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Elasticity of substitution and the slowdown of Italian productivity

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Elasticity of substitution and the slowdown of Italian productivity*

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Abstract

The aim of this paper is to investigate the roots of the slowdown in the Italian total factor productivity (TFP). The analysis focusses on the specific pattern of technical progress in determining the dynamics of the TFP. This analysis can not be done with Cobb–Douglas technology but requires the employment of a CES function that allows distinguishing between the direction and the bias of technical progress. We employ a CES specification embodying both labor- and capital-augmenting technical change, with a \( \sigma \) less than 1. We obtain three main results. 1) There seems to have been a structural break around the mid-1990s in the direction and bias of technological change; 2) The first half of the sample features a labor-augmenting technical change and a capital bias; 3) In the second part of the sample, both these characteristics seem to disappear, and the evolution of factor endowments assumes a key role. This fact may be seen as one of the potential causes of the stagnation in Italian productivity.

**JEL classification**: C30; E 22; E23; O33.

**Keywords**: CES production function; Elasticity of substitution; Factor-augmenting technical progress and ICT technical change.

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1 Introduction

In this paper we build on the dynamic model of the Italian economy of Saltari et al. (2012, 2013). The main result of those papers is that the weakness of the Italian economy in the last two decades has been the total factor productivity (TFP) slowdown. The aim of this paper is to investigate the roots of this slowdown. The analysis focusses on the specific pattern of technical progress in determining the dynamics of TFP. Of course, this analysis can not be done with the Cobb-Douglas technology, where technical progress is only Hicks neutral, but requires a CES production function which allows distinguishing between the direction and the bias of technical progress.\footnote{In this paper we do not address the microeconomic aspects of the TFP slowdown. For a recent survey of the microeconomic aspects of technological bias and the correlated induced innovations, see Acemoglu (2014).}

Unlike most of the literature, this investigation employs a CES specification with both labor- and capital-augmenting technical change. While for labor input we keep the traditional constant growth rate representation, for capital we impose a particular structure with Information and Communication Technologies (ICT) capital playing a key role. Moreover, in this exercise we do not calibrate the parameters of the CES production function but use our estimated values. It should be noted that the estimated elasticity of substitution ($\sigma$) is less than 1. Such a value is by now well-grounded in the empirical literature (see for instance León-Ledesma 2010; for a critical discussion of the traditional methodology of estimating the elasticity of substitution, see Federici and Saltari 2014). On theoretical grounds, a $\sigma > 1$ implies that any amount of output can be produced with either zero amount of capital or zero amount of labor, which is clearly absurd (note that the Cobb-Douglas almost shares this last property). The data on the Italian economy refers to the period 1981:Q4–2005:Q2.\footnote{The dataset is available from the authors upon request.}

We obtain three main results. 1) There seems to have been a structural break in the direction and bias of technological change around the mid-1990s, i.e., at the mid-point of the sample; 2) The first half features a labor-augmenting technical change and a capital bias; 3) In the last part of the sample, both these characteristics seem to disappear, and the evolution of factor endowments assumes a key role. The disappearance of the contribution from technical progress may be viewed as one of the potential causes of the stagnation in Italian productivity.

This paper is organized as follows. The next section briefly recalls
our production function and normalizes it. Section 3 compares the Cobb–Douglas and CES computation of TFP; it also discusses the determinants of technological progress. Section 4 describes the evolution of the direction and factor bias. Section 5 concludes.

2 The technology

Our theoretical framework is one of dynamic disequilibrium with traditional and ICT (communication equipment, hardware and software) investment functions, skilled and unskilled labour sectors, and price determination under imperfect competition (for details, see Saltari et al. 2012).

The production technology is given by the following CES aggregate production function

\[ Y_t = \beta_3 \left[ (C_t^\gamma K_t) \right]^{-\beta_1} + \left( \beta_2 e^{\mu K_t^\gamma} L \right)^{-\beta_1} \]  

In Equation (1), \( \beta_3 \) is a measure of the TFP and \( \beta_1 \) defines the elasticity of substitution through the relation \( \sigma = \frac{1}{1+\beta_1} \). Moreover, we have two factor-augmenting technical progress. The efficiency of traditional capital is augmented by ICT capital, \( C \), with a weighting factor equal to \( \gamma \), a proxy of the relative share of the ICT in total capital. As for labor-augmenting technical progress, we follow the bulk of the literature in assuming that it grows at a constant rate \( \mu = \lambda_K + \gamma \lambda_C \), where \( \lambda_K \) and \( \lambda_C \) are the rates of technical progress in the use of capital \( K \) and innovative (i.e., ICT) capital, \( C \), with \( \beta_2 \) as a scaling factor. That way, labor efficiency partly depends on the growth of ICT capital through \( \gamma \lambda_C \). Thus, unlike most of the literature, labor efficiency is closely linked to capital efficiency. Finally, \( L \) denotes employment, defined as a Cobb-Douglas function of the skilled and unskilled labor components.

