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Optimal monetary policy and the time-dependent price and wage Phillips curves: An international comparison*

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Abstract. We investigate the behavior of central banks in seven advanced economies, focusing on how observed monetary policies align with optimal ones as determined by model-consistent welfare measures. Our approach stands out by emphasizing the importance of inertia's impact on the output gap and the dynamics of prices and wages. We incorporate inertia into our model using duration-dependent adjustments. By integrating this aspect into a simple New Keynesian model, our analysis aims to identify shared patterns and distinctive features in the monetary policy approach of central banks across different countries.

JEL classification: E31, E32, C11.

Keywords: duration-dependent adjustments, intrinsic inflation persistence, DSGE models, hybrid Phillips curves, and optimal policy

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1. Introduction

Inflation is back after at least three decades of moderate price and wage changes. The recent surge and enduring nature of inflation rates have brought the theme of price stability to the forefront of monetary policy discussions. This paper provides an international perspective on the matter. Our study centers on the connection between inertia and monetary policy, aiming to conduct a comparative analysis of observed and optimal monetary policies in a context where inertial processes influence economic variables. Through a cross-country comparison, we aim to shed light on common trends and specific features of different central banks' approaches to monetary policy.

Economists have studied time-dependent price and wage adjustments in macroeconomic models.[1] These adjustments can significantly affect the nature of macroeconomic distortions. Furthermore, using time-dependent pricing models to introduce intrinsic inflation persistence has also become essential for understanding inflation dynamics in sticky price and wage models. Against this backdrop, our paper investigates the effects of time-dependent price and wage adjustments on the nature of macroeconomic distortions in sticky price and wage models with intrinsic inflation persistence induced by time-dependent pricing models. We aim to comprehensively analyze the welfare costs generated by these adjustments and derive a model-consistent welfare measure to explain them. Moreover, the study will examine the impact of the slopes of price and wage hazard functions on the expectations of forward-looking agents and their decisions. Finally, we will investigate whether and how these effects impact monetary policy and highlight the implications for central bank policy in countries with different macroeconomic structures.

Overall, this paper contributes to the existing literature by providing a detailed analysis of the effects of time-dependent price and wage adjustments on macroeconomic distortions and welfare costs. The study will also shed light on monetary policy's role in managing the effects of these adjustments and offer insights for policymakers and researchers. Borrowing from Di Bartolomeo *et al*. (2020), who used a generalized model of Erceg *et al*. (2000) that accounts for price and wage hazard functions to estimate and analyze seven industrialized economies' price and wage structures, we extend the country comparison to an optimal monetary policy analysis Precisely, we provide a second-order Taylor approximation of the expected value of the intertemporal utility function from

[1] Among others, Cecchetti (1986), Coenen *et al*. (2007), Sheedy (2007, 2010), Woodford (2009), Midrigan (2011), Yao (2016), Di Bartolomeo and Di Pietro (2017, 2018). As noted by Woodford (2009), it should be stressed that the forecast of a significant positive hazard rate is the empirical findings of Eichenbaum *et al*. (2011). For wages, see Barattieri *et al*. (2014), Di Bartolomeo *et al*. (2020), Grigsby *et al*. (2021). A critical discussion of the evidence is provided in Di Bartolomeo and Di Pietro (2017).

their model to compute a consistent welfare measure. Moreover, Bayesian estimates from their model framework have been used as observed policy-relevant dynamics to perform our counterfactual exercise. Finally, an optimization algorithm was developed to narrate which alternative history we would have observed if different central banks had maximized welfare-based criteria.

Our analysis focuses on intrinsic inflation persistence and its relationship with hazard functions, which measure the probability of changing a price and the duration of price stickiness. Sheedy (2007, 2010) introduced the general approach of considering intrinsic inflation persistence through time-dependent pricing mechanisms, which Di Bartolomeo and Di Pietro (2017) later generalized to wage dynamics. The shape of the hazard function, whether upward or downward sloping, plays a crucial role in determining the likelihood of introducing a new price for goods or services that have remained unchanged for an extended period.

We also build upon a second literature strand that examines optimal monetary policy by deriving a welfare criterion based on time-dependent models for both price and wage adjustments. This strand includes studies that have developed a general approach to approximating welfare around the steady state (such as Rotemberg and Woodford, 1997; Woodford, 2003; Benigno and Woodford, 2005, 2012.) Our approach, based on Woodford's (2003) method, introduces a subsidy that counteracts distortions arising from market power so that a zero-inflation policy yields an efficient level of output in the steady state. Di Bartolomeo and Di Pietro (2018) examined the link between time-dependent mechanisms and optimal monetary policy but focused only on prices, ignoring wage adjustments.

The remainder of the paper is organized as follows. Section 2 describes a simple New Keynesian model augmented with duration-dependent price and wage adjustments and derives the model-consistent welfare function. It also discusses the related economic implications. Our research strategy and methodology are outlined in Section 3. Results are presented in Section 4. Finally, Section 5 concludes.

2. The theoretical framework

2.1 The model[2]

We consider a small-scale New Keynesian DSGE model. The demand side of the economy is characterized by many identical households, each composed of a continuum of members that supply different labor services and consume goods. The supply side is populated by a continuum of firms

[2] See Di Bartolomeo *et al*. (2020) for a detailed model derivation. A technical appendix is also available upon request.

that produce differentiated goods using household labor. The economy is characterized by monopolistic competition in the goods and labor markets. Firms and households are price and wage-setters and reset prices and wages periodically with a time-dependent probability. Nominal price and wage rigidities are modeled according to time-dependent mechanisms. The model is expressed as a deviation from its long-run level.

The aggregate demand (1) is derived from the Euler equation. It inversely conveys the output gap (y_t) to the real interest rate (r_t). Formally,

$$y_t = \frac{1}{1+h} E_t y_{t+1} + \frac{h}{1+h} y_{t-1} - \frac{1-h}{\sigma(1+h)} (r_t + E_t z_{t+1} - z_t) \tag{1}$$

where h is the habit parameter; σ is the relative risk aversion coefficient; z_t is a preference shock that evolves according to $z_t = \rho_z z_{t-1} + \varepsilon_t^z$ with $\rho_z \in [0,1)$ and $\varepsilon_t^z \sim N(0, \sigma_z^2)$. The real interest rate is the difference between the nominal one (i_t) and the expected inflation rate ($E_t \pi_{t+1}^p$).

The supply side is described by the Phillips curve (2) that positively relates inflation (π_t^p) to the real marginal cost (mc_t). Formally,

$$\pi_t^p = \psi_p \pi_{t-1}^p + \beta[1 + (1-\beta)\psi_p] E_t \pi_{t+1}^p - \beta^2 \psi_p E_t \pi_{t+2}^p + k_p \Theta_p (mc_t + \zeta_t) \tag{2}$$

where β is the discount factor; ζ_t is a supply (or additive price markup) shock that evolves according to $\zeta_t = \rho_\zeta \zeta_{t-1} + \varepsilon_t^\zeta$ with $\rho_\zeta \in [0,1)$ and $\varepsilon_t^\zeta \sim N(0, \sigma_\zeta^2)$; $\Theta_p = \frac{1-\delta}{1-\delta+\delta\varepsilon_p}$ with δ representing the labor weight in the Cobb-Douglas-production function and ε_p is the elasticity of substitution between goods.

The two parameters ψ_p and k_p in (2) define the slope and the intercept of the Phillips curve and derives from the slope (φ_p) and intercept (α_p) of the price hazard function. They are as follows:

$$\begin{cases} \psi_p = \dfrac{\varphi_p}{(1-\alpha_p) - \varphi_p[1 - \beta(1-\alpha_p)]} \\ k_p = \dfrac{(\alpha_p + \varphi_p)[1 - \beta(1-\alpha_p) + \beta^2 \varphi_p]}{(1-\alpha_p) - \varphi_p[1 - \beta(1-\alpha_p)]} \end{cases} \tag{3}$$

It is worth noting that α_p is the probability of resetting the price at time *t* of a firm that has set its price at *t*–1, while φ_p is the slope of the hazard, which can be flat as in Calvo (1983) pricing (when $\varphi_p = 0$), upward-sloping (when $\varphi_p > 0$), or downward-sloping (when $\varphi_p < 0$). The parameters also define the unconditional probability of a price reset, i.e., $\alpha_p + \varphi_p$, and the unconditional expected duration of price stickiness, i.e., $\left(1-\varphi_p\right)/\left(\alpha_p+\varphi_p\right)$.[3]

The marginal cost (mc_t) and the production function (y_t) are:

$$mc_t = \omega_t + n_t - y_t \tag{4}$$

$$y_t = a_t + (1-\delta)n_t \tag{5}$$

where n_t are the hours worked and ω_t is the real wage; a_t is a production disturbance that evolves as $a_t = \rho_a a_{t-1} + \varepsilon_t^a$ with $\rho_a \in [0,1)$ and $\varepsilon_t^a \sim N(0, \sigma_a^2)$.

