$\boldsymbol{\mathcal{W}}_n \quad = \quad \min_{x_n} \left\{ \frac{2}{A_1 \rho} \ln \left(E_n \left[\exp \left(\frac{A_1 \rho}{2} (c^*_n \, + \, e^{-r\Delta} \boldsymbol{\mathcal{W}}_{n+1}) \right) \right] \right) \right\} , \tag{3.2}$$

[7]The optimization criterion (3.1) put forward by Vitale (2015) is similar to that proposed by Hansen and Sargent (1994, 1995). The two criteria differ because in Hansen and Sargent's the per-period cost, c_n, is deterministic and hence outside the expectation operator, while in (3.1) is stochastic.

[8]The functional form $\ln(E[\exp(\frac{\rho}{2}\boldsymbol{\mathcal{X}})])$ is monotonically increasing and convex in $\boldsymbol{\mathcal{X}}$. In the optimization criterion in (3.1) $\boldsymbol{\mathcal{X}} \equiv c_n + e^{-r\Delta}\boldsymbol{\mathcal{C}}_{n+1}$, which is convex in x_n and $g_{n-1} - \hat{g}_{n-1}$. Then, we see that the optimization criterion in (3.1) is convex in x_n and $g_{n-1} - \hat{g}_{n-1}$. This means that the optimization criterion is well-defined, in that it is convex in the control variable x_n and state variable $g_{n-1} - \hat{g}_{n-1}$.

where $c_n^* = -\pi_n^*$. Then, to simplify algebra we introduce the rescaled risk-aversion coefficient $\rho^* = \rho A_1$ and employ the modified optimization criterion (3.2).

As in Section 2 we concentrate on linear stationary equilibria. In this respect we can now establish the main result of this Section which describes the optimal trading strategy of a risk-averse insider endowed with the recursive risk-sensitive preferences represented by the optimization criterion (3.2).

Lemma 2 *Assume that the market maker sets the transaction price according to equation (1.3) and formulates his expectation of the risky asset's underlying profitability according to equation (1.4). Then, the trading strategy of the insider is such that in any period n:*

1) her optimal market order is found solving the double recursion

$$-B(g_{n-1} - \hat{g}_{n-1})^2 \;=\; \min_{x_n} \max_{\epsilon_n} \left[c_n^* - \frac{1}{\rho^*} \frac{\epsilon_n^2}{\sigma_\epsilon^2} - e^{-r\Delta} B(g_n - \hat{g}_n)^2 \right] ; \qquad (3.3)$$

2) the optimization criterion is a quadratic form in $g_{n-1} - \hat{g}_{n-1}$,

$$\mathcal{W}_n \;=\; -B(g_{n-1} - \hat{g}_{n-1})^2 - C \, , \qquad (3.4)$$

with B and C positive constants.

Proof. See the Appendix.

Importantly, the double recursion in (3.3) is equivalent to

$$B(g_{n-1} - \hat{g}_{n-1})^2 \;=\; \max_{x_n} \min_{\epsilon_n} \left[\pi_n^* + \frac{1}{\rho^*} \frac{\epsilon_n^2}{\sigma_\epsilon^2} + e^{-r\Delta} B(g_n - \hat{g}_n)^2 \right] ,$$

which coincides with that in (2.1) for $\epsilon_n = \sigma_\epsilon w_n$ and $\rho^* = \theta$. Therefore, as a Corollary of Lemma 2 we conclude that the optimal trading strategy of a risk-averse insider endowed with the recursive risk-sensitive preferences represented by the optimization criterion (3.2) corresponds to the robust one of the ambiguity-averse insider considered in Section 1.2. This proves Proposition 2 which posits that an equivalence holds between the formulation with an ambiguity-averse insider and that in which such an insider is risk-averse.

Proposition 2 *The stationary linear equilibrium with a risk-averse insider is observationally equivalent to that with an ambiguity-averse insider, in that it is characterized by the same trading and pricing strategies on the part of the insider and the market maker.*

While observationally equivalent the two equilibria are not identical, in that ambiguity-aversion and risk-aversion represent very different attitudes towards uncertainty. In fact, the risk-averse insider is concerned with the volatility of her trading profits, in that she cannot anticipate the liquidity traders' market order, but is not uncertain about their expected value. On the contrary, the ambiguity-averse insider is uncertain about what this expected value is, in that she is unsure about the market maker's pricing rule. However, Proposition 2 suggests that the impact of ambiguity- and risk-aversion on the strategies of the insider and of the market maker, and hence on the characteristics of the market for the risky asset, is the same. This implies that in the next Section θ can equivalently represent a measure of either ambiguity- or risk-aversion.

4 Ambiguity/Risk-aversion and Market Quality

We are now interested in investigating the impact of ambiguity/risk-aversion on the trading activity of the insider and on the characteristics, such as its efficiency and liquidity, of the market for the risky asset. We start by considering a limit case in which trading approaches a continuous auction, as we have analytical results.

4.1 Continuous-time Trading

For Δ, the time interval between two consecutive auctions, converging to 0 the continuous-time limit of the stationary linear equilibrium described in Section 2 is reached. When $\Delta \downarrow 0$ trading approaches a continuous auction, as traders can trade the risky asset at any time. Then, the following Proposition holds.

Proposition 3 *In the continuous-time limit the stationary linear equilibrium illustrated in Proposition 1 exists, it is unique and it is characterized by the following asymptotic behavior of the* *coefficients which identify it:*

$$\lim_{\Delta\downarrow 0} \frac{\beta}{\sqrt{\Delta}} = \left(2\kappa + r + \theta\sigma_g\sigma_l\right)^{1/2} \frac{\sigma_l}{\sigma_g}, \tag{4.1}$$

$$\lim_{\Delta\downarrow 0} \frac{\Sigma_g}{\sqrt{\Delta}} = \frac{1}{\left(2\kappa + r + \theta\sigma_g\sigma_l\right)^{1/2}} \sigma_g^2, \tag{4.2}$$

$$\lim_{\Delta\downarrow 0} \frac{\lambda_d}{\sqrt{\Delta}} = \frac{\nu}{\left(2\kappa + r + \theta\sigma_g\sigma_l\right)^{1/2}} \frac{\sigma_g^2}{\sigma_d^2}, \tag{4.3}$$

$$\lim_{\Delta\downarrow 0} \lambda_x = \frac{\sigma_g}{\sigma_l}, \tag{4.4}$$

$$\lim_{\Delta\downarrow 0} B = \frac{1}{2}\frac{\sigma_l}{\sigma_g}. \tag{4.5}$$

Proof. See the Appendix.

By continuity Proposition 3 suggests that for Δ small enough the system of equations (2.4), (2.3), (2.5), (2.6) and (2.7) does have a solution. In addition, inspection of the asymptotic behavior of the coefficients identyfing the stationary linear equilibrium of Proposition 1 proves the following Corollary, which illustrates some important implications for the impact of ambiguity/risk-aversion on the insider's trading activity and on market quality.

Corollary 2 *In the continuous-time limit, an ambiguity/risk-averse insider will trade more aggressively than her risk-neutral (expected-profit maximizer) counterpart, increasing the speed with which private information is impounded into the asset's price and benefiting market efficiency.*

Indeed, we immediately see that for $\Delta \downarrow 0$ the limit of $\beta/\sqrt{\Delta}$ is larger, while that of $\Sigma_g/\sqrt{\Delta}$ is smaller, for $\theta > 0$ than for the value of θ consistent with Chau and Vayanos' formulation ($\theta = 0$). Furthermore, the limit of $\lambda_d/\sqrt{\Delta}$ is smaller for $\theta > 0$. This indicates that in the continuous-time limit an ambiguity/risk-averse insider finds it optimal to trade more aggressively than the risk-neutral counterpart studied by Chau and Vayanos, choosing a larger trading intensity for the market orders she submits ($\lim_{\Delta\downarrow 0} \beta/\sqrt{\Delta}$ is larger) and revealing to the market maker a larger proportion of her private information. Consequently, with an ambiguity/risk-averse insider the market is more efficient ($\lim_{\Delta\downarrow 0} \Sigma_g/\sqrt{\Delta}$ is smaller) and the market maker learns more about the underlying profitability from order flow than from the dividend yield ($\lim_{\Delta\downarrow 0} \lambda_d/\sqrt{\Delta}$ is smaller).

Another interesting Corollary of Proposition 3 is the following.

Corollary 3 *In the continuous-time limit, market liquidity, as measured by the market depth $1/(A_1\lambda_x)$, is unaffected by the insider's degree of ambiguity/risk-aversion.*

Prima facie this result may appear to contradict the asymptotic behavior of the insider's trading intensity. In fact, in the limit $\beta/\sqrt{\Delta}$ takes a larger value when the insider is more ambiguity/risk-averse. Then, one wonders how the market can be equally liquid for different values of θ. Indeed, as the insider trades more aggressively and places larger market orders when θ is larger, *ceteris paribus* adverse selection should induce the market maker to reduce market liquidity. However, Σ_g is also smaller and hence the market maker's uncertainty on the fundamental value of the risky asset is attenuated. Corollary 3 indicates that in the continuous-time limit these two contrasting effects on the liquidity coefficient λ_x (that positive of a larger β and that negative of a smaller Σ_g) exactly compensate each other, so that market liquidity is unaffected by the insider's degree of ambiguity/risk-aversion.

A third Corollary of Proposition 3 is the following.