The model allows us to estimate, among other things, the parameters of the production function for the sample period 1981:Q4-2005:Q2. For the reader’s convenience, the estimates of the parameters of the production function are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \beta_1 )</th>
<th>( \sigma = \frac{1}{1+\beta_1} )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \gamma )</th>
<th>( \lambda_K )</th>
<th>( \lambda_C )</th>
<th>( \mu = \lambda_K + \gamma \lambda_C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.52</td>
<td>0.66</td>
<td>27.07</td>
<td>0.87</td>
<td>0.05</td>
<td>0.00134</td>
<td>0.0365</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Notice that \( \beta_3, \lambda_K, \lambda_C \) and thus \( \mu \) are all expressed on a quarterly basis. Thus, for instance the yearly growth rate of labor efficiency is about 1.2%.
2.1 Normalization

We normalize the production function so that the variables are independent of the unit of measure, i.e., are in index number form. Moreover, it defines specific ‘families’ of CES functions whose members all share the same base period but are distinguished by the elasticity of substitution (and only the elasticity of substitution). Finally, normalization is necessary for a number of reasons, such as securing the basic property of CES production (the strictly positive relation between the elasticity of substitution and the level of output; see Grandville 2009), and is useful to determine the direction and bias of technical progress (Acemoglu 2002).

We set the base period for the normalization at the middle of the sample, i.e., $t = 48$ corresponding to 1991:Q4, and denote it by the index 0. Normalization implies that all the variables are expressed in terms of their baseline values, i.e., $K_0, L_0$ and $Y_0$.

To normalize the production function, we start with our production function written as:

$$Y_t = \beta_3 \left[ (KIT_t)^{-\beta_1} + \left( \beta_2 e^{\mu (t-t_0)} L_t \right)^{-\beta_1} \right]^{-\frac{1}{\beta_1}}$$  \hspace{1cm} (2)

where $t_0$ is the base period used for normalization, and to simplify notation we set $KIT = C^\gamma K$.

Under imperfect competition, factor compensation is subject to a constant mark-up, denoted by $\beta_{13}$, so that in any period $t$ the following relation holds:

$$(i_t KIT_t + w_t L_t) \beta_{13} = Y_t$$

where $i_t$ is the real interest rate and $w_t$ is the wage rate.

In the reference period, capital compensation is:

$$i_0 = \frac{1}{\beta_{13}} \frac{\partial Y_0}{\partial KIT_0} = \frac{\beta_3 - \beta_1}{\beta_{13}} \left( \frac{Y_0}{KIT_0} \right)^{1+\beta_1}$$

so that total capital compensation over total factor income, or the capital share ($\pi_0$), in the base period is:

$$\pi_0 = \frac{i_0 KIT_0}{Y_0} \beta_{13} = (\beta_3)^{-\beta_1} \left( \frac{Y_0}{KIT_0} \right)^{\beta_1}$$  \hspace{1cm} (3)

Proceeding in the same way for the labor share and substituting in (2), we get the normalized production function:

$$Y_t = \left[ \pi_0 (KIT_t)^{-\beta_1} + (1 - \pi_0) LIT_t^{-\beta_1} \right]^{-\frac{1}{\beta_1}}$$  \hspace{1cm} (4)
where output, labor and capital are already expressed in index form, and 
\[ LIT = \left(e^{\mu (t-t_0)}L_t\right)^{-\beta_1} \]. In the normalized production function the only 
crucial parameter is \( \beta_1 \).

Of course, in the Cobb–Douglas case (where \( \beta_1 = 0 \)), the production 
function becomes:
\[ Y_t^{CD} = (K_{IT_t})^{\pi_0} (LIT_t)^{1-\pi_0}. \]

### 3 Technical progress

The rate of growth of output is determined by the time log derivative of 
Equation (4):

\[
\frac{\dot{Y}_t}{Y_t} = \varepsilon_{Y,KIT} \left( \frac{\dot{K}_t}{K_t} + \gamma \frac{\dot{C}_t}{C_t} \right) + \varepsilon_{Y,LIT} \left( \frac{\dot{L}_t}{L_t} + \mu \right)
\]

\[
= \pi_0 \left( \frac{Y_t}{K_{IT_t}} \right)^{\beta_1} \left( \frac{\dot{K}_t}{K_t} + \gamma \frac{\dot{C}_t}{C_t} \right) + (1 - \pi_0) \left( \frac{Y_t}{L_{IT_t}} \right)^{\beta_1} \left( \frac{\dot{L}_t}{L_t} + \mu \right)
\]