The equilibrium of the labor market is defined by the wage Phillips curve (6) that conveys wage inflation (π_t^w) negatively to the gap between the real wage and the marginal substitution rate between labor and consumption (mrs_t). Formally, we can write:

$$\pi_t^w = \psi_w \pi_{t-1}^w + \beta[1+(1-\beta)\psi_w]E_t\pi_{t+1}^w - \beta^2\psi_w E_t\pi_{t+2}^w - \frac{k_w(\omega_t - mrs_t)}{1+\varepsilon_w\gamma} \tag{6}$$

where γ is the inverse of Frisch elasticity; ε_w is the elasticity of substitution between workers' services. Note that we can define: $\omega_t - \omega_{t-1} = \pi_t^w - \pi_t^p$. Similarly, parameters ψ_w and k_w derive from the slope (φ_w) and intercept (α_w) of the wage hazard function:

$$\begin{cases} \psi_w = \dfrac{\varphi_w}{(1-\alpha_w) - \varphi_w[1-\beta(1-\alpha_w)]} \\ k_w = \dfrac{(\alpha_w+\varphi_w)[1-\beta(1-\alpha_w)+\beta^2\varphi_w]}{(1-\alpha_w) - \varphi_w[1-\beta(1-\alpha_w)]} \end{cases} \tag{7}$$

The interpretation of α_w and φ_w is as in (3). Therefore, the unconditional probability of a wage reset is $\alpha_w + \varphi_w$ and the unconditional expected duration of wage stickiness is $(1-\varphi_w)/(\alpha_w+\varphi_w)$. It is easy to verify that for $\varphi_w = 0$, equation (6) collapses to a flat Calvo's wage-adjustment mechanism (Erceg *et al*., 2000.)[4]

[3] See Sheedy (2007).

[4] It is worth noting that the model we use inherits the structure from Erceg *et al*. (2000). The introduction of persistence does not affect this. Therefore, monetary policy cannot achieve the Pareto-optimal equilibrium in our framework unless both wages and prices are completely flexible. This setup presents a trade-off in stabilizing the output gap, price inflation, and wage inflation. Achieving the Pareto optimum is only feasible if wages or prices are flexible. Consequently, strict price inflation targeting falls short of being optimal compared to policies considering either the output gap or wage inflation (for detail, see Galì, 2008: Chapter 6.)

The marginal rate of substitution between labor and consumption is derived from the household's utility function and can be written as follows:

$$mrs_t = \frac{\sigma}{1-h}(y_t - hy_{t-1}) + \gamma n_t - z_t \tag{8}$$

The model is closed by the specification of the monetary policy that is set according to a simple Taylor rule (Taylor, 1999):

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left(\delta_\pi \pi_t^p + \delta_y y_t\right) + \varepsilon_t^i \tag{9}$$

where $\delta_\pi > 0$ is the feedback coefficient of the monetary policy; $\delta_y > 0$ is the feedback coefficient on output gap; $\rho_i \in [0,1)$ is a smoothing parameter that captures policy inertia; $\varepsilon_t^i \sim N(0, \sigma_i^2)$ is a white noise (i.e., policy innovation.)[5]

2.2 The welfare loss function

To compute the paths of policy-relevant variables under commitment and discretion regimes and evaluate optimal monetary policy across countries, we derive a model-consistent welfare-loss function by second-order Taylor approximating the expected value of the intertemporal utility function.

The period utility is:[6]

$$U_t(C_t, C_{t-1}, N_t(j)) = \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t(j)^{1+\gamma}}{1+\gamma} \tag{10}$$

By a second-order approximation, once we account for the aggregate resource constraint, i.e., $c_t = y_t$, (10) can be written as:

$$u_t \simeq U_c Y \left[\tfrac{1-h\beta}{1-h}\left(y_t + \tfrac{y_t^2}{2}\right) - \tfrac{\sigma}{2}\left(\tfrac{y_t - hy_{t-1}}{1-h}\right)^2\right] + U_n N\left[\int_0^1 n_t\,(j)dj + \tfrac{1+\gamma}{2}\int_0^1 n_t^2\,(j)dj\right] \tag{11}$$

where the symbol $\simeq$ indicates that an approximation is accurate up to the second order and steady state values are denoted by upper case letters.

The second-order approximation of the aggregate employment, $N_t \equiv \int_0^1 N_t\,(j)dj$, is

[5] We use a simple Taylor rule that generally captures monetary policy. However, alternative rules might perform better in this context. A counterfactual of the optimality of rules is beyond the scope of this work; for a more detailed discussion, see Erceg *et al*. (2000) and Galì (2008). We focus on optimal policies according to a linear-quadratic approach (Rotemberg and Woodford, 1997; Woodford, 2003; Benigno and Woodford, 2005, 2012.).

[6] We used the fact that $C_t = \int_0^1 C_t\,(j)dj$. Since the same is not true for $N_t(j)$, we keep the index j for the labor supply.

$$n_t + \frac{1}{2}n_t^2 \simeq \int_0^1 n_t\,(j)dj + \frac{1}{2}\int_0^1 n_t^2\,(j)dj \tag{12}$$

Using the approximation of the labor-demand equation, we obtain

$$\int_0^1 n_t^2\,(j)dj \simeq n_t^2 + \varepsilon_w^2 var_j w_t(j) \tag{13}$$

Manipulating together (12) and (13), we rewrite (11), as

$$u_t \simeq U_c Y\left[\frac{1-h\beta}{1-h}\left(y_t + \frac{y_t^2}{2}\right) - \frac{\sigma}{2}\left(\frac{y_t - hy_{t-1}}{1-h}\right)^2\right] + U_n N\left[n_t + \frac{1+\gamma}{2}n_t^2 + \frac{\varepsilon_w^2\gamma}{2}var_j w_t(j)\right] \tag{14}$$

Now we derive a relation between aggregate employment and output:[7]

$$N_t = \int_0^1\int_0^1 N_t\,(i,j)djdi = \Delta_{w,t}\Delta_{p,t}\left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\delta}} \tag{15}$$

where $\Delta_{w,t} \equiv \int_0^1\left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} dj$ and $\Delta_{p,t} \equiv \int_0^1\left(\frac{P_t(i)}{P_t}\right)^{\frac{-\varepsilon_p}{1-\delta}} di$ measure the degree of wage and price dispersion, respectively.

Log-linearizing (15), under the normalization $A = 1$, we get:

$$(1-\delta)n_t = y_t - a_t + \frac{\varepsilon_p}{2\Theta_p}var_i\{p_t(i)\} + \frac{(1-\delta)\varepsilon_w}{2}var_j\{w_t(j)\} \tag{16}$$

where $var_i\{p_t(i)\}$ and $var_j\{w_t(j)\}$ indicate the cross-sectional variance of prices and wages, respectively.

Substituting (16) into (14), we obtain

$$\begin{aligned} u_t \simeq U_c Y&\left[\frac{1-h\beta}{1-h}\left(y_t + \frac{y_t^2}{2}\right) - \frac{\sigma}{2}\left(\frac{y_t - hy_{t-1}}{1-h}\right)^2\right] + \\ &+ \frac{U_n N}{1-\delta}\left[y_t + \frac{\varepsilon_p var_i\{p_t(i)\}}{2\Theta_p} + \frac{\varepsilon_w var_j\{w_t(j)\}}{2\Theta_w} + \frac{(1+\gamma)y_t^2}{2(1-\delta)}\right] \end{aligned} \tag{17}$$

where $\Theta_w = (1-\delta)(1+\varepsilon_w\gamma)$.

Accounting for an efficient steady state,[8] i.e., $-U_n/U_c = MPN = (1-\delta)Y/N$, and using (17), after some algebra, we get:

[7] See Galì (2008, p. 142).

[8] We assume an output or employment subsidy that offsets the distortions due to the market powers so that the steady state under a zero-inflation policy involves an efficient output level. The approach can be generalized to the case of a

$$w_t \simeq -\frac{1}{2} N^{1+\gamma} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{\sigma}{\lambda} + \frac{\gamma+\delta}{1-\delta} \right) y_t^2 + \frac{\sigma h}{\lambda} (h y_{t-1}^2 - 2 y_t y_{t-1}) \right] + \right.$$
$$\left. + \frac{\varepsilon_p}{\Theta_p} \sum_{t=0}^{\infty} \beta^t \left[var_i\{p_t(i)\} \right] + + \frac{\varepsilon_w}{\Theta_w} \sum_{t=0}^{\infty} \beta^t \left[var_j\{w_t(j)\} \right] \right\} \quad (18)$$

where $\lambda = (1 - h\beta)(1 - h)$.

To obtain an expression for $var_j\{w_t(j)\}$ and $var_i\{p_t(i)\}$, we exploit that the log-aggregate-wage level evolves as $\log \ W_t = \sum_{h=0}^{\infty} \theta_{w,h} \log \ W_{t-h}^*$, where W_t^* is the reset wage and $\theta_{w,h}$ the share of workers posting a wage which last change was h periods ago. Thus, the wage level is a weighted average of past reset wages and the share of workers using such wages at t and the same holds for the log-aggregate-price level.[9]

Exploiting the above log adjustments and the properties of the variance, we approximate the discounted sum of price or wage dispersion Δ_t^i for $i \in \{p, w\}$ as:

$$\sum_{t=0}^{\infty} \beta^t \Delta_t^i \simeq \frac{1}{d_i} \sum_{t=0}^{\infty} \beta^t \left[\left(\pi_t^i - \varphi_i \pi_{t-1}^i \right)^2 - (\alpha_i + \varphi_i)(\pi_t^i)^2 \right] \quad (19)$$

where $d_i = [1 - \beta(1 - \alpha_i - \varphi_i)](\alpha_i + \varphi_i) \in (0,1)$.