Corollary 4 *In the continuous-time limit, the effects of ambiguity/risk-aversion and time-discounting on the insider's trading activity and on efficiency and liquidity are similar but distinct.*

This is because θ and r enter additively in the expressions for the limit behavior of β and Σ_g, while neither appears in that for the limit behavior of λ_x. As time-discounting and ambiguity/risk-aversion enter separately into the objective function in the constraint program (1.7) and into the optimization criterion (3.2) they have distinct effects on the insider's trading activity and on market quality. Such effects are however similar in that, as also shown in the next Section when discussing the properties of the discrete-time formulation, they both favor early resolution of uncertainty.

Proposition 3 allows to derive an interesting comparative static result pertaining to the impact of liquidity trading on market efficiency. Thus, Chau and Vayanos show that when the insider is an expected-profit maximizer the volume of liquidity trading, measured by σ_l^2, does not affect market efficiency. On the contrary, from Proposition 3 we see that for $\theta > 0$ $\lim_{\Delta\downarrow 0} \Sigma_g/\sqrt{\Delta}$ is smaller when σ_l^2 is larger, so that the following Corollary holds.

Corollary 5 *In the continuous-time limit, with an ambiguity/risk-averse insider, market efficiency is increasing with the volume of liquidity trading.*

This holds because the variability of the profits of the risk-averse insider and the Knightian uncertainty about such profits of her ambiguity-averse counterpart increase with σ_l^2.[9] Then, with a larger volume of liquidity trading the insider will choose to reveal a larger proportion of her private information to offset the negative impact of a larger σ_l^2 on the variability/uncertainty of her payoffs.

4.2 Discrete-Time Trading

It is important to establish whether the conclusions drawn for the continuous-time limit are also valid when trading takes place at equally-spaced-in-time call auctions. As mentioned, the system of equations (2.4), (2.3), (2.5), (2.6) and (2.7) for $\Delta > 0$ does not have an explicit solution and a numerical procedure is called for. In what follows the numerical procedure illustrated in Section 2 is used for a benchmark parametric configuration proposed by Chau and Vayanos. In particular, in their calibration they employ data for Coca-cola stock. For this stock estimated values for the volatility of dividends, σ_d, and of the underlying profitability, σ_g, are respectively 1.06 and 0.62. The estimated values for the mean-reverting coefficients for the processes governing the dividend yield, ν, and the underlying profitability, k, are respectively 1.47 and 0. The continuously compounding interest rate, r, is 2 percent.

In their calibration the standard deviation of liquidity trading, σ_l, is normalized to 1, as this parameter does not influence the efficiency of the market. Proposition 3 indicates that such result does not survive the introduction of ambiguity/risk-aversion, as now the volume of liquidity trading affects the speed with which the insider reveals her private information. For easy of comparison we will maintain their benchmark choice for σ_l but we will also discuss what happens when we modify it. Finally, we experiment with different values for the coefficient θ and for the time interval between subsequent auctions, Δ. For θ we consider values between 0 and 1, while for Δ we choose values ranging from 1/252, for daily trading, to 1/120960, for minute-by-minute trading.

[9] In order to see how the volume of liquidity trading affects the Knightian uncertainty of the ambiguity-averse insider, consider that $\lim_{\sigma_l \downarrow 0} \sigma_\epsilon = 0$, so that when σ_l^2 drops to zero the dynamics of the excess profitability, $g_n - \hat{g}_n$, and that of the normalized profits, π_n^*, become deterministic both in the approximating specification (1.5) and in the distorted one (1.6). When this happens Knightian uncertainty dissipates and hence ambiguity-aversion has no impact on the insider's trading strategy. In fact, for $\sigma_\epsilon \downarrow 0$ the argmin in the double-recursion (2.1) becomes $w_n^{\min} = 0$ and the insider's trading strategy collapses to that of her risk-neutral (profit-maximizer) counterpart.

[Figure 1 about here.]

In Figure 1 we represent the dependence of the equilibrium coefficients β, Σ_g, λ_x and λ_d on the frequency of trading, $1/\Delta$, for the benchmark choice of the parameters. Thus, the ratios $\Sigma_g/\sqrt{\Delta}$ (top, left panel), $\beta/\sqrt{\Delta}$ (top, right panel) and $\lambda_d/\sqrt{\Delta}$ (bottom, right panel) and the coefficient λ_x (bottom, left panel) are plotted against a trading frequency varying from the daily to the minute-by-minute one and compared to their continuous-time limits.

Comparing the actual behavior of these coefficients to their asymptotic counterparts we see that convergence to the continuous-time limit is achieved fairly rapidly. This is particular evident for the liquidity coefficient λ_x, which already at the daily frequency is less than 10 percent away from its asymptotic value. Interestingly, this plot also suggests that market liquidity decreases with the trading frequency.

[Figure 2 about here.]

From Proposition 3 we concluded that in the continuous-time limit an ambiguity/risk-averse insider will trade more aggressively, revealing a larger proportion of her private information and increasing the efficiency of the market, than her risk-neutral (expected-profit maximizer) counterpart. In Figure 2 we plot the dependence of the equilibrium coefficients Σ_g (top, left panel), β (bottom, left panel), λ_x (top, right panel) and λ_d (bottom, right panel) on θ, for the benchmark parametric constellation and for four different values of Δ, corresponding to the weekly, daily, hourly and minute-by-minute frequency of trading. The dependence of the four coefficients on θ is clear-cut and consistent with the implications of Proposition 3 for the continuous-time limit.

In particular, for all four values of Δ, the more ambiguity/risk-averse the insider is, the larger her trading intensity, β. As with a larger θ more information on the underlying profitability of the risky asset is conveyed by the insider's market orders, the market maker gives more weight to order flow (λ_x is larger) and less to the dividend yield (λ_d is smaller) in formulating his expectation of g_n. Since with a larger θ more information is elicited from order flow, the market maker is less uncertain about the underlying profitability of the risky asset, so that the conditional variance of g_n, Σ_g, is smaller. Consequently, the asset market is more efficient and less liquid.

As the frequency of trading, $1/\Delta$, rises the volume of liquidity trading observed in a single auction, $\sigma_l^2\Delta$, drops and so does the insider's trading intensity, β. Nevertheless, the asymptotic behavior of β indicates that $\beta/\sqrt{\Delta}$ converges to a constant for $\Delta \downarrow 0$. This implies that, as the number of auctions in one year is $1/\Delta$, the overall volume of trading by the insider in a given spell of time augments when the trading frequency rises. Consequently, as $1/\Delta$ increases order flow becomes more informative and the market becomes more efficient (Σ_g is smaller) and less liquid (λ_x is larger).

Quantitatively, the impact of ambiguity/risk-aversion is substantial, in particular when considering a large trading frequency. Thus, for hourly trading we see that the conditional variance Σ_g more than halves for θ varying from 0 to 1. The percentage drop is even more pronounced for minute-by-minute trading.

[Figure 3 about here.]

From Proposition 3 we have concluded that in the continuous-time limit an ambiguity/risk-averse insider trades more aggressively when the volume of liquidity trading is larger, revealing an even larger proportion of her private information to the marker maker. In Figure 3 we plot the dependence of the coefficients Σ_g (top, left panel), β (bottom, left panel), λ_x (bottom, middle panel) and λ_d (bottom, right panel) on θ for two values of σ_l^2 and for two values of Δ.

Consistently with Corollary 5 we see that for $\theta > 0$ the market maker's conditional variance of the underlying profitability of the risky asset, Σ_g, is smaller when the volume of liquidity trading is larger. Indeed, with a larger volume of liquidity trading the insider chooses a larger trading intensity β for all values of θ, since her market orders are more easily disguised among those of the liquidity traders. However, for $\theta > 0$ with a larger volume of liquidity trading the insider chooses such a large trading intensity as to reveal a larger proportion of her private information. This confirms our claim that the irrelevance of the liquidity conditions on the efficiency of the asset market established by Chau and Vayanos hinges on the assumption that the insider maximizes the expected value of her discounted profits. Our result is general as it applies to both the continuous-time limit and the formulation with a finite trading frequency.

In Figure 3 we also plot the half-life of the market maker's prediction error of the risky asset's fundamental value, $t_{0.5}$ (top, middle panel), as well as the half-life of such prediction error relative to the benchmark scenario with no insider trading, $t_{0.5}/t_{0.5,0}$ (top, right panel). In n the market maker's prediction error is equal to the difference between the fundamental value and the transaction price he sets, $v_n - p_n$. The half-life $t_{0.5}$ indicates the time it must elapse (while $t_{0.5}/\Delta$ is the number of auctions which must be run) before such value is expected to halve, i.e. $t_{0.5}$ is such that $E_n[v_{n+t_{0.5}/\Delta} - p_{n+t_{0.5}/\Delta}] = \frac{1}{2}(v_n - p_n)$, while $t_{0.5,0}$ is the corresponding value in the absence of insider trading. This half-life measures the speed of convergence of the transaction price to the fundamental value and can be considered an alternative measure of market efficiency which indicates the actual speed with which private information is diffused in the market.