where \( \varepsilon_{Y,KIT} = \frac{\partial Y}{\partial K_{IT}}/K_{IT} \) and \( \varepsilon_{Y,LIT} = \frac{\partial Y}{\partial L_{IT}}/L_{IT} \) are the elasticities of 
output with respect to inputs in efficiency units. In this framework, the 
capital-augmenting technical change is \( \pi_0 \left( \frac{Y_t}{K_{IT_t}} \right)^{\beta_1} \gamma \frac{\dot{C}_t}{C_t} \), while the labor-
augmenting factor is \( (1 - \pi_0) \left( \frac{Y_t}{L_{IT_t}} \right)^{1+\beta_1} \mu \). Intuitively, the contribution 
of each input-augmenting factor to the rate of growth of output can be split into 
two components: one is the pure technical progress \( \left( \gamma \frac{\dot{C}_t}{C_t}, \mu \right) \); the other is the 
sensitivity of output to technical change \( (\pi_0 \left( \frac{Y_t}{K_{IT_t}} \right)^{\beta_1}, (1 - \pi_0) \left( \frac{Y_t}{L_{IT_t}} \right)^{\beta_1}) \).

In the Cobb–Douglas case, \( \beta_1 = 0 \), and the elasticities are simply the income 
shares.

It is worth noticing that, unlike the traditional specification, capital-
augmenting technical progress depends on the dynamics of the stock of ICT 
capital. This choice of capital-augmenting technical progress is motivated 
by the key role played by ICT in the dynamics of productivity in 
industrialized countries at least since the 1990s. The relevance of ICT is particularly 
important for Italy (although in a negative sense). However, by the impossibility 
theorem of Diamond et al. (1978), we cannot separately identify this 
role from that of the elasticity of substitution unless one imposes a specific 
structure on technical change. In defining this structure, we abandon the 
traditional specification of a constant rate of growth of technical progress.
In particular, our model assumes that the efficiency of the traditional capital stock is augmented by ICT capital according to a weighting factor equal to $\gamma$. Since labour-augmenting is defined as $\mu = \lambda K + \gamma \lambda C$, the same factor also increases labour efficiency. This way, we are assuming that ICT investment also improves labour productivity. As far as we know, this specification of technical progress was first introduced in the Kaldor (1957) growth model.\(^3\)

### 4 The advantage of using a CES production function

The contribution of technical progress to the growth of output (the result of growth accounting exercises) is computed using the Cobb–Douglas production function through the Solow residual. To see the relevance of the elasticity of substitution, let us compare the computation of TFP using the Cobb–Douglas production function with that obtained with the CES. To this end, we calibrate Equation (5) with the three key parameter estimates shown in Table 1 ($\sigma, \gamma, \mu$):\(^4\)

$$TFP_{CES} = \frac{\dot{Y}_t}{Y_t} - \left( \varepsilon_{Y,KIT} \frac{\dot{K}_t}{K_t} + \varepsilon_{Y,LIT} \frac{\dot{L}_t}{L_t} \right)$$  \hfill (6)

\(^3\)Kaldor is explicit in affirming that one specific characteristic of his growth model is that “... it eschews any distinction between changes in techniques (and in productivity) which are induced by changes in the supply of capital relative to labour and those induced by technical invention or innovation—i.e., the introduction of new knowledge. The use of more capital per worker (whether measured in terms of the value of capital at constant prices, in terms of tons of weight of the equipment, mechanical power, etc.) inevitably entails the introduction of superior techniques” (p. 595).

\(^4\)Employing the observed data for capital, labour and output and our parameter estimates, the capital share for the Italian economy in the reference period, using Equation (3), is

$$\pi_0 = (\beta_3)^{-\beta_1} \left( \frac{Y_t}{K T_0} \right)^{\beta_1} = 0.25$$

so that the labour income share is

$$1 - \pi_0 = 0.75$$

Since these estimates are quite close to those present in several different databases (such as OECD, EU KLEMS, AMECO), we decided to adopt these values of the income shares for the reference period.
In the Cobb–Douglas case, the \( TFP_{CD} \) becomes

\[
TFP_{CD} = \frac{Y^{CD}_t}{Y^{CD}_t} - \left( \pi_0 \frac{K_t}{K_t} + (1 - \pi_0) \frac{L_t}{L_t} \right)
\]

The result of these two growth accounting exercises is illustrated in Figure 1. A notable feature of the graph is that in the first part of the sample period, the TFP from the Cobb–Douglas lies above that of the CES, while in the second part, they essentially overlap.