Finally, noting that up to a second-order approximation $\frac{\varepsilon_p}{2\Theta_p} var_i\{p_t(i)\} \simeq (1-\delta)^2 log\Delta_t^p$ and $\frac{1-\varepsilon_w}{2} var_j\{w_t(j)\} \simeq (1-\delta)^2 log\Delta_t^w$, we substitute (19) into (18) to get our welfare measure with internal habit and duration-dependent-price and wage adjustments:

$$w_t \simeq -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(\pi_t^p - \varphi_p \pi_{t-1}^p \right)^2 - \left(\alpha_p + \varphi_p \right) \left(\pi_t^p \right)^2}{d_p \Theta_p \varepsilon_p^{-1}} + \frac{\sigma (y_t - h y_{t-1})^2}{\lambda} + \right.$$
$$\left. + \frac{(\gamma+\delta) y_t^2}{1-\delta} + \frac{(\pi_w^w - \varphi_w \pi_{t-1}^w)^2 - (\alpha_w + \varphi_w)(\pi_t^w)^2}{d_w \Theta_w \varepsilon_w^{-1}} \right\} \quad (20)$$

Equation (20) shows that welfare is a quadratic expression of the output gap, price, and wage inflation, that is, in matrix form:

distorted steady state. However, this introduces further complications (see Benigno and Woodford, 2005a, 2005b, 2012).

[9] See the Appendix A.

$$w_t \simeq -\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t \begin{bmatrix}\pi_t^p\\ \pi_t^w\\ y_t\\ \pi_{t-1}^p\\ \pi_{t-1}^w\\ y_{t-1}\end{bmatrix}\begin{bmatrix}\Gamma_1^p & 0 & 0 & \Gamma_3^p & 0 & 0\\ 0 & \Gamma_1^w & 0 & 0 & \Gamma_3^w & 0\\ 0 & 0 & \Gamma_1^y & 0 & 0 & \Gamma_3^y\\ \Gamma_3^p & 0 & 0 & \Gamma_2^p & 0 & 0\\ 0 & \Gamma_3^w & 0 & 0 & \Gamma_2^w & 0\\ 0 & 0 & \Gamma_3^y & 0 & 0 & \Gamma_2^y\end{bmatrix}\begin{bmatrix}\pi_t^p\\ \pi_t^w\\ y_t\\ \pi_{t-1}^p\\ \pi_{t-1}^w\\ y_{t-1}\end{bmatrix}^T \tag{21}$$

where $\Gamma_1^i = \frac{\varepsilon_i(1-\alpha_i-\varphi_i)}{d_i\Theta_i}$, $\Gamma_2^i = \frac{\varepsilon_i\varphi_i^2}{d_i\Theta_i}$, $\Gamma_3^i = -\frac{\varepsilon_i\varphi_i}{d_i\Theta_i}$ for $i \in \{p, w\}$ are the terms that depend on the hazard slopes; and $\Gamma_1^y = \frac{\sigma}{\lambda} + \frac{\gamma+\delta}{1-\delta}$, $\Gamma_2^y = \frac{h^2\sigma}{\lambda}$ and $\Gamma_3^y = -\frac{h\sigma}{\lambda}$ are the terms that do not depend on the hazard parameters. It is worth noting that the parameters associated with the output gap are the same found in Galì (2008) and Erceg *et al.* (2000) once one accounts for habits.

3. The role of intrinsic price and wage inflation persistence

3.1 Some theory

Welfare losses depend on fluctuations in the gap between output and its efficient level. However, they are also affected by price and wage dispersion, which depend on the hazard shapes and the selection effect. In fact, to the extent that the dispersion increases, the misallocation of resources also increases and, consequently, the welfare loss worsens. Focusing on the relation among price dispersion, hazard shapes, and the selection effect, this section explores the nature of macro distortions induced by vintage-dependent pricing models, making explicit how welfare costs are generated by time-dependent price and wage adjustments. Furthermore, it derives some general properties of the welfare loss (21). Specifically, by looking closely at (21), we can derive three propositions and related implications.

We can explain the rationale of the weights of the welfare cost (21) by considering the relationship between the selection effect and the price dispersion. For this purpose, following Sheedy (2010), it is helpful to write the weights of (21) in terms of the selection effect ($s_i = \varphi_i$) and of the unconditional resetting probability ($q_i = \alpha_i + \varphi_i$),[10] i.e., $\tilde{\Gamma}_1^i = \frac{1-q_i}{[1-\beta(1-q_i)]q_i}$; $\tilde{\Gamma}_2^i = \frac{s_i^2}{[1-\beta(1-q_i)]q_i}$; $\tilde{\Gamma}_3^i = -2\,\frac{s_i}{[1-\beta(1-q_i)]q_i}$ for $i \in \{p, w\}$. These weights are normalized by the factor $\varepsilon_i\Theta_i^{-1}$. Henceforth, we only refer to price dispersion for brevity, but the same applies to wages.

First, note that if prices are flexible (i.e., $q_i = 1$), then $q_i = \alpha_i$ and $\varphi_i = 0$. Therefore, in this

[10] The selection effect (s_i) and the unconditional resetting probability (q_i) depend on the hazard shape but are not independent of each other. A discussion of the loss weights in terms of the hazard independent parameters (α_i and φ_i) is provided in Appendix B.

case, $\tilde{\Gamma}_1^i = \tilde{\Gamma}_2^i = \tilde{\Gamma}_3^i = 0$ reflecting the fact that price dispersion is always zero. Conversely, if prices are sticky, all weights fall in q_i. The intuition is straightforward.

Focusing on the sticky price case, we observe that the losses associated with the price dispersion depend on current inflation, as in Calvo (1983). However, if the selection effect is non-zero, the cost depends on two additional components: inflation persistence and the sign of the price trend. Let us look closely at the weights of (21) to grasp the economic intuition behind each of them. We know that a positive selection (upward-sloping hazard) effect implies that the probability of changing prices in each period is higher for older than newer ones, and firms' catch-up is more likely than roll-back. The opposite occurs for an adverse selection effect (negative hazard.) In the case of Calvo (1983), the two effects offset one another.

The coefficient of the square of the current inflation ($\tilde{\Gamma}_1^i$) measures the impact of price stickiness on price dispersion, which is driven by the unconditional probability of not adjusting prices $(1 - q_i)$. The intuition is the same as in standard sticky price models.

The coefficient of the square of the lagged inflation ($\tilde{\Gamma}_2^i$) captures the costs associated with the intrinsic inflation persistence that characterizes time-dependent price adjustments. The time-dependent adjustment mechanism somehow assumes that the later the price adjustment, the higher the medium-term costs of established habits once the price update has been verified.

Assume that the past inflation is not zero ($\pi_{t-1}^i \neq 0$), but shocks vanish if the hazard is flat, catch-up and roll-back effects offset each other, and current inflation is zero. Conversely, suppose the catch-up and roll-back effects do not compensate; in that case, a positive (negative) net impact implies that more firms will tend to adjust too much (too little), increasing the dispersion in the price adjustment process. Specifically, the positive (negative) selection effect creates intrinsic persistence (overshooting) in the price adjustment process. In both cases, there is a greater dispersion in prices and costs regarding welfare loss compared to the case of the flat hazard.

Let us describe the intuition of the relationship between price dispersion, inflation persistence, and the selection effect with an example provided by Figure 1. We compare three different selection effects (s), which correspond to a flat, positive, and negative slope of the hazard function. In all cases, the value of the unconditional adjusting probability (q) is chosen to obtain the same initial impact on inflation to a supply shock occurring at time 1 and lasting one period.[11]

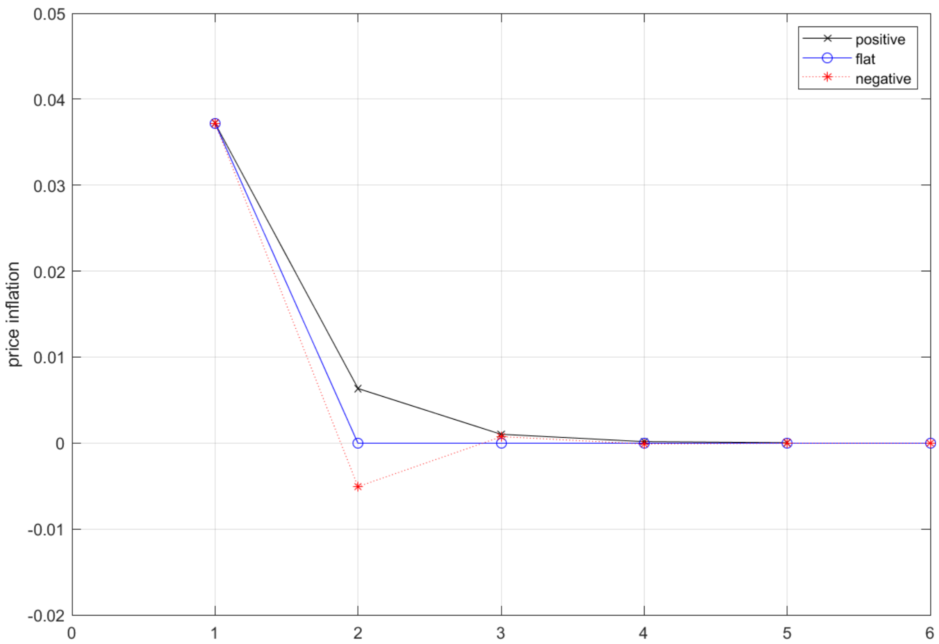
The initial impact of the shock is always an increase in inflation. All firms that adjust prices adjust them upwards to fill the gap between the actual and desired prices. In the period after the temporary inflationary shock has occurred, some agents are randomly selected to adjust their prices.