This half-life is fairly high (close to one year) when no insider operates in the asset market and it is much smaller when an insider enters it. In addition, consistently with results in Figure 2, we see that with a larger frequency of trading this half-life is smaller. Indeed, as the insider's trading intensity, β, is of order $\Delta^{1/2}$ the information content of order flow (more precisely, the signal-to-noise ratio in order flow) augments with the trading frequency. Also in line with results unveiled by Figure 2, we see that the half-life dramatically falls when θ augments. Thus, for the hourly frequency and the large volume of liquidity trading the half-life is 0.0469 (i.e. about 12 days) when $\theta = 0$ and it is 0.0163 (i.e. about 4 days) when $\theta = 1$. In brief, Figure 3 proposes even more compelling evidence of how important the attitude of the insider towards uncertainty is in determining the efficiency of the market.

4.3 Ambiguity/Risk-aversion and Earlier Resolution of Uncertainty

Corollary 2 and the numerical analysis in Section 4.2 show that the larger θ is, the more aggressive the insider's trading strategy and the more efficient the market for the risky asset are. Tallarini (2000) suggests how to rationalize this result, as he finds that the recursive risk-sensitive preferences described in Section 3 are characterized by a coefficient of relative risk-aversion which exceeds the inverse of the inter-temporal elasticity of substitution. Thanks to this property it can then be established that such preferences induce earlier resolution of uncertainty *vis-à-vis* the case of expected utility (see Kreps and Porteus (1978) and Epstein and Zin (1989)).

This implies that when θ is larger the risk-averse insider considered in Section 3 is willing to accept smaller expected profits in order to reduce her uncertainty. In fact, within Chau and Vayanos' model the insider faces a trade-off between the expected value of her current profits and the variability of her future ones. Then, when risk-averse, she is actually willing to forgo part of her expected current profits to reduce their future variability, consistently with the prescription that an agent endowed with the recursive risk-sensitive preferences introduced in Section 3 prefers early resolution of uncertainty.

Because of Proposition 2 this explanation of the behavior of the risk-averse insider extends to the formulation with an ambiguity-averse insider. Indeed, we conclude that in a dynamic context an ambiguity-averse insider also favors early resolution of uncertainty and trades in order to make the environment in which she operates less uncertain. In general, we argue that when either risk- or ambiguity-averse the insider will be willing to trade more aggressively than she would if she were simply maximizing her expected profits. Consequently, she will reveal more information to the market maker, as this will make the prices at which she will trade less volatile and her profits less uncertain. Since the variability of such profits increases with the volume of liquidity trading, when σ_l^2 is larger the impact on ambiguity/risk-aversion on the insider's trading strategy is even stronger.

These conclusions are analogous to those derived by Holden and Subrahmanyam (1994) in their analysis of the impact of risk-aversion within Kyle's sequential auction model (Kyle, 1985), as they find that a risk-averse insider reveals her private information at a faster pace than her risk-neutral counterpart. Indeed, even within Kyle's model the insider finds it convenient to sacrifice part of her trading profits in order to reduce their variability.

Our analysis is however not a mere replica of Holden and Subrahmanyam's, in that in the analytical framework we employ, differently from Kyle's, there is an infinite horizon, future payoffs are discounted and the risky asset's fundamental value is subject to stochastic shocks. Then, because of time-discounting, we cannot adopt the CARA utility function Holden and Subrahmanyam employ to introduce risk-aversion in Chau and Vayanos' framework. In fact, if we were to combine the CARA utility function with time-discounting the optimal trading strategy of the insider would fail to be time-invariant.[10] Consequently, in our analysis we rely on the recursive preferences described in Section 3.

The impact of ambiguity-aversion on traders' behavior we unveil contrasts with the *portfolio inertia* typically exhibited in models of asset markets with ambiguity-averse investors (Dow and Ribeiro da Costa Werlang, 1992; Caskey, 2009; Condie and Ganguli, 2011, 2014; Ozsoylev and Werner, 2011; Easley, O'Hara, and Yang, 2014; Mele and Sangiorgi, 2015). In these models ambiguity-averse investors fail to trade if prices are not sufficiently favorable as to overcome their Knightian uncertainty on their payoffs. As these traders are less aggressive *vis-à-vis* their expected-utility maximizer counterparts, ambiguity-aversion reduces price informativeness and market efficiency.

[10]See Whittle (1990; Chapter 18.1) and Hansen and Sargent (1994) on the interplay between the exponential transformation of the CARA utility function and time-discounting.

Such disparity stems from several facets of our analysis which are novel with respect to the existing literature on ambiguity-aversion in asset markets. In particular, according to our formulation: i) Knightian uncertainty pertains to the pricing process and not to fundamentals or to signals received on such fundamentals; ii) the ambiguity-averse trader acts strategically; and iii) she solves a dynamic rather than a static optimization exercise, so that she considers the future implications of her trading decisions.

4.4 Trading Volume, Trading Frequency and Market Quality

An implication of our analysis pertains to the access of unsophisticated traders to securities markets. Retail traders and institutional investors, such as pension funds, insurance companies and mutual funds, represent the bulk of this class of traders in securities markets as they trade securities for liquidity, hedging and diversification motives. As they do not possess privileged information on the fundamentals of the securities they trade, their transactions do not present any information content. However, they contribute to the liquidity of securities markets and according to our analysis increase their efficiency. Indeed, in Sections 4.1 and 4.2 we have seen that as the volume of liquidity trading increases so does market efficiency. Consequently, technological innovations and normative changes in securities markets which facilitate market participation of unsophisticated traders and increase trading volume should benefit market quality and ought to be favored by regulators and exchanges.

A second implication of our analysis is that technological innovations which increase the pace of the market have important effects on the quality of securities markets. Indeed, from the analysis of Figures 1 to 3 we have concluded that as the frequency of trading increases the market for the risky asset becomes more efficient and less liquid, as the insider finds it optimal to trade at a faster speed. In fact, as with a larger trading frequency the signal-to-noise ratio in order flow is larger, an increase in the pace of the market presents opposite effects on liquidity and efficiency. On the one hand, more severe adverse-selection induces the market maker to increase λ_x, so that transaction costs augment and liquidity deteriorates; on the other hand, as more information is conveyed through the trading process, market efficiency improves.

Such findings contribute to the recent debate among regulators, practitioners and researchers on the impact of high frequency trading on liquidity, efficiency and other features of securities markets. Interestingly, several empirical studies on the impact of high frequency traders in securities markets (notably Carrion (2013), Hasbrouck and Saar (2013), Brogaard *et al.* (2014))

find that they tend to improve market efficiency, consistently with the implications of our analysis. However, these studies also find that, differently from the conclusions of our analysis, high frequency traders also benefit market liquidity.

Concluding Remarks

We have studied the trading strategy of an agent who possesses some private information on the fundamental value of a risky asset but who is also uncertain about the beliefs of a market maker who sets the corresponding transaction price. As she cannot determine the exact probability distribution of the profits her trades will generate, the insider faces Knightian uncertainty and selects a robust trading strategy. In this way we have investigated the impact of ambiguity-aversion on her trading strategy and on market quality.

The results of our analysis can be summarized as follows:

- The robust trading strategy of an ambiguity-averse trader is identified via a max-min choice mechanism, according to which she selects as her market orders those which maximize her expected profits against those market maker's beliefs which penalize her most.
- This robust trading strategy is equivalent to that of an insider who does not face any Knightian uncertainty and who is endowed with risk-sensitive recursive preferences.
- When trading in the market for the risky asset is continuous, the intensity of trading on the part of the insider is increasing in her degree of ambiguity-aversion. This implies that when the insider is more ambiguity-aversion, the market for the risky asset is more efficient.
- The impact of ambiguity-aversion on the insider's behavior and on market quality is exacerbated with a larger volume of liquidity trading in the market for the risky asset.
- The same conclusions hold when trading takes place at equally-spaced-in-time auctions.
- When the frequency of trading augments the market for the risky asset becomes more efficient but less liquid as the insider trades at a faster pace.

These results, which contrast with the *portfolio inertia* typically exhibited in models of asset markets with ambiguity-averse investors, are the consequence of the earlier resolution of uncertainty favored by ambiguity/risk-averse agents.

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Appendix

- **Conditional Entropy of Approximating and Distorted Specifications**.

Under the approximating specification in n $z_n \mid z_{n-1} \sim N(\mu_n, \sigma_\epsilon^2)$, with $\mu_n = (1-(\kappa+\lambda_d\nu)\Delta)z_{n-1} - \lambda_x x_n$, as x_n is known to be function of z_{n-1}, while under the distorted one $z_n \mid z_{n-1} \sim N(\mu_n+\sigma_\epsilon w_n, \sigma_\epsilon^2)$. Then, the *relative entropy* $I(f_a, f_d)(z_{n-1})$ is equal to $\frac{1}{2}w_n^2$. In fact,

$$f_a(z_n \mid z_{n-1}) \;=\; \frac{1}{\sqrt{\pi}\sigma_\epsilon}\exp\left[-\frac{1}{2}\left(\frac{z_n-\mu_n}{\sigma_\epsilon}\right)^2\right] \quad \text{and} \quad f_d(z_n \mid z_{n-1}) \;=\; \frac{1}{\sqrt{\pi}\sigma_\epsilon}\exp\left[-\frac{1}{2}\left(\frac{z_n-\mu_n-\sigma_\epsilon w_n}{\sigma_\epsilon}\right)^2\right] .$$

Thus,

$$\frac{f_d(z_n \mid z_{n-1})}{f_a(z_n \mid z_{n-1})} \;=\; \exp\left[\frac{1}{2}\frac{1}{\sigma_\epsilon^2}\Big((z_n-\mu_n)^2 \;-\; \Big((z_n-\mu_n-\sigma_\epsilon w)^2\Big)\Big)\right] .$$