**Figure 1** The dynamics of TFP

A plausible interpretation is that our estimated \( \sigma \) is about two-thirds, while the Cobb–Douglas technology has a \( \sigma \) equal to 1. It follows that, from the property of general means (see Grandville 2009), the Cobb–Douglas output and correspondingly its growth rate is higher than in our CES case. In addition, there is a different weighting of the input growth rates in the two functions: the Cobb–Douglas uses fixed weights (equal to the income shares), while the CES uses the output-factor elasticities. As we will see, the gap between the two TFPs, and its narrowing until it vanishes at about the middle of the sample period, can be explained by splitting the TFP into its components.
4.1 The decomposition of TFP

Whereas the Cobb–Douglas allows the computation of TFP only residually, a further advantage of the CES function is the possibility of decomposing the TFP. This decomposition can best be done if we come back to our original framework. The tools are the output elasticities with respect to the inputs, which represent a key feature of the CES production function. Indeed, they allow distinguishing between the contributions to the output growth rate of the different factors in factor-augmenting technical change. To appreciate the relevance of this property, we analyze the pattern of technical change of the Italian economy.

Let us start with the labor contribution to technical change, $\varepsilon_\nu \cdot \mu$. Its dynamics is represented in Figure 2.

**Figure 2** Labor-augmenting technical change

It is straightforward to see that the labor contribution features two quite distinct patterns: in the first half of the sample period (1981:4–1994:2), labor augmentation is steadily increasing. It is more troubling to detect a clear behavior in the second half. Indeed, it remains approximately constant. Hence, in the mid-1990s there seems to be a structural break. The occurrence of such a break is confirmed by a simple Chow’s breakpoint test. How sensitive is this result to changes in the value of $\sigma$? As a robustness check of the break timing, we tried values of $\sigma$ closer or equal to 1 without finding any relevant differences.

A regime shift seems to be confirmed by the development of capital-
augmentation, $\varepsilon_{Y,KIT-\gamma \frac{C_t}{L_t}}$. Its time evolution is quite volatile with a number of peaks; indeed, a test based on global information criteria indicate the existence of multiple breaks. However, a simple visual inspection of Figure 3 shows that the relevant break occurs around the middle of the 1990s.

**Figure 3** Capital-augmenting technical change

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### 5 Factor bias

The CES production function also sheds light on another aspect, the factor bias, which is defined by the ratio of the marginal productivities of the inputs (not in efficiency units). From Equation (4), we have:

$$\frac{\partial Y/\partial K}{\partial Y/\partial L} = \frac{\pi_0}{1 - \pi_0} \left( \frac{e^{\mu (t-t_0)}}{C_t} \right)^{\beta_1} \left( \frac{L_t}{K_t} \right)^{1+\beta_1}.$$

Technical progress is biased towards a factor if it increases its marginal product more than the other factor’s. Following Acemoglu (2002), the bias can be divided into two parts. One is the traditional substitution effect, determined by the relative endowments of the two inputs, that favors the scarcer factor. The other component, that can be referred to as the technical change effect, depends on the relative weight of the factor-augmenting technical change. This second effect is absent in the Cobb–Douglas case.

The bias is clearly linked to the size of the elasticity of substitution. In our case, where $\beta_1 = 0.52$ ($\sigma = 0.66$), the dominance of labor-augmenting
technical change in the first half of the sample implies that technical change is capital biased. Intuitively, the presence of capital bias means that technical change favors capital input.

In Figure 4 the contribution of technical change to capital bias is given by the positive vertical distance separating the CES and the Cobb–Douglas (which includes only the substitution effect). Looking at the graph, it is worth noting that although present, the capital bias progressively reduces until it vanishes in the middle of the 1990s. To clarify this point, the vertical distance, a measure of the contribution of technical progress, is graphed in Figure 5.
In fact, the graph clearly shows not only the disappearance of technical change but also verifies the occurrence of a structural break around the middle of the 1990s seen above. As in our technology representation (4), technical change is predominantly driven by ICT investment (see the definition of $\mu$ and of the capital-augmenting factor), the disappearance of the technical change contribution can be viewed as a failure to effectively employ innovative technologies in the Italian economy.

6 Conclusions

Most analyses of the current economic Italian stagnation focus on a TFP slowdown without delving into its causes. In this paper, we tried to take a step further, looking at the determinants of TFP. To this end, we used our previous CES specification and estimated parameters. We find evidence of a structural break in the mid-1990s in the impact and nature of technical change. Labor augmentation and capital bias were found to have been dominant in the first half of the sample period, while no evidence of technological progress of any type seems to be present in the second half. We believe that these results can be relevant not only for theoretical purposes but also for policy choices. This task is left for future research.
References


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