[11] Figure 1 is illustrative and can be built similarly for wage inflation.

Among them, those who have previously adjusted are charging a higher price and thus aim to roll back, while the others are posting a too low price and aim to catch up.

In Calvo's world, the impact on aggregate inflation of the two groups (roll-back and catch-up firms) is the same, but the sign is the opposite and, as a result, the inflation is zero, that is, if nothing else occurs and the shock is dampened. Instead, if the probability of adjusting the prices of catch-up firms is larger than the probability of roll-back ones, price increases dominate falls. Therefore, inflation remains positive (i.e., intrinsic inflation persistence.) Conversely, when the selection effect is negative, falls in prices dominate increases leading to negative inflation (adjustment implies inflation overshooting.) Due to either the positive or negative selection effect, the price adjustment process across firms increases the price dispersion in period 2 compared to Calvo's price-setting framework.

Figure 1 – Selection effect, inflation persistence, and price dispersion with different slopes for the hazard function.



Regarding the last component, the sign coefficient of the cross-product ($\tilde{\Gamma}_3^i$) depends on the sign of s_i and it is related to the direction of the trend component of price dynamics (i.e., the sign of $\pi_t^i \pi_{t-1}^i$). In detail, if the selection effect is positive ($s_i > 0$): *a)* dispersion increases when there is a reversal in the price trend (i.e., $\pi_t^i \pi_{t-1}^i < 0$); *b)* conversely, dispersion falls when the prices continue to move in their initial direction (no trend reverting, i.e., $\pi_t^i \pi_{t-1}^i > 0$). The opposite occurs when the selection effect is negative ($s_i < 0$).

The intuition about the trend effects on the welfare loss is as follows. Price dispersion depends on the gap between actual prices and the desired ones. When prices increase (or decrease), desired prices move in the same direction as the last price adjustment. Therefore, if the share of adjusting firms that catch up is greater than that of adjusting firms that roll back, the gap between desired and actual prices would be reduced; a positive selection effect reduces the welfare loss. Conversely, when we observe a price trend reverting, desired prices move opposite to the last price adjustment, and the contrary occurs. When prices are still increasing or decreasing, a positive selection effect reduces the price dispersion and the welfare cost. The price dispersion increases when the price trend reverts, and the welfare loss magnifies. We can summarize our findings in a proposition.[12]

Proposition 1. Welfare costs generated by time-dependent price and wage adjustments depends on three components: i) the price dispersion; ii) the intrinsic inflation persistence; iii) the trend component of price dynamics. For any given selection effect, the costs i) and ii) are always increasing in the (price and wage) stickiness, which is measured by the complement to the unconditional probability of adjusting prices. The strength and the sign of the components associated with the inflation persistence and trend depend on the selection effect. If the selection effect is positive the cost increases when there is a reversal in the price trend and vice versa. The opposite occurs when the selection effect is negative.

Second, equation (21) nests many common welfare losses. By assuming $\varphi_p = \varphi_w = 0$ (flat slopes), $h = 0$ (no habits), and $\varepsilon_w \to \infty$ (flexible wage), the loss function (21) simplifies to a loss consistent with a forward-looking sticky price in Calvo's economy, i.e., the three-equation textbook model described, e.g., in Galì (2008: Chapter 4).[13] Similarly, posing $\varphi_p = \varphi_w = 0$, the loss (21) approximates welfare in a sticky-price/wage model of the kind of Erceg *et al.* (2000) or Benigno and Woodford (2005b)—with habits (for $h \in [0,1]$) or without (for $h = 0$). Finally, if φ_p and φ_w are different from zero, equation (21) generalizes the sticky price/wage model to the case of upward or downward hazard function in price and wage setting. This finding is summarized in the following proposition.[14]

Proposition 2. By an appropriate parametrization, the welfare criterion (21) is consistent with any

12 We refer to Appendix C for a proof.

[13] See also Woodford (2003, Chapter 6).

[14] See Appendix C for a formal poof.

combination of hazard slopes for price or wage adjustments, stickiness, and habits.

Third, the effects of changes on welfare in the shape of the price hazard are independent of changes in the shape of the wage hazard. Specifically, the price-hazard function affects the welfare loss only through price dispersion, which in turn only depends on the price dynamics, although with more complex interactions between inflation and its variation compared to Calvo's model. The same applies to the wage-hazard function and can be summarized in the proposition below.[15]

Proposition 3. The price hazard affects welfare independently of the wage hazard and vice versa. The rationale of the result is that price and wage dispersion are independent.

Note that our welfare loss does not generalize the losses consistent with alternative models introducing intrinsic inflation inertia (e.g., Galì and Gertler, 1999; Steinsson 2003; or Di Bartolomeo *et al*., 2016.) In these cases, the price dispersion and, consequently, the welfare-based losses also depend on the output gap dynamics.

The above proposition has interesting policy implications. In our setup, the welfare costs of price stickiness are independent of those stemming from wage stickiness. Therefore, policies designed to fix the costs of price stickiness are not substitutes for those related to wage rigidities in the labor market and vice versa. A way to reduce the costs of non-constant hazards is partial indexation. However, indexation should be built by accounting for the firms' last price spell to compensate for the impact of non-zero selection effects. The partial indexation should be directly proportional to the time elapsed since the last spell when the selection effect is positive and vice versa when negative.

3.2 Hazard slopes and optimal monetary policy

This section illustrates how varying slopes in hazard functions impact optimal policies. Numerical simulations are necessary to analyze monetary policy effects. We approach this by considering three distinct scenarios: flat (baseline), positive-sloping, and negative-sloping Phillips curves, all with the same average price duration.[16] This method is applied to both price and wage adjustments.[17]

[15] We provide further details in the Appendix C.

[16] Alternative approaches would have been to compare the effects of different selection effects (s_i), keeping constant the unconditional probability of resetting the prices (q_i) or focusing on the hazard independent parameters. See Appendix D.

[17] We concentrate on three cases where both curves are flat, positively, or negatively sloped. This focus is based on our earlier finding that price hazards impact welfare independently of wage hazards and vice versa. Different combinations lead to the same qualitative findings. Results are available upon request.

According to Klenow and Malin (2011), the average duration for price adjustments is 3 quarters, while for wage adjustments, it is 3.8 quarters, as found by Barattieri e*t al*. (2014).[18] These values fix the average durations for prices and wages in our illustrations.

The calibration of the three scenarios is then as follows.[19] i) The Calvo Phillips curve for the price is flat ($\varphi_p = 0$). We set $\alpha_p = 0.33$ to match a duration equal to 3. Similarly, the flat curve for wage adjustments requires $\varphi_w = 0$ and $\alpha_w = 0.26$ to match a duration equal to 3.8. ii) The second scenario implies positive slopes for the hazard functions governing price and wage adjustments. We consider the most representative case where the slopes are steep (consistently with the fixed average durations.) This is obtained by assuming $\alpha_p = 0$ and $\alpha_w = 0$, then $\varphi_p = 0.25$ and $\varphi_w = 0.21$ match the observed average durations. iii) The last scenario considers negative sloped hazard functions. We assume that both curves have the same negative slope. So, we fix $\varphi_p = \varphi_w = -0.1$, which is consistent with a probability of reset prices (wages) in the first period equal to $\alpha_p = 0.46$ ($\alpha_w = 0.39$).[20]

The rest of the parameters are calibrated as standard as possible. They are coherent with Christiano *et al.* (2005). The discount factor (β) and the production function parameter (δ) are calibrated to 0.99 and 0.36 to meet the observed real interest rate in the steady state and the long-run labor share in industrialized economies. The elasticity of substitution between goods is calibrated at $\varepsilon_p = 6$, implying an average markup of 20%, while the elasticity between workers' types is set to $\varepsilon_w = 21$, implying an average markup of 5%. Both parameters characterizing the utility function σ and γ are calibrated to 1, while the habit parameter is set to 0.65.

Our results are illustrated in Figures 2 and 3, which refer to the cases of price and wage markup shock, respectively. They report the IRFs of the output gap and price (wage) inflation. We consider the discretionary (commitment) regime in the left (right) panel. In each panel, we consider the three scenarios introduced above.

The paths broadly reflect the results documented by the existing literature. Figure 2 shows that discretion highlights the trade-off between stabilizing the deviations of the output gap or changes

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[18] There is a certain variability between the microeconomic estimates of duration that naturally depend on the samples and the reference periods used. In our estimations (cf. Table 3), price duration ranges from 2.15 (Canada) to 5.68 (France). Wage duration ranges from 1.51 (Italy) to 4.32 (France). We checked the robustness of our results to different assumptions about durations. Results are available upon request.

[19] Remember that the Phillips curve is governed by two parameters: the intercept α_i and the slope φ_i, which imply an average duration equal to $(1 - \varphi_i)/(\alpha_i + \varphi_i)$. See equation (6).

[20] This scenario was also calibrated to obtain an extreme situation with high but plausible values for initial price adjustments.

in prices. Commitment induces deflation following inflationary cost-push shocks to stabilize the expectations and improve the current trade-off between inflation and the output gap.[21] We observe similar dynamics in the case of wage markup shocks (see Figure 3.)

Let us explore the differences implied by the alternative scenarios. In each policy regime, the IRFs exhibit similar dynamics under the different slopes of the hazard. Looking at the case of discretion, the more the hazard is steep, the lower the impact of the shock on both inflation and the output gap, and the less costly in terms of welfare equivalent to permanent consumption decline, as illustrated below. The same occurs in the commitment regime, where monetary policies that induce lower deflation (positive slope) to stabilize expectations are more effective in stabilizing the output gap. In all the cases, the figures show that the economy’s resilience to shocks positively correlates to the slopes of the hazards for a given average duration of prices. As shown, the dynamics of the main macroeconomic variables implied by the different slopes of the hazard are quite similar and in line with the theory. However, it is crucial to evaluate their effects on welfare.