Since

$$(z_n-\mu_n)^2 \;=\; (z_n-\mu_n-\sigma_\epsilon w_n)^2 \;+\; \sigma_\epsilon^2 w_n^2 \;+\; 2\sigma_\epsilon w_n(z_n-\mu_n-\sigma_\epsilon w_n)\,,$$

$$\frac{f_d(z_n \mid z_{n-1})}{f_a(z_n \mid z_{n-1})} \;=\; \exp\left[\frac{1}{2}w_n^2 \;+\; \frac{w_n}{\sigma_\epsilon}(z_n-\mu_n-\sigma_\epsilon w_n)\right] .$$

Then,

$$\log\left(\frac{f_d(z_n \mid z_{n-1})}{f_a(z_n \mid z_{n-1})}\right) \;=\; \frac{1}{2}w_n^2 \;+\; \frac{w_n}{\sigma_\epsilon}(z_n-\mu_n-\sigma_\epsilon w_n)\,,$$

so that

$$\begin{aligned} I(f_a, f_d)(z_{n-1}) \;&=\; \int \log\left(\frac{f_d(z_n \mid z_{n-1})}{f_a(z_n \mid z_{n-1})}\right)\, f_d(z_n \mid z_{n-1})dz_n \\ &=\; \int \frac{1}{2}w_n^2\, f_d(z_n \mid z_{n-1})dz_n \;+\; \frac{w_n}{\sigma_\epsilon}\int (z_n-\mu_n-\sigma_\epsilon w_n)\, f_d(z_n \mid z_{n-1})dz_n \;=\; \frac{1}{2}w_n^2\,. \end{aligned}$$

- **Proof of Lemma 1**.

Let us solve the modified Bellman equation (2.1). To simplify the algebra we rewrite it as follows

$$B(g_{n-1}-\hat{g}_{n-1})^2 \;=\; \max_{x_n}\min_{\xi_n}[\,\pi_n^* \;+\; \frac{1}{\theta}\,\frac{1}{\sigma_\epsilon^2}\xi_n^2 \;+\; e^{-r\Delta}\; B(g_n-\hat{g}_n)^2\,] \quad \text{with} \quad \xi_n \equiv \sigma_\epsilon\, w_n\,.$$

Under the assumption that $g_n - \hat{g}_n$ respects equation (1.6), taking the derivative of the expression in brackets with respect to ξ_n we have that as $1 + e^{-r\Delta}\theta\sigma_\epsilon^2 B > 0$,

$$\begin{aligned} \xi_n^{\min} &= \psi_g(g_{n-1} - \hat{g}_{n-1}) \;+\; \psi_x x_n \,, \\ \text{where} \qquad \psi_g &= -\,e^{-r\Delta}\left(1 - (\kappa + \nu\lambda_d)\Delta\right) \frac{\theta\sigma_\epsilon^2}{1 + e^{-r\Delta}\theta\sigma_\epsilon^2 B}\, B \,, \\ \psi_x &= -\frac{1}{2}(1 - 2e^{-r\Delta}\,\lambda_x B) \frac{\theta\sigma_\epsilon^2}{1 + e^{-r\Delta}\theta\sigma_\epsilon^2 B} \,. \end{aligned}$$

Inserting into the expression in brackets that for $\xi_n^{\min}$ and maximizing with respect to x_n we find, after some tedious algebra, that if $4\lambda_x(1 - e^{-r\Delta}\lambda B) + \theta\sigma_\epsilon^2 > 0$, a minimum is reached for $x_n = \beta(g_n - \hat{g}_n)$, with

$$\beta \;=\; \frac{\big(1 - (\kappa + \nu\lambda_d)\Delta\big)(1 - 2e^{-r\Delta}\lambda_x B)}{2\lambda_x(1 - e^{-r\Delta}\lambda_x B) \;+\; \frac{1}{2}\theta\sigma_\epsilon^2} \,.$$

Plugging this expression back into into that in brackets, after some long but straightforward algebra, we find that $\max_{x_n} \min_{\xi_n} [\pi_n^* + \frac{1}{\theta}\frac{1}{\sigma_\epsilon^2}\xi_n^2 \;+\; e^{-r\Delta}\; B(g_n - \hat{g}_n)^2] = B(g_{n-1} - \hat{g}_{n-1})^2$, where

$$\begin{aligned} B \;\;=\;\; & \frac{(1 - 2e^{-r\Delta}\lambda_x B)\big(1 - (\kappa + \nu\lambda_d)\Delta\big)^2 \lambda_x}{4[(1 - e^{-r\Delta}\lambda_x B)\lambda_x + \frac{1}{4}\theta\sigma_\epsilon^2]^2} \;+\; e^{-r\Delta}\, \frac{\big(1 - (\kappa + \nu\lambda_d)\Delta\big)^2 \lambda_x^2}{4[(1 - e^{-r\Delta}\lambda_x B)\lambda_x + \frac{1}{4}\theta\sigma_\epsilon^2]^2}\, B \\ & \;+\; \frac{\big(1 - (\kappa + \nu\lambda_d)\Delta\big)^2}{16[(1 - e^{-r\Delta}\lambda_x B)\lambda_x + \frac{1}{4}\theta\sigma_\epsilon^2]^2}\, \theta\sigma_\epsilon^2 \,. \end{aligned}$$

Rearranging this expression it is found that B solves the cubic equation

$$\left[\left(1 + \frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2\right) \;-\; e^{-r\Delta}\lambda_x B\right] \left(4e^{-r\Delta}\lambda_x^2\, B^2 \;-\; 4\lambda_x\, B\,\left(1 + \frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2\right) \;+\; \big(1 - (\kappa + \nu\lambda_d)\Delta\big)^2\right) \;=\; 0\,.$$

This possesses three roots. However, $\left(1 + \frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2\right) \;-\; e^{-r\Delta}\lambda_x B = 0$ entails that $4\lambda_x(1 - e^{-r\Delta}\lambda_x B) + \theta\sigma_\epsilon^2 = 0$ and hence it violates the second order condition of the maximization with respect to x_n. The second part of this equation presents two roots,

$$B_\pm \;\;=\;\; \frac{1}{2\lambda_x}\frac{1}{e^{-r\Delta}} \left(1 \;+\; \frac{1}{4\lambda_x}\theta\sigma_\epsilon^2 \;\pm\; \left[\left(1 \;+\; \frac{1}{4\lambda_x}\theta\sigma_\epsilon^2\right)^2 - e^{-r\Delta}\big(1 - (\kappa + \nu\lambda_d)\Delta\big)^2\right]^{1/2}\right) \,.$$

Now consider that

$$\beta\lambda_x \;=\; \frac{\big(1 - (\kappa + \nu\lambda_d)\Delta\big)(1 - 2e^{-r\Delta}\lambda_x B)}{2(1 - e^{-r\Delta}\lambda_x B) \;+\; \frac{1}{2}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2} \,.$$

Since, given the expression for λ_d in equation (2.5)

$$1 - (\kappa + \nu\lambda_d)\Delta \;=\; (1 - \kappa\Delta)\, \frac{\Sigma_g\beta^2\sigma_d^2 \;+\; \sigma_d^2\,\sigma_l^2\Delta}{\Sigma_g^2(\beta^2\sigma_d^2 \;+\; \nu^2\,\sigma_l^2\Delta^2) \;+\; \sigma_d^2\,\sigma_l^2\Delta} \;>\; 0\,,$$

the condition $\beta\lambda_x > 0$ implied by equation (2.6) is equivalent to

$$\frac{(1 - 2e^{-r\Delta}\lambda_x B)}{2(1 - e^{-r\Delta}\lambda_x B) \, + \, \frac{1}{2}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2} \; > \; 0 \, .$$

It is immediate to check that for B_+ the numerator in this ratio is negative, while the denominator is positive, so that this constraint is violated. Instead, for B_- both numerator and denominator are positive and the constraint is satisfied. □

• Proof of Corollary 1.
For $\theta \downarrow 0$

$$\begin{aligned} \beta \quad &\to \quad \frac{\big(1 - (\kappa + \nu\lambda_d)\Delta\big)(1 - 2e^{-r\Delta}\lambda_x B)}{2\lambda_x(1 - e^{-r\Delta}\lambda_x B)} \, , \\ B \quad &\to \quad \frac{e^{r\Delta}}{2\lambda_x}\left(1 \, - \, \left[1 - e^{-r\Delta}\big(1 - (\kappa + \nu\lambda_d)\Delta\big)^2\right]^{1/2}\right) \, . \end{aligned}$$

As $\sigma_\epsilon^2 = (\lambda_d^2\sigma_d^2 + \sigma_g^2 + \lambda_x^2\sigma_u^2)\Delta$ these expressions correspond to those derived by Chau and Vayanos. □

• Proof of Lemma 2.
To prove this Lemma we first need to establish a preliminary result.