Figure 2 – Optimal responses to a price-markup shock with different slopes for the price hazard function (discretion and commitment.)

[21] The future deflation is incorporated into current inflationary expectations, leading to lower expectations. The lower expectations in turn, improve the current policy trade-off between inflation and the output gap. The benefits of lower current inflation outweigh the (future) loss associated with carrying out the deflationary period. The more the agents look forward, the more effective the commitment. Discretion suffers from a stabilization bias since, in such a case, any (eventual) deflationary promise from central bankers is not credible, i.e., it will renege in the next period. Hence, compared to commitment, discretion is typically characterized by insufficient inertia in the central banker’s policy actions and by an excessive stabilization of the output at the expense of inflation.

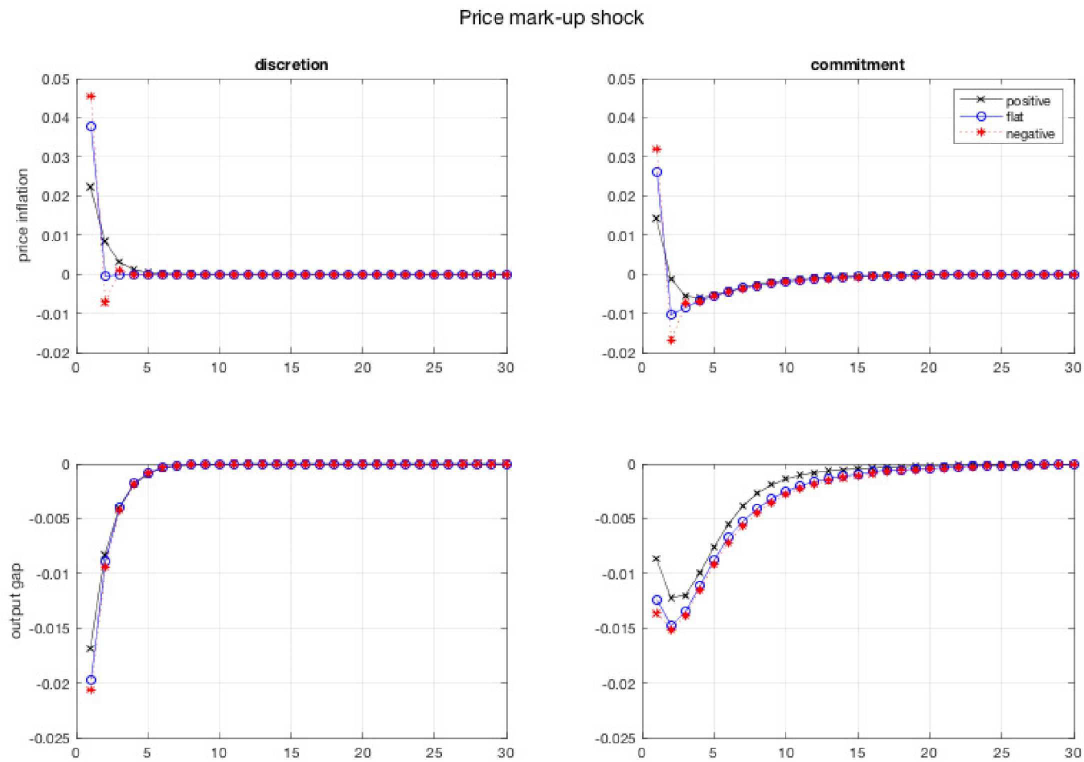
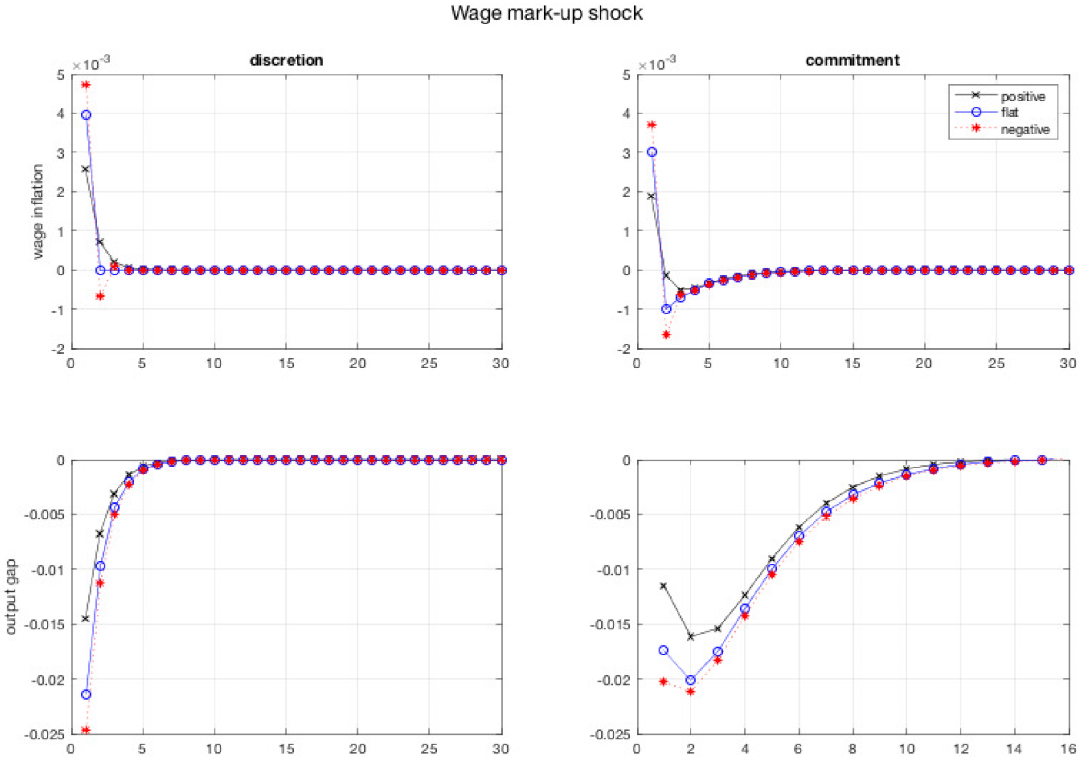


Figure 3 – Optimal responses to a wage-markup shock with different slopes for the wage hazard function (discretion and commitment.)



The impact on welfare is described in Table 1. The table shows the variation of the welfare loss to a benchmark identified as the commitment in the case of Calvo pricing (flat hazard.) Specifically—as, e.g., Ravenna and Walsh (2011)—we report the change in the loss relative to a welfare-based optimal commitment with a flat hazard, where losses are expressed as a percent of steady-state consumption. This representation can use the standard Calvo's case as the baseline calibration.

The impact of the volatilities associated with the different cases on welfare is now evident. Differences in welfare costs associated with different hazard slopes are not negligible. As the hazard slope increases, the impact of the price markup shock on both inflation and the output gap diminishes, leading to lower welfare costs when optimal policies are enacted. In both policy regimes, the economy's resilience to shocks is positively correlated to the slopes of the hazards for a given average duration of prices. In response to a price markup shock, implementing the optimal policy under the negative selection effect in both the commitment and discretion regimes entails almost double the losses obtained under a positive selection effect. Commitment marginal gains over discretion are also affected by the steepness of the hazards. These tend to be lower as the hazard slope becomes steep. Shocks in the wage markup show similar qualitative results.

Table 1. Variations in welfare losses relative to the welfare-based optimal commitment with a flat hazard.

Price markup shock	Hazard slope		
	Negative	Flat	Positive
Commitment	0.137	0.000	-0.391
Discretion	0.694	0.524	0.055
Wage markup shock	Hazard slope		
	Negative	Flat	Positive
Commitment	0.213	0.000	-0.311
Discretion	0.574	0.328	-0.033

The intuition behind our result is as follows. The performance of monetary policy in stabilizing price and wage shocks largely depends on the ability of monetary authorities to affect current choices by influencing expectations, as agents are forward-looking, and both prices and wages are sticky.[22] For any given average price or wage duration, the hazard slope affects the distribution of future expectations, thus influencing their persistence. Given a flat hazard associated with a given duration, by increasing its slope, the number of price-setters who expect to adjust prices in the future increases, but then the number of those who expect to adjust their prices in the short term must fall to keep a constant duration. As a result, the stickiness of prices/wages decreases and, with it, the economic volatility. The opposite verifies for a negative slope.[23]

4. Empirical methodology: Monetary policy counterfactuals

Optimal policies are evaluated within a "what if" exercise. We perform a counterfactual policy analysis using Di Bartolomeo *et al*. (2020)[24] as the baseline scenario. This analysis focuses on determining what the Central Banks would have done if, all else being equal, they had aimed to maximize a welfare function based on our model, considering the historical shocks that have been identified.

Our exercise is thus implemented in a two-step procedure.

[22] It is worth noting that this also occurs under discretion because of inflation inertia and output persistence (see, e.g., Steinsson, 2003.)

[23] A formal proof of the above observation independent of the calibration used is provided in Appendix D.

[24] Di Bartolomeo *et al*. (2020) estimate the model for 7 countries, assuming a Taylor rule (with interest rate estimated persistence) to describe the observed monetary policy. We refer to them for further details related to the estimation strategy.

1. We employ the estimates to calibrate the model's initial conditions for seven countries: Australia, Canada, France, Germany, Italy, the United Kingdom, and the United States. Four macroeconomic variables for each country, namely the real GDP, price inflation, real wage, and nominal interest rate have been used to run the estimations. The sample size covers the period from the 1980s to the beginning of the recent financial crisis.[25] A more extended sample is considered for the US starting from the 1960s. Specifically, we use a) the deep parameters characterizing the slopes and initial values of price and wage Phillips curves; b) the historical-shock innovation series.[26] Note that estimations include the monetary policy rule (9), which captures the observed behavior of the central bank.
2. We simulate the model (1)-(8) by assuming that the central bank minimizes the welfare loss (20) and faces the historically estimated shock dynamics. We consider both commitment and discretion. To this extent, we developed an algorithm based on Soderlind (1999), which accounts for 1) multiple shocks, 2) changes in the current state of the economy, and 3) past promises for the case of commitment.[27] It is worth noting that we assume that the monetary authorities react to the estimated shock dynamics. The assumption is equivalent to considering that the central banker only knows what he would have known at every moment the policy was implemented.

Commitment and discretion are two extreme cases of monetary policy conduct. To determine the central bankers' relative attitude towards one of these two monetary regimes, we rely on a metric assessing the forecasting accuracy as resulted from simulations conducted under commitment and discretion. It involves comparing the forecast error generated by the commitment simulation to that of the discretion simulation and determining whether the central bank's conduct aligns more closely with commitment or discretion based on which simulation yields a relatively smaller (or greater) error.

In both commitment and discretion, for assessing the performance in forecasting accuracy, we utilize the Root Mean Square Error (RMSE.)[28] This metric is highly effective for comparative analysis as it enables distinct and measurable comparisons between different models. The RMSE approach of

[25] The rationale for closing the sample in 2008 is essentially to avoid dealing with the zero-lower bound reached by the nominal interest rate in that year, as discussed by Di Bartolomeo *et al*. (2020).

[26] See the following subsection for details.

[27] Details are available in Appendix E.

[28] See, among others, Adolfson *et al.* (2007) and Diebold *et al.* (2017.)

squaring errors also lends it a robustness against outliers.[29]- Specifically, in our multivariate context, we utilize a summary measure that considers the joint forecasting performance of commitment or discretion, which involves computing a multivariate statistic by dividing the inverse of the log determinant of the variance-covariance matrix of forecast errors by 2 to convert from variance to standard error and by the number of variables to obtain an average figure (Del Negro *et al*., 2007.) This forecasting statistic for multiple variables operates as a composite of the RMSE for each variable, distinctively incorporating the forecast error correlations, which is not the case with a basic weighted average. We also analyze the forecasting accuracy for individual variables by employing the RMSE, derived from the aggregate forecast error across the specified time horizon.

5. Optimal monetary policy and counterfactual analysis

5.1 Estimated Phillips Curves

Based on Di Bartolomeo *et al*. (2020) estimates, Table 2 highlights the differences across countries in the backward components of the curves and the sacrifice ratios. Both are relevant for policy decisions. On the one hand, persistence in the inflation process means excessive price increases can persist in the economic system without further external shocks once inflation picks up. On the other hand, flat Phillips Curves imply that the sacrifice ratio is high, i.e., bringing down excessive inflation by contracting aggregate demand is costly for the central bank. High sacrifice ratios and persistence imply that the central bank becomes less powerful once inflation is entrenched in the economy.

Price and wage inflation are relatively persistent in Canada. Conversely, these are relatively not persistent in France. The two countries represent two extreme cases. Price inflation is also not very persistent in the United States, where wage variations are moderately persistent. In addition to Canada, wage variations are relatively persistent in Australia and Italy. Price trends are moderately persistent in Italy and Germany and, to a lesser extent, in Australia and the UK. The sacrifice ratios for France are relatively high for both prices and wages. The other countries show minor differences. The sacrifice ratio of price (wage) inflation in the United States is relatively high (low.)

Table 2 – Estimated Phillips Curves

Country	Prices			Wages		
	forward	backward	sacrifice ratio	forward	backward	sacrifice ratio
Australia	0.992	0.207	4.23	0.993	0.319	2.53

[29] It is important to note limitations, such as its sensitivity to outliers potentially skewing results and the context-dependent suitability of relying on squared errors.

Canada	0.993	0.313	3.10	0.993	0.368	2.10
France	0.991	0.129	6.37	0.991	0.124	4.50
Germany	0.992	0.251	3.64	0.992	0.282	2.59
Italy	0.992	0.261	3.62	0.993	0.373	2.16
UK	0.992	0.207	4.25	0.992	0.210	2.50
US	0.991	0.179	4.86	0.992	0.236	1.79

Note: The backward component of the Phillips curve is associated with *t*–1 (the component associated with *t*–2 is obtained by multiplying by $-\beta^2$.) See equations (2) and (6). The sacrifice ratios are the inverse of the slope of the Phillips curves.

Drawing from the estimated Phillips Curves, Table 3 reports the unconditional probability of a price reset which is expressed, as previously discussed, by q_i and the unconditional expected duration of price stickiness which is $D_i = \frac{1-\varphi_i}{\alpha_i+\varphi_i}$, for $i \in \{p, w\}$. As the hazard slope roughly matches q_i, the table provides evidence in favor of an upward-sloping hazard curve both for prices and wages, confirming that the time-dependent mechanism accounts for the intrinsic inflation persistence for both. Except for France, countries in the sample exhibit similar duration for both prices and wages. The outcomes are in line with the micro evidence.[30] Furthermore, despite some degree of variability,[31] the results are also similar to estimates based on macroeconomic data.[32] As mentioned, France behaves as an outlier since prices and wages duration are longer than one year, i.e., 5.7 and 4.3 quarters for prices and wages, respectively. Opposite cases are Italy and Canada, performing with a lower duration for both prices and wages. Germany mimics the Italian price structure but has a lower slope for the wage hazard function leading to a higher wage expected duration, i.e., 2.2. The expected US price and wage duration is 4 and 2.2 quarters, respectively. Finally, UK has a smaller (greater) price (wage) setting structure than the US.

Table 3 – Countries' Phillips Curves: Estimated unconditional probability and duration.

Countries	q_p	D_p	q_w	D_w
Australia	0.27	2.87	0.36	1.97
Canada	0.32	2.15	0.42	1.62
France	0.15	5.68	0.21	4.32
Germany	0.27	2.84	0.34	2.20

[30] As emphasized in Section 3.1, Klenow and Malin (2011) found an average price duration of 3 quarters, while Barattieri *et al.* (2014) found 3.8 quarters average duration for wages.

[31] Depending on the sample and period used for the analysis.

[32] Among others, for the United States, Christiano *et al*. (2005) found, e.g., 2.5 and 2.8 for price and wage durations; Rabanal and Rubio-Ramirez (2005) found 4.2 and 2.3; Galì *et al*. (2011) found 2.3 and 1.8; Di Bartolomeo and Di Pietro (2017) found 3.7 and 2.0.

Italy	0.27	2.71	0.43	1.51
UK	0.23	3.41	0.33	2.48
US	0.20	4.02	0.34	2.18

5.2 Optimal policy design across countries

Now, we examine the alternative history that would have unfolded if monetary policy had been conducted according to an optimizing welfare-based criterion. To accomplish this, we extrapolate the historical values of the relevant shocks from our estimates and calculate the optimal response of the monetary authority to these shocks. Using an optimization algorithm,[33] we conduct two simulations—one for commitment and another for discretion—and obtain the counterfactual trajectory of the economy.

Our findings are presented in Figure 4 and Table 4. Figure 4 illustrates the trajectory of welfare-relevant variables (output, price, and wage inflation) obtained from the simulations of discretion and commitment alongside their observed values. Table 4 provides a summary of the standard deviations for these variables. Additionally, it includes the standard deviations observed in the data for comparison.

Compared to the observed behavior of macroeconomic variables, we find that a discretion regime generally leads to lower variability of wage inflation in most countries in our sample, except for Germany and Italy. On the other hand, due to its credibility, a commitment regime grants the central banker more influence over expectations, resulting in greater overall stabilization of inflation and inflation expectations over longer periods. This behavior leads to lower price and wage inflation variability in all countries in our sample under a commitment regime.

However, for some countries (Australia, Canada, and Italy), the optimal output gap variability in the commitment regime is slightly larger than the observed variability. Consequently, the observed variability of price inflation is higher than what would be consistent with the commitment policy regime but significantly lower than that associated with discretion across all countries.

[33] As described in Section 3.1 and further detailed in Appendix E.

Figure 4 – Observed and simulated dynamics of the welfare-relevant variables

Note: The figure shows a) by column output gap, price inflation, and wage inflation; b) by row Australia, Canada, France, Germany, Italy, the UK, and the US.