Lemma 3 *If $Q(\mathbf{x}, \boldsymbol{\epsilon})$ is a quadratic form in the vectors $\mathbf{x}$ and $\boldsymbol{\epsilon}$ which admits the saddle point value $\max_{\mathbf{x}} \min_{\boldsymbol{\epsilon}} Q(\mathbf{x}, \boldsymbol{\epsilon})$, then the following holds*

$$\min_{\mathbf{x}} \int \exp\left[-\frac{1}{2}\, Q(\mathbf{x}, \boldsymbol{\epsilon})\right] d\boldsymbol{\epsilon} \;\propto\; \exp\left[-\frac{1}{2}\, \max_{\mathbf{x}}\, \min_{\boldsymbol{\epsilon}}\, Q(\mathbf{x}, \boldsymbol{\epsilon})\right] \, .$$

Proof.
Consider the quadratic form $Q(\mathbf{x}, \boldsymbol{\epsilon})$ in the vectors $\mathbf{x}$ and $\boldsymbol{\epsilon}$, where

$$Q(\mathbf{x}, \boldsymbol{\epsilon}) \; = \; \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\epsilon} \end{pmatrix}' \begin{pmatrix} \mathbf{Q}_{\mathbf{x}\,\mathbf{x}} & \mathbf{Q}_{\mathbf{x}\,\boldsymbol{\epsilon}} \\ \mathbf{Q}_{\boldsymbol{\epsilon}\,\mathbf{x}} & \mathbf{Q}_{\boldsymbol{\epsilon}\,\boldsymbol{\epsilon}} \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\epsilon} \end{pmatrix} \, .$$

Assume Q admits a minimum in $\boldsymbol{\epsilon}$ in that $\mathbf{Q}_{\boldsymbol{\epsilon}\,\boldsymbol{\epsilon}}$ is positive definite. Then, the following holds

$$\int \exp\left[-\frac{1}{2}\, Q(\mathbf{x}, \boldsymbol{\epsilon})\right] d\,\boldsymbol{\epsilon} \quad \propto \quad \exp\left[-\frac{1}{2}\, \min_{\boldsymbol{\epsilon}}\, Q(\mathbf{x}, \boldsymbol{\epsilon})\right] \, . \tag{A.1}$$

In fact, for $\hat{\boldsymbol{\epsilon}}$ the vector $\boldsymbol{\epsilon}$ minimizing Q, we can write $Q(\mathbf{x}, \boldsymbol{\epsilon}) = Q(\mathbf{x}, \hat{\boldsymbol{\epsilon}}) + (\boldsymbol{\epsilon} - \hat{\boldsymbol{\epsilon}})'\mathbf{Q}_{\boldsymbol{\epsilon}\,\boldsymbol{\epsilon}}(\boldsymbol{\epsilon} - \hat{\boldsymbol{\epsilon}})$. Consider that as $\mathbf{Q}_{\boldsymbol{\epsilon}\,\boldsymbol{\epsilon}}$ is positive definite and invertible, the minimum of Q with respect to $\boldsymbol{\epsilon}$ is obtained for

$\hat{\boldsymbol{\epsilon}} = -\mathbf{Q}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}^{-1}\,\mathbf{Q}_{\boldsymbol{\epsilon}\mathbf{x}}\,\mathbf{x}$ and is equal to $Q(\mathbf{x}, \hat{\boldsymbol{\epsilon}}) = \mathbf{x}'[\mathbf{Q}_{\mathbf{x}\mathbf{x}} \,-\, \mathbf{Q}_{\mathbf{x}\boldsymbol{\epsilon}}\,\mathbf{Q}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}^{-1}\,\mathbf{Q}_{\boldsymbol{\epsilon}\mathbf{x}}]\mathbf{x}$. Thus,

$$\begin{aligned} Q(\mathbf{x}, \boldsymbol{\epsilon}) - Q(\mathbf{x}, \hat{\boldsymbol{\epsilon}}) \quad &= \quad \boldsymbol{\epsilon}'\,\mathbf{Q}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}\,\boldsymbol{\epsilon} \,+\, \boldsymbol{\epsilon}'\,\mathbf{Q}_{\boldsymbol{\epsilon}\mathbf{x}}\,\mathbf{x} \,+\, \mathbf{x}'\,\mathbf{Q}_{\mathbf{x}\boldsymbol{\epsilon}}\,\boldsymbol{\epsilon} \,+\, \mathbf{x}'\,\mathbf{Q}_{\mathbf{x}\boldsymbol{\epsilon}}\,\mathbf{Q}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}^{-1}\,\mathbf{Q}_{\boldsymbol{\epsilon}\mathbf{x}}\mathbf{x} \\ &= \quad \boldsymbol{\epsilon}'\,\mathbf{Q}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}\,\boldsymbol{\epsilon} \,-\, \boldsymbol{\epsilon}'\,\mathbf{Q}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}\,\hat{\boldsymbol{\epsilon}} \,-\, \hat{\boldsymbol{\epsilon}}'\,\mathbf{Q}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}\,\boldsymbol{\epsilon} \,+\, \hat{\boldsymbol{\epsilon}}'\,\mathbf{Q}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}\,\hat{\boldsymbol{\epsilon}} \\ &= \quad (\boldsymbol{\epsilon} \,-\, \hat{\boldsymbol{\epsilon}})'\,\mathbf{Q}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}\,(\boldsymbol{\epsilon} \,-\, \hat{\boldsymbol{\epsilon}})\,. \end{aligned}$$

As $Q(\mathbf{x}, \hat{\boldsymbol{\epsilon}}) = \min_{\boldsymbol{\epsilon}} Q(\mathbf{x}, \boldsymbol{\epsilon})$ is a constant in the integral in equation (A.1), we find that

$$\int \exp\left[-\frac{1}{2}\,Q(\mathbf{x}, \boldsymbol{\epsilon})\right] d\,\boldsymbol{\epsilon} \quad = \quad \exp\left[-\frac{1}{2}\,\min_{\boldsymbol{\epsilon}}\,Q(\mathbf{x}, \boldsymbol{\epsilon})\right] \times \int \exp[-\,\frac{1}{2}\,(\boldsymbol{\epsilon} - \hat{\boldsymbol{\epsilon}})'\mathbf{Q}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}(\boldsymbol{\epsilon} - \hat{\boldsymbol{\epsilon}})]\,d\,\boldsymbol{\epsilon}\,.$$

Therefore, the constant of proportionality in equation (A.1) is $\int \exp(\,-\,\frac{1}{2}\,\boldsymbol{\Delta}'\mathbf{Q}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}\boldsymbol{\Delta})\,d\,\boldsymbol{\Delta} = (2\pi)^{m/2}\text{det}(\mathbf{Q}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}})^{-1/2}$, where m is the dimension of $\boldsymbol{\epsilon}$, and hence it is independent of $\mathbf{x}$. Then, suppose that we solve the program $\min_{\mathbf{x}} \int \exp\left[-\,\frac{1}{2}\,Q(\mathbf{x}, \boldsymbol{\epsilon})\right]$. Assume that Q admits a saddle point with respect to $\boldsymbol{\epsilon}$ and $\mathbf{x}$, so that $\max_{\mathbf{x}}\,\min_{\boldsymbol{\epsilon}}\,Q(\mathbf{x}, \boldsymbol{\epsilon})$ exists. From equation (A.1)

$$\min_{\mathbf{x}} \int \exp\left[-\frac{1}{2}\,Q(\mathbf{x}, \boldsymbol{\epsilon})\right] d\,\boldsymbol{\epsilon} \quad \propto \quad \min_{\mathbf{x}}\,\exp\left[-\frac{1}{2}\,\min_{\boldsymbol{\epsilon}}\,Q(\mathbf{x}, \boldsymbol{\epsilon})\right] \;=\; \exp\left[-\frac{1}{2}\,\max_{\mathbf{x}}\,\min_{\boldsymbol{\epsilon}}\,Q(\mathbf{x}, \boldsymbol{\epsilon})\right]\,.\square$$

It is worth noting this result applies also when Q is a non-homogeneous quadratic form, which depends on $\mathbf{x}$ and $\boldsymbol{\epsilon}$, alongside a third vector $\mathbf{z}$, insofar it admits a saddle point $\max_{\mathbf{x}} \min_{\boldsymbol{\epsilon}} Q(\mathbf{x}, \boldsymbol{\epsilon}, \mathbf{z})$.

Then, let us introduce the *stress* function proposed by Vitale (2015).

Definition 1 *The (discounted) stress function in* n *is* $\boldsymbol{\mathcal{S}}_n \,\equiv\, c_n^* \,-\, \frac{1}{\rho^*}(\epsilon_n)^2/\sigma_\epsilon^2 \,+\, e^{-r\Delta}\,\boldsymbol{\mathcal{W}}_{n+1}$.