Table 4 – Observed and simulated standard deviations for the welfare-relevant variables

	Australia	Canada	France	Germany	Italy	UK	US
	Observed in the data						
Output gap	0.0102	0.0111	0.0066	0.0095	0.0083	0.0132	0.0119
Price inflation	0.0093	0.0090	0.0082	0.0056	0.0071	0.0072	0.0058
Wage inflation	0.0096	0.0101	0.0082	0.0056	0.0081	0.0072	0.0060
	Simulated under discretion						
Output gap	0.0310	0.0244	0.0309	0.0501	0.0305	0.0072	0.0197
Price inflation	0.0193	0.0333	0.0291	0.0393	0.0185	0.0160	0.0181
Wage inflation	0.0055	0.0068	0.0076	0.0294	0.0119	0.0038	0.0038
	Simulated under commitment						
Output gap	0.0147	0.0112	0.0057	0.0076	0.0118	0.0104	0.0098
Price inflation	0.0063	0.0056	0.0016	0.0046	0.0042	0.0069	0.0034
Wage inflation	0.0011	0.0007	0.0004	0.0007	0.0017	0.0008	0.0001

The impact on welfare is described in Table 5. Following, e.g., Ravenna and Walsh (2011), the table reports the welfare losses expressed as a percent of steady-state consumption.[34] The table reports the estimated losses (column (1)) and welfare losses obtained in the alternative polity scenarios (column (2)-(4).) It also reports the welfare gap between the observed loss and the counterfactual one both for discretion and commitment (column (3)-(5).)

The estimated welfare losses show that Italy and Germany got lower estimated welfare losses over the period under analysis, unlike Australia, France, and the UK, where the observed monetary policy experienced more considerable welfare costs. Counterfactual exercises show that optimal monetary policy under commitment is always less costly regarding welfare losses, expressed in terms of their welfare equivalent permanent consumption reduction confirming the general result provided by Levine *et al.* (2008). The gain of commitment over discretion becomes large in France, Canada, and the US. Moreover, its relative gain over the observed policy is particularly relevant for France. The case of Italy is interesting as the welfare losses obtained under the observed monetary regime are almost equivalent to those that would have been observed if the central bank had acted under discretion. Moreover, discretionary policies outperform in terms of welfare losses the observed Taylor-based rule in four out of seven countries in the sample, namely Australia, Canada, Germany, and the UK.

[34] It is worth noting that welfare loss in the last column does not depend on the estimations of the Taylor rule parameters, but it only depends on the observed dynamics of the relevant macro variables (cf. Table 4.)

Table 5 – Observed and simulated welfare losses (in percent of steady-state consumption)

Country	Observed	Discretion		Commitment	
	(1)	(2)	(3)	(4)	(5)
Australia	6.42	2.55	3.87	0.33	6.09
Canada	3.11	2.80	0.31	0.18	2.93
France	4.84	7.93	-3.09	0.02	4.82
Germany	1.59	0.61	0.98	0.07	1.52
Italy	0.65	0.66	-0.01	0.08	0.57
UK	4.59	1.54	3.05	0.27	4.32
US	2.82	5.03	-2.21	0.34	2.48

Table 5 shows a comparison between the different regimes in terms of welfare loss. However, this comparison can be misleading, as differences across countries depend on different historical shocks, so relative comparisons are only partially indicative of the behavior of central bankers. In the following subsection, we focus on the latter.

4.3 Central bankers' attitudes

Central bank theory emphasizes the relationship between policy regimes and expectations, as the latter is crucial in transmitting monetary policies to the economy. There are two opposing policy regimes: commitment and discretion. They differ in the central bank's ability to constrain itself contingently to observed shocks. A central banker who operates under commitment creates persistence in policy decisions to influence the actions of forward-looking agents.[35] However, in a model with intrinsic price persistence through the Phillips Curves, any central bank's actions, regardless of the regime, will tend to persist and influence expectations. Identifying the effects of different policy regimes in a Phillips Curve model with forward and backward components is, therefore, challenging.

Our approach involves assessing the consistency between the simulated and observed data. While expecting either policy regime to perfectly predict the data is unrealistic, we aim to determine how closely the observed policy aligns with one of the central banker's two policy regimes by measuring relative deviations. To achieve this, we evaluate the joint forecasting performance of each policy regime's simulation and compare it to the observed outcomes. We utilize the inverse log determinant of the variance-covariance matrix of forecast errors, divided by two to convert from variance to standard error, and then divide by the number of variables to obtain an average figure (Del Negro *et al*., 2007.)

[35] The central banker's constraint will affect current variables to the extent that the agents' action is forward-looking.

Our results are reported in Table 6. In the first row, we display our multivariate statistic in the case of the commitment regime. This statistic is always higher than the one associated with discretion, as shown in row (2), which reports the percent improvement of forecasting of commitment compared to discretion. Similarly, rows (3)-(5) present the change for the differences in performance for the specific single variables relevant to welfare (output gap and price and wage inflation.) For such a comparison, we use the change in the RMSE. Percentage improvements (positive entries) of forecasting of commitment over discretion are computed by taking the relative difference multiplied by 100.

Table 6 – Commitment vs. discretion (RMSE)

	Australia	Canada	France	Germany	Italy	UK	US.
Commitment	-0.0059	-0.0057	-0.0052	-0.0055	-0.0055	-0.0058	-0.0053
Change (%)*							
Multivariate	8.6	7.9	14.5	12.9	13.1	4.5	10.8
Output gap	43.9	59.2	77.3	19.6	67.4	-14.5	63.3
Price inflation	-8.0	58.0	64.3	69.8	40.8	15.7	39.9
Wage inflation	30.1	-11.7	49.3	16.8	11.7	24.0	37.8

Note (*). For each cell, we (successfully) tested that the change implied by the commitment to discretion was statistically different from zero.

The impact of the ECB's action has been similar across France, Italy, and Germany. Indeed, in all euro area countries, the conduct of the central banker can be cataloged as a commitment, which has greater explanatory power over the data (more than 10%) for all the three variables relevant to welfare than discretion. For instance, commitment outperforms discretion in explaining the observed data by 12.9% in Germany, i.e. there is a positive percentage improvement in forecasting accuracy of commitment relative to discretion.

Regarding RMSE, the commitment outperforms the forecasted output gap, price inflation, and wage inflation in Germany and the US. However, in the US, the commitment is more oriented towards output stabilization than inflation, while for the EU countries the monetary policy can be defined as strongly inflation oriented. A rough measure of this difference is the gap between the improvement in the output gap and inflation in RMSEs, which is 24.3 p.p. for the US and -50.2 p.p. for Germany.

It is worth noting that the analysis was carried out individually for the three-euro area countries, which may not accurately reflect the common monetary policy of the ECB. However, as the ECB was not active for the entire period under analysis and the sample does not include all the euro area countries, estimating a common monetary policy would not only be beyond the scope of this paper, but would also give an incomplete and misleading picture.[36]

6. Conclusions

We explored the implications for optimal monetary policy of macro distortions induced by intrinsic price and wage inflation inertia stemming from vintage-dependent pricing models. To this extent, we derived a model-consistent welfare-loss function by second-order Taylor approximating the households' utility function. We considered both price and wage stickiness assuming aggregate price and wage hazard functions, which allow that the probability of changing prices for a price- or wage-setter may be conditional on the length of the current price or wage spell.

The impact of macro distortions caused by intrinsic inflation inertia on optimal monetary policy is found theoretically significant and cannot be disregarded. Welfare depends on output variability and price/wage dispersion. However, in the case of vintage-dependent-price-adjustment models, price dispersion depends not only on current inflation but also on two other factors related to its intrinsic inertia: inflation persistence and the sign of the price trend. The rationale for including these factors lies in the selection effects described by Sheedy (2010). Compared to the well-known Calvo's adjustment, we found that very different losses should be associated with different assumptions about the hazard slopes since when these are not flat, more complex dynamics affect price dispersion. After illustrating the properties of the welfare loss of vintage-dependent price and wage adjustments, we examine optimal policies. Comparing the outcomes from the hazard function with different signs of the slope, we found that the performance of the policy response to cost shocks is positively related to the selection effect, i.e., to the slope of the hazard for any given average duration of price or wage spells. Welfare differences are not negligible.

After illustrating and discussing the property of the welfare loss consistent with inflation inertia stemming from the vintage-dependent adjustment process, we estimated Phillips Curves for seven countries to test the practical relevance of our argument. We examine the alternative history that would have unfolded if monetary policy had been conducted based on an optimizing welfare-based criterion. The empirical exercise emphasizes the importance of persistence in the inflation process and sacrifice ratios for making policy decisions. Our results reveal that implementing an optimal monetary policy within a commitment framework invariably reduces welfare losses, as

[36] It remains that we must exercise caution in drawing conclusions that suggest a monetary authority in the euro area has successfully reduced inflation and stabilized the economy, while the policy stance of the Fed appears to be less focused on inflation control.

evidenced by the corresponding decrease in permanent consumption. The advantages of commitment compared to discretion are especially notable in countries like France, Canada, and the United States. Additionally, we evaluate the attitudes of central bankers towards commitment and discretion by examining the forecasting accuracy of simulations for each policy regime and contrasting them with actual outcomes. This approach enables us to ascertain the extent to which the observed policy is consistent with either of the two central banker policy regimes. Considering the caveats mentioned in the previous section, the empirical findings imply that the central bank policy in the euro area tends to adhere to a commitment regime with a strong emphasis on inflation stabilization. In contrast, the US monetary policy is more inclined towards stabilizing output.

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Appendix A – Price and wage dispersion

We need an expression for both the price and wage dispersion to derive the welfare function. Only the wage dispersion derivation will be provided in what follows since the proof is equivalent to prices.[37] We write the cross-sectional mean and variance as $\bar{W}_t \equiv E_j w_t(j)$ and $\Delta_t^w = var_j[w_t(j)]$, where we define $w_t(j) = \log \ W_t(j)$.

The aggregate wage level evolves according to $w_t(j) = \sum_{l=0}^{\infty} \theta_w \log \ W_{t-l}^*$ and thus the cross-sectional mean can be written as

$$\bar{W}_t = E_j[w_t(j)] = (1-\alpha_w)\left[E_j w_{t-1}(j)\right] - \varphi_w\left[E_j w_{t-2}(j)\right] + (\alpha_w + \varphi_w) w_t^* \tag{A.1}$$

It follows that

$$\bar{W}_t - \bar{W}_{t-1} = (1-\alpha_w)\left[E_j w_{t-1}(j) - \bar{W}_{t-1}\right] - \varphi_w\left[E_j w_{t-2}(j) - \bar{W}_{t-1}\right] + \\ +(\alpha_w + \varphi_w)[w_t^* - \bar{W}_{t-1}] \tag{A.2}$$

from which we obtain:

$$w_t^* - \bar{W}_{t-1} = \frac{\pi_t^w - \varphi_w \pi_{t-1}^w}{\alpha_w + \varphi_w} \tag{A.3}$$

The cross-sectional variance $\Delta_t^w = var_j[w_t(j) - \bar{W}_{t-1}]$ is

$$E_j([w_t(j) - \bar{W}_{t-1}]^2) - (E_j[w_t(j) - \bar{W}_{t-1}])^2 = E_j([w_t(j) - \bar{W}_{t-1}]^2) - (\pi_t^w)^2 \tag{A.4}$$

where the first r.h.s. term, $E_j([w_t(j) - \bar{W}_{t-1}]^2)$, is[38]

$$(1-\alpha_w)E_j([w_{t-1}(j) - \bar{W}_{t-1}]^2) - \varphi_w E_j([w_{t-2}(j) - \bar{W}_{t-1}]^2) + \tag{A.5}$$

[37] Further details in case of the price dispersion derivation are provided by Di Bartolomeo and Di Pietro (2017). For price adjustments, see instead Sheedy (2007).

[38] See equation (A.1).

$$+(\alpha_w+\varphi_w)[w_t^*-\bar{W}_{t-1}]^2$$

By using $\Delta_{t-1}^w = E_j([w_{t-1}(j)-\bar{W}_{t-1}]^2)$, $\Delta_{t-2}^w = E_j([w_{t-2}(j)-\bar{W}_{t-1}]^2)$ and plugging (A.1) and (A.4) in (A.3), we obtain:

$$\Delta_t^w \simeq (1-\alpha_w)\Delta_{t-1}^w - \varphi_w \Delta_{t-2}^w + \frac{(\pi_t^w - \varphi_w \pi_{t-1}^w)^2}{\alpha_w+\varphi_w} - (\pi_t^w)^2 \tag{A.6}$$

Integrating (A.6), the degree of wage dispersion at any point of time from some initial period on is given by

$$\Delta_t^w \simeq (1-\alpha_w-\varphi_w)^{t+1}\Delta_{-1}^w + \sum_{s=0}^{t}(1-\alpha_w-\varphi_w)^{t-s}\left[\frac{(\pi_s^w-\varphi_w\pi_{s-1}^w)^2}{\alpha_w+\varphi_w}-(\pi_s^w)^2\right] \tag{A.7}$$

By discounting (A.7), over all periods $t \geq 0$, we get

$$\sum_{t=0}^{\infty}\beta^t\,\Delta_t^w = \sum_{t=0}^{\infty}\beta^t\left\{\sum_{s=0}^{t}(1-\alpha_w-\varphi_w)^{t+1}\left[\frac{(\pi_s^w-\varphi_w\pi_{s-1}^w)^2}{\alpha_w+\varphi_w}-(\pi_s^w)^2\right]\right\} =$$

$$= \frac{1}{1-\beta(1-\alpha_w-\varphi_w)}\sum_{t=0}^{\infty}\beta^t\left[\frac{(\pi_t^w-\varphi_w\pi_{t-1}^w)^2}{\alpha_w+\varphi_w}-(\pi_t^w)^2\right] \tag{A.8}$$

Appendix B – Comparative statics on the loss weights

Differentiating the weights of the loss (21) allows us to compute the marginal effects of changes in the shape of the hazard (α_i and φ_i) on them. Note that changes in α_i reflect those in the unconditional probability of resetting the prices, q_i. Conversely, changes in φ_i do not directly map into variations in the selection effect, s_i, as these also affect the unconditional probability of resetting the prices.

After some algebra, we can obtain for $i \in \{p, w\}$:

$$\begin{cases} \dfrac{\partial \Gamma_1^i}{\partial \alpha_i} < 0;\ \dfrac{\partial \Gamma_2^i}{\partial \alpha_i} < 0;\ \dfrac{\partial \Gamma_3^i}{\partial \alpha_i} > 0 \\ \dfrac{\partial \Gamma_1^i}{\partial \varphi_i} < 0;\ sign\left(\dfrac{\partial \Gamma_2^i}{\partial \varphi_i}\right) = sign(\varphi_i);\ \dfrac{\partial \Gamma_3^i}{\partial \varphi_i} < 0 \ \text{ if } \varphi_i < \dfrac{\left(\alpha_i\beta(1-\beta+\alpha_i\beta)\right)^{\frac{1}{2}}}{\beta} \end{cases}$$

The weights associated with the output are independent of the hazard shape.

The effects of a change in the hazard slope on economic volatility might be ambiguous.

1. If the calibration matches the observed prices/wages average durations, steeper hazards lead to lower variability for output and price and wage inflation. As shown in the main text, in such a case, an increase in φ_i raises the unconditional probability of resetting prices/wages reducing the stickiness of prices/wages to keep constant the average duration and the

economic volatility.

2. Instead, fixing q_i, an increase in φ_i raises the prices/wages average duration supporting a significant intrinsic persistence that leads to more volatile economic outcomes on average. Therefore, comparing the effects of different selection effects (s_i) keeping constant the unconditional probability of resetting the prices (q_i), the economic volatility implied by optimal policies increases in the slope of the hazard function. The rationale is the same as just explained. Keeping constant q_i, an increase of s_i raises the average duration of prices/wages and, therefore, volatility on average. However, to increase the selection effect, s_i, we need to consider a steeper slope of the hazard; in turn, this tends to reduce the unconditional duration, and therefore, it is then needed to reduce α_i in the hazard to keep constant q_i.

Appendix C

Proposition 1. Welfare costs generated by time-dependent price and wage adjustments depends on three components: i) the price dispersion; ii) the intrinsic inflation persistence; iii) the trend component of price dynamics. For any given selection effect, the costs i) and ii) are always increasing in the (price and wage) stickiness, which is measured by the complement to the unconditional probability of adjusting prices. The strength and the sign of the components associated with the inflation persistence and trend depend on the selection effect. If the selection effect is positive the cost increases when there is a reversal in the price trend and vice versa. The opposite occurs when the selection effect is negative.

Proof. Consider the loss $L(t, t-1) = \tilde{\Gamma}_1 \pi_t^2 + \tilde{\Gamma}_2 \pi_{t-1}^2 + \tilde{\Gamma}_3 s \pi_t \pi_{t-1} = \frac{g\pi_t^2 + s^2\pi_{t-1}^2 - 2s\pi_t\pi_{t-1}}{(1-\beta g)(1-g)}$, which measures the social cost for a couple of values of price (or wage) inflation observed at t and $t-1$ (π_t and π_{t-1}) as a function of the complement of the unconditional probability of resetting price (g), i.e., price stickiness, and the selection effect (s). Note that price and wage inflation equally and independently affect the loss (17), so we can discuss the point in general (we ignore subindex i.) The degree of stickiness q is mapped in $\tilde{\Gamma}_1$, $\tilde{\Gamma}_2$, and $\tilde{\Gamma}_3$. The first part of the proposition follows by observing the fact that $\frac{\partial \tilde{\Gamma}_1}{\partial g} = \frac{1-\beta g^2}{(1-\beta g)^2(1-g)^2} > 0$ and, for any given selection effect (s), by observing $\frac{\partial \tilde{\Gamma}_2}{\partial g} \propto \frac{\partial \tilde{\Gamma}_3}{\partial g} \propto \frac{1+\beta-2\beta g}{(1-\beta g)^2(1-g)^2} > 0$ as $1+\beta-2\beta g > 0$, being between zero and one. It is easy to note that the strength and the sign of the components associated with ii) and iii) depend on the selection effect. Both increase in magnitude in s, while the sign of the latter depends on the trend ($\pi_t \pi_{t-1}$.) ■

Proposition 2. By an appropriate parametrization, the welfare criterion (21) is consistent with any combination of hazard slopes for price or wage adjustments, stickiness, and habits.

Proof. By assuming $\varphi_p = \varphi_w = 0$ (flat slopes), $h = 0$ (no habits), and $\varepsilon_w \to \infty$ (flexible wage), it becomes that $\tilde{\Gamma}_2^i = \tilde{\Gamma}_3^i = 0$ for $i \in \{p, w\}$, $\Gamma_2^y = \Gamma_3^y = 0$ and $\Gamma_1^w = 0$, therefore the loss (21) can be rewritten exactly as in Galì (2008: Chapter 4, p.81): $w_t \simeq -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\Gamma_1^y y_t^2 + \Gamma_1^p \pi_t^p\right]$, where $\Gamma_1^p = \frac{\varepsilon_p}{\Theta_p}$ and $\Gamma_1^y = \left(\sigma + \frac{\gamma+\delta}{1-\delta}\right)$. It follows that the loss function (21) simplifies to a loss consistent with