Proof of Lemma 2.
Let assume that in $n\,+\,1$ the optimization criterion $\boldsymbol{\mathcal{W}}_{n+1}$ is a quadratic form in $g_n \,-\, \hat{g}_n$, so that two positive constants, B_{n+1} and C_{n+1}, exist such that $\boldsymbol{\mathcal{W}}_{n+1} = -B_{n+1}(g_n - \hat{g}_n)^2 - C_{n+1}$. Because the exponential function is monotonic we have that $\exp(\frac{\rho^*}{2}\boldsymbol{\mathcal{W}}_n) = \min_{x_n} E_n\left[\exp\left(\frac{\rho^*}{2}(c_n^* + e^{-r\Delta}\,\boldsymbol{\mathcal{W}}_{n+1})\right)\right]$. Since i) $g_n \,-\, \hat{g}_n$ is linearly dependent on ϵ_n via equation (1.5), ii) $c_n^* \,=\, -(g_n \,-\, \hat{g}_n)x_n$ and iii) the optimization criterion $\boldsymbol{\mathcal{W}}_{n+1}$ is assumed to be a quadratic form in $g_n \,-\, \hat{g}_n$, then the distribution of $c_n^* \,+\, e^{-r\Delta}\,\boldsymbol{\mathcal{W}}_{n+1}$ depends on that of ϵ_n and hence, given that $\epsilon_n \sim N(0, \sigma_\epsilon^2)$,

$$\min_{x_n}\,E_n\left[\exp\left(\frac{\rho^*}{2}(c_n^* \,+\, e^{-r\Delta}\,\boldsymbol{\mathcal{W}}_{n+1})\right)\right] \quad = \quad \left(2\pi\sigma_\epsilon^2\right)^{-1/2}\,\min_{x_n} \int \exp\left(\rho^*\,\frac{\boldsymbol{\mathcal{S}}_n}{2}\right)\,d\epsilon_n\,.$$

Now, since $\boldsymbol{\mathcal{W}}_{n+1}$ is assumed to be a quadratic form in $g_n - \hat{g}_n$ and this is linear in ϵ_n, x_n and $g_{n-1} - \hat{g}_{n-1}$, $\boldsymbol{\mathcal{W}}_{n+1}$ can be expressed as a quadratic form in ϵ_n, x_n and $g_{n-1} \,-\, \hat{g}_{n-1}$. Similarly, c_n^* is a quadratic form in ϵ_n, x_n and $g_{n-1} \,-\, \hat{g}_{n-1}$ and so is $\boldsymbol{\mathcal{S}}_n$. Thus, if the stress in n admits a saddle point, in that $\min_{x_n}\,\max_{\epsilon_n} \boldsymbol{\mathcal{S}}_n$ exists, then $-\boldsymbol{\mathcal{S}}_n$ admits a saddle point in the statement of Lemma 3. Exploiting this

Lemma

$$
\begin{aligned}
\min_{x_n} \int \exp\left(\rho^* \frac{\boldsymbol{\mathcal{S}}_n}{2}\right) d\,\epsilon_n \quad &= \quad \min_{x_n} \int \exp\left(-\frac{1}{2}\underbrace{(-\rho^* \boldsymbol{\mathcal{S}}_n)}_{Q(x_n,\epsilon_n)}\right) d\epsilon_n \\
&= \quad K_n \exp\left(-\frac{1}{2}\max_{x_n}\min_{\epsilon_n}(-\rho^* \boldsymbol{\mathcal{S}}_n)\right) \;=\; K_n \exp\left(\frac{\rho^*}{2}\min_{x_n}\max_{\epsilon_n} \boldsymbol{\mathcal{S}}_n\right),
\end{aligned}
$$

where, using the result outlined in the proof of Lemma 3, we establish that $K_n = (2\pi/q_{\epsilon_n\epsilon_n})^{1/2}$ with $q_{\epsilon_n\epsilon_n}$ equal to the second derivative of $-\rho^* \boldsymbol{\mathcal{S}}_n$ with respect to ϵ_n. This implies that

$$
\min_{x_n} E_n\left[\exp\left(\frac{\rho^*}{2}(c_n^* + e^{-r\Delta}\,\boldsymbol{\mathcal{W}}_{n+1})\right)\right] = (\sigma_\epsilon^2 q_{\epsilon_n\epsilon_n})^{-1/2} \exp\!\left(\frac{\rho^*}{2}\min_{x_n}\max_{\epsilon_n} \boldsymbol{\mathcal{S}}_n\right).
$$

This implies that *extremizing* $\boldsymbol{\mathcal{S}}_n$, i.e. maximizing it with respect to ϵ_n and minimizing the resulting function with respect to x_n, we find that in period n: i) the saddle point pins down the optimal market order for the insider; ii) the *extremized* stress, equal to the saddle point value $\min_{x_n}\max_{\epsilon_n} \boldsymbol{\mathcal{S}}_n$, is a quadratic form in $g_{n-1} - \hat{g}_{n-1}$, $-B_n(g_{n-1} - \hat{g}_{n-1})^2 - e^{-r\Delta}C_{n+1}$, and iii) because $\exp(\frac{\rho^*}{2}\boldsymbol{\mathcal{W}}_n) = \min_{x_n} E_n\left[\exp\left(\frac{\rho^*}{2}(c_n^* + e^{-r\Delta}\,\boldsymbol{\mathcal{W}}_{n+1})\right)\right]$, the optimization criterion $\boldsymbol{\mathcal{W}}_n$ is a quadratic form in $g_{n-1} - \hat{g}_{n-1}$ equal to the *extremized* stress plus a constant independent of $g_{n-1} - \hat{g}_{n-1}$,

$$
\boldsymbol{\mathcal{W}}_n \;=\; -\gamma_n \;+\; \min_{x_n}\max_{\epsilon_n} \boldsymbol{\mathcal{S}}_n\,, \quad \text{where} \quad \gamma_n \;=\; \frac{1}{\rho^*}\ln(\sigma_\epsilon^2 q_{\epsilon_n\epsilon_n})\,.
$$

In general the saddle point for $\boldsymbol{\mathcal{S}}_{n+j}$ must be derived in all future dates ($n+1$, $n+2$, $\cdots$, $n+j$, $\cdots$) before it can be found in n to determine the optimal market order x_n. However, with a stationary trading strategy we simply need to find a fix point in the double recursion implied by the extermination of the stress. In fact, with a stationary strategy we see that $B_n = B_{n+1} = B$, $C_n = C_{n+1} = C$ and $\gamma_n = \gamma = \frac{1}{\rho^*}\ln(\sigma_\epsilon^2 q_{\epsilon_n\epsilon_n})$, with $q_{\epsilon_n\epsilon_n} = \frac{1}{\sigma_\epsilon^2} + e^{-r\Delta}\rho^* B$. Then, we have $\boldsymbol{\mathcal{W}}_n = -\gamma + \min_{x_n}\max_{\epsilon_n}\left\{c_n^* - \frac{1}{\rho^*}(\epsilon_n)^2/\sigma_\epsilon^2 - e^{-r\Delta}B(g_n - \hat{g}_n)^2 \;-\; e^{-r\Delta}C\right\}$, so that a stationary trading strategy implies that the following fixed points hold,

$$
-B(g_{n-1} - \hat{g}_{n-1})^2 \;=\; \min_{x_n}\max_{\epsilon_n}\left\{c_n^* \;-\; \frac{1}{\rho^*}\frac{\epsilon_n^2}{\sigma_\epsilon^2} \;-\; e^{-r\Delta}B(g_n - \hat{g}_n)^2\right\} \quad \text{and} \quad C \;=\; \gamma \;+\; e^{-r\Delta}C\,.\square
$$

• **Proof of Proposition 3.**
Let us introduce b and S_g such that $\beta = b\sqrt{\Delta}$ and $\Sigma_g = S_g\sqrt{\Delta}$. Chau and Vayanos show that for $\Delta \downarrow 0$ equation (2.7) converges to $S_g^2 b^2 = \sigma_g^2\sigma_l^2$. Then, consider that plugging equation (2.4) into (2.3) and

readjusting we find that

$$\frac{\beta\lambda_x}{(1-(\kappa+\nu\lambda_d)\Delta)} \;=\; \frac{\left[\left(1+\frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2\right)^2 \;-\; e^{-r\Delta}\big(1-(\kappa+\nu\lambda_d)\Delta\big)^2\right]^{1/2} \;-\; \frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2}{1 \;+\; \left[\left(1+\frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2\right)^2 \;-\; e^{-r\Delta}\big(1-(\kappa+\nu\lambda_d)\Delta\big)^2\right]^{1/2} \;+\; \frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2}$$

Plugging equations (2.5) and (2.6) into the left hand side of this equation we find that this is equal to $\beta^2\Sigma_g/(\beta^2\Sigma_g+\sigma_l^2\Delta)$, so an equilibrium value for β is found when it solves the following equation

$$\frac{\beta^2\Sigma_g}{\beta^2\Sigma_g+\sigma_l^2\Delta} \;=\; \frac{\left[\left(1+\frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2\right)^2 \;-\; e^{-r\Delta}\big(1-(\kappa+\nu\lambda_d)\Delta\big)^2\right]^{1/2} \;-\; \frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2}{1 \;+\; \left[\left(1+\frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2\right)^2 \;-\; e^{-r\Delta}\big(1-(\kappa+\nu\lambda_d)\Delta\big)^2\right]^{1/2} \;+\; \frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2}\,. \tag{A.2}$$

Now, (A.2) can also be written as follows

$$\frac{S_g b^2}{S_g b^2\sqrt{\Delta}+\sigma_d^2} \;=\; \frac{\left[\frac{1}{\Delta}\left(1+\frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2\right)^2 \;-\; \frac{1}{\Delta}e^{-r\Delta}\big(1-(\kappa+\nu\lambda_d)\Delta\big)^2\right]^{1/2} \;-\; \frac{1}{4}\frac{1}{\sqrt{\Delta}}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2}{1 \;+\; \left[\left(1+\frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2\right)^2 \;-\; e^{-r\Delta}\big(1-(\kappa+\nu\lambda_d)\Delta\big)^2\right]^{1/2} \;+\; \frac{1}{4}\frac{1}{\lambda_x}\theta\sigma_\epsilon^2}\,. \tag{A.3}$$

Using equations (2.5) and (2.6) we can write

$$1-(\kappa+\nu\lambda_d)\Delta \;=\; (1-\kappa\Delta)\,\frac{\Sigma_g\beta^2\sigma_l^2 \;+\; \sigma_d^2\sigma_l^2\Delta}{\Sigma_g(\beta^2\sigma_l^2 \;+\; \nu^2\sigma_l^2\Delta^2) \;+\; \sigma_d^2\sigma_l^2\Delta}\,. \tag{A.4}$$

Inspection of this equation allows to write $1-(\kappa+\nu\lambda_d)\Delta = (1-\kappa\Delta)(1+o(\Delta^{3/2}))$ where $o(\Delta^{3/2})$ denotes a function of Δ of order 3/2. Similarly, considering the definition of σ_ϵ^2 and equation (2.6), we can write $\frac{1}{\lambda_x}\sigma_\epsilon^2 = (\lambda_x\sigma_l^2+\frac{1}{\lambda_x}\sigma_g^2)\Delta+o(\Delta^{3/2})$. Exploiting these expressions we can write the right hand side of equation (A.3) as follows

$$\frac{\left[\frac{1}{\Delta}\left(1 \;-\; e^{-r\Delta} \;+\; 2\kappa\Delta e^{-r\Delta} \;+\; \frac{1}{2}\theta\big(\lambda_x\sigma_l^2+\frac{1}{\lambda_x}\sigma_g^2\big)\Delta \;+\; o(\Delta^{3/2})\right)\right]^{1/2} \;-\; \frac{1}{4}\theta\big(\lambda_x\sigma_l^2 \;+\; \frac{1}{\lambda_x}\sigma_g^2 \;+\; o(\Delta^{1/2})\big)\sqrt{\Delta}}{1 \;+\; \left[\left(1-\; e^{-r\Delta}+2\kappa\Delta e^{-r\Delta}+\frac{1}{2}\theta\big(\lambda_x\sigma_l^2+\frac{1}{\lambda_x}\sigma_g^2\big)\Delta \;+\; o(\Delta^{3/2})\right)\right]^{1/2} \;+\; \frac{1}{4}\theta\big(\lambda_x\sigma_l^2+\frac{1}{\lambda_x}\sigma_g^2+o(\Delta^{1/2})\big)\Delta}\,.$$

Exploiting the Hôpital's rule and considering that for $\Delta\downarrow 0$ λ_x converges to $\frac{S_g b}{\sigma_l^2}$, we find that, the right hand side of equation (A.3) converges to $2\kappa+r+\frac{1}{2}\theta\frac{1}{S_g b}(S_g^2b^2+\sigma_g^2\sigma_l^2)$, while the left hand side converges to $\frac{S_g b^2}{\sigma_d^2}$. In brief, we find that for $\Delta\downarrow 0$ equations (2.7) and (A.2) converge to the following

$$\begin{aligned} S_g^2b^2 &= \sigma_g^2\sigma_l^2\,, \\ \frac{S_g b^2}{\sigma_d^2} &= 2\kappa \;+\; r \;+\; \frac{1}{2}\,\theta\frac{1}{S_g b}(S_g^2b^2 \;+\; \sigma_g^2\sigma_l^2)\,. \end{aligned}$$

From these equations we see that

$$\lim_{\Delta\downarrow 0} b \;=\; \Big(2\kappa \,+\, r \,+\, \theta\sigma_l\sigma_g\Big)^{1/2}\,\frac{\sigma_l}{\sigma_g}\,, \quad \lim_{\Delta\downarrow 0} S_g \;=\; \frac{1}{\Big(2\kappa \,+\, r \,+\, \theta\sigma_l\sigma_g\Big)^{1/2}}\,\sigma_g^2\,.$$

In addition, $\lim_{\Delta\downarrow 0}\lambda_x = \lim_{\Delta\downarrow 0}\frac{S_g b}{\sigma_l^2} = \frac{\sigma_g}{\sigma_l}$. As for λ_d, notice that it can be written as

$$\lambda_d \;=\; \frac{(1-\kappa\Delta)\,S_g\,\nu\sigma_l^2\sqrt{\Delta}}{S_g\sqrt{\Delta}\,(b^2\sigma_d^2+\nu^2\sigma_l^2\Delta)+\sigma_d^2\sigma_l^2}\,,$$

so that $\lim_{\Delta\downarrow 0}\frac{\lambda_d}{\sqrt{\Delta}} \;=\; \lim_{\Delta\downarrow 0}\frac{S_g\nu}{\sigma_d^2} \;=\; \nu\,\frac{\sigma_g^2}{\sigma_d^2}\Big(2\kappa \,+\, r \,+\, \theta\sigma_l\sigma_g\Big)^{-1/2}$. In addition, since $\lim_{\Delta\downarrow 0}\frac{1}{\lambda_x}\sigma_\epsilon \;=\; 0$, $\lim_{\Delta\downarrow 0} 1-(\kappa+\nu\lambda_d)\Delta = 1$ and $\lim_{\Delta\downarrow 0} e^{r\Delta} = 1$, it is found that $\lim_{\Delta\downarrow 0} B = \lim_{\Delta\downarrow 0}\frac{1}{2}\frac{1}{\lambda_x} = \frac{1}{2}\frac{\sigma_l}{\sigma_g}$.

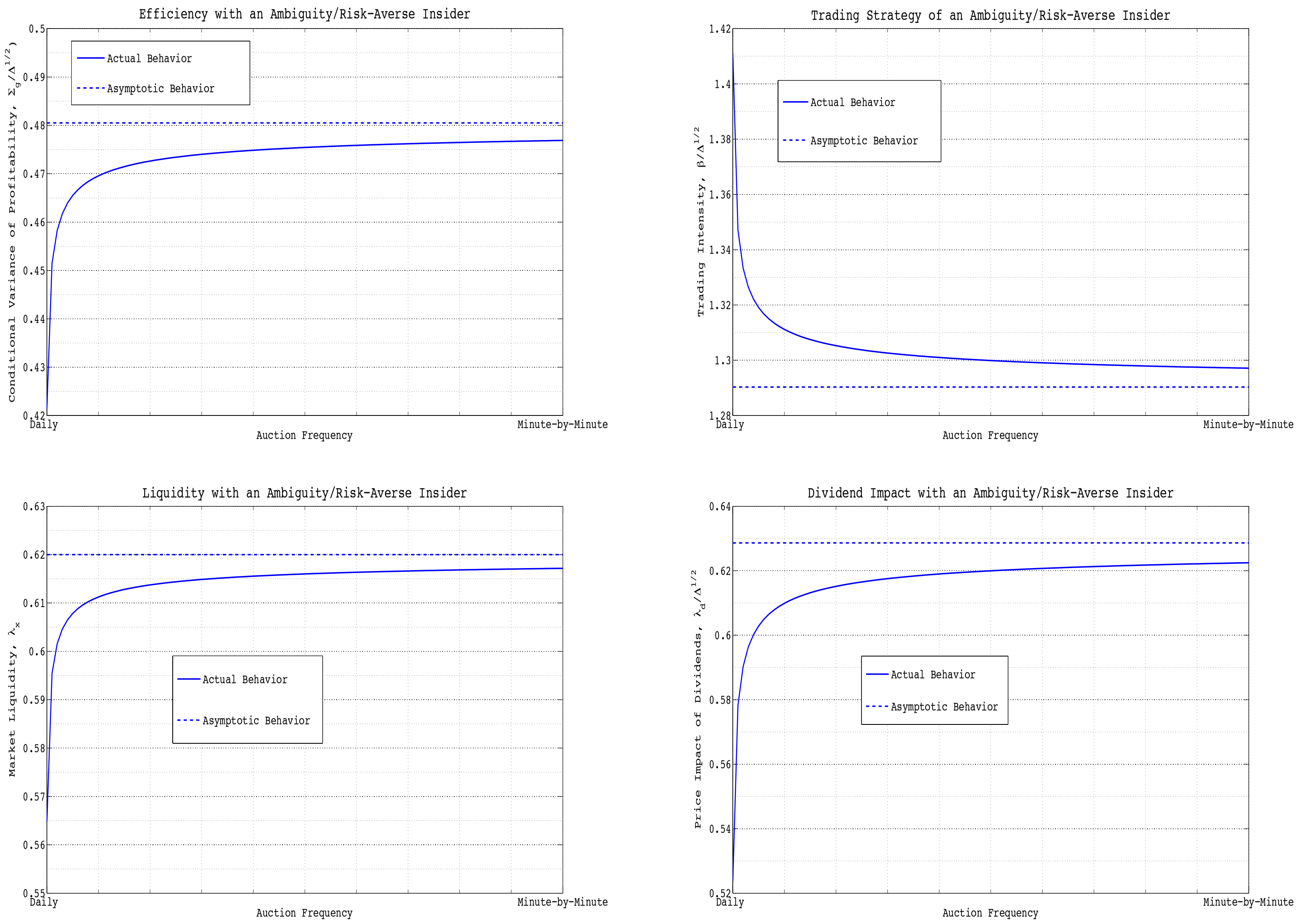

Figure 1: The Convergence to asymptotic values for the conditional variance of the underlying profitability ($\Sigma_g/\sqrt{\Delta}$; top, left panel), the insider's trading intensity ($\beta/\sqrt{\Delta}$; top, right panel), the liquidity coefficient (λ_x; bottom, left panel) and the impact of the dividend yield ($\lambda_d/\sqrt{\Delta}$; bottom, right panel) for $\sigma_l = 1$, $\sigma_g = 0.62$, $\sigma_d = 1.06$, $\nu = 1.47$, $\kappa = 0$, $r = 0.02$ and $\theta = 1$ ($\rho^* = 1$).

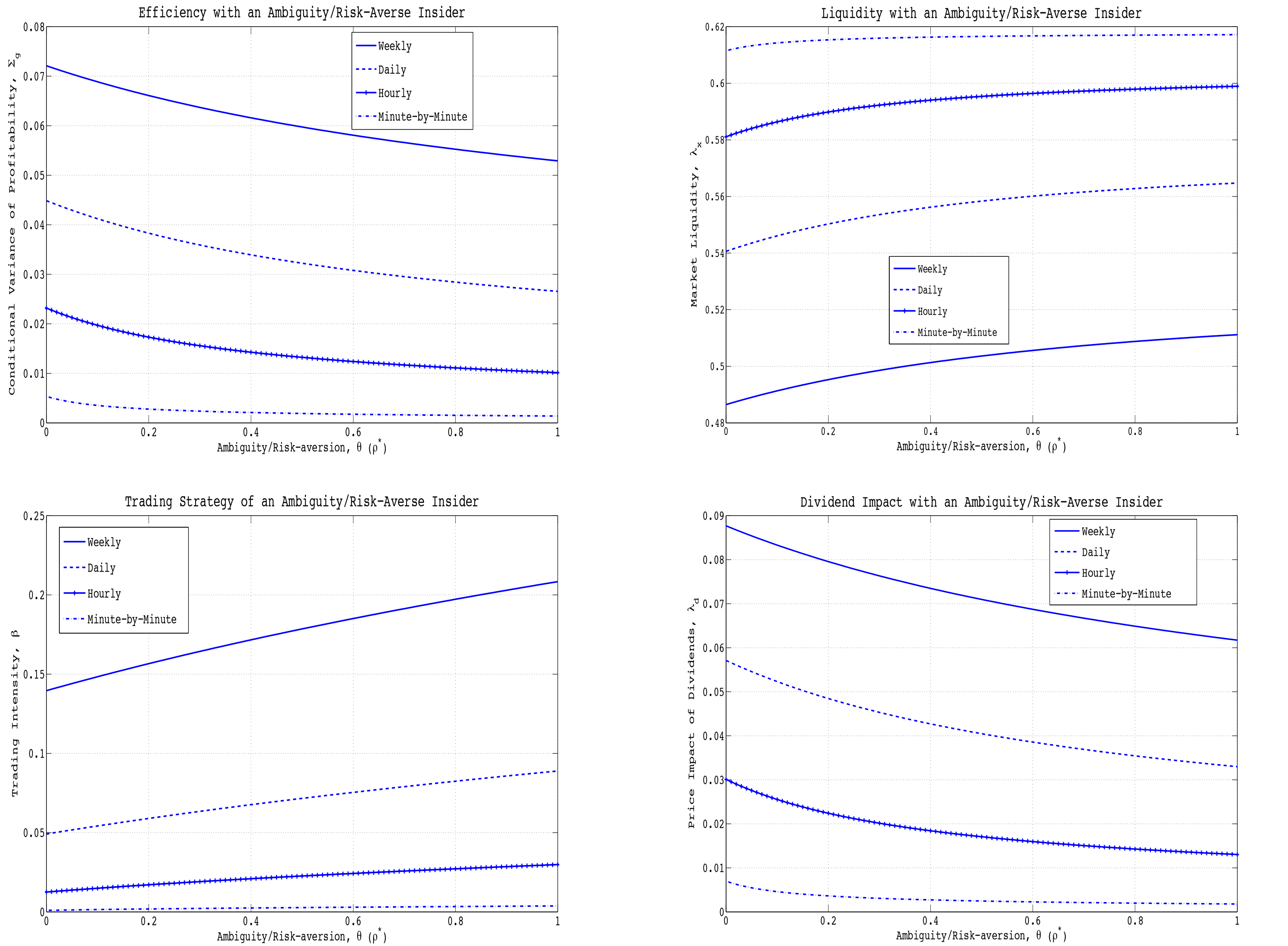

Figure 2: The dependence of the conditional variance of the underlying profitability (Σ_g; top, left panel), the insider's trading intensity (β; bottom, left panel), the liquidity coefficient (λ_x; top, right panel) and the impact of the dividend yield (λ_d; bottom, right panel) on the ambiguity/risk-aversion coefficient, θ (ρ^*), for $\sigma_l = 1$, $\sigma_g = 0.62$, $\sigma_d = 1.06$, $\nu = 1.47$, $\kappa = 0$, $r = 0.02$ and four different trading frequencies.

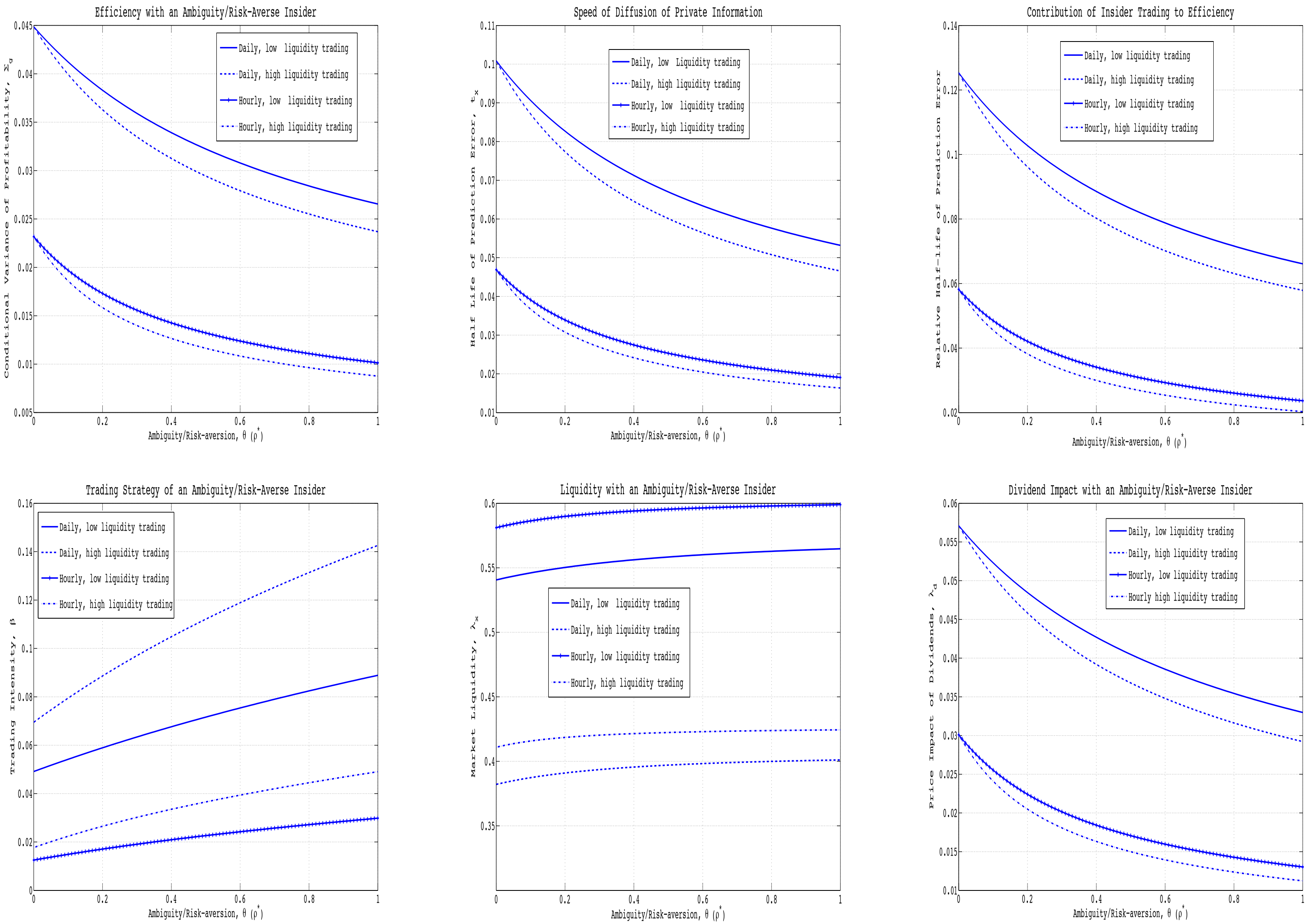

Figure 3: The dependence of the conditional variance of underlying profitability (Σ_g; top, left panel), the half-life of prediction error of the market maker ($t_{0.5}$; top, middle panel), the half-life of the prediction error of the market maker relative to the benchmark scenario with no insider ($t_{0.5}/t_{0.5,0}$; top, right panel), the insider's trading intensity (β; bottom, left panel), the liquidity coefficient (λ_x; bottom, middle panel) and the impact of the dividend yield (λ_d; bottom, right panel) on the ambiguity/risk-aversion coefficient, θ (ρ^*), for $\sigma_g = 0.62$, $\sigma_d = 1.06$, $\nu = 1.47$, $\kappa = 0$, $r = 0.02$, two different trading frequencies ($\Delta = 1/252$ and $\Delta = 1/2016$) and two different volumes of liquidity trading ($\sigma_l^2 = 1$ and $\sigma_l^2 = 2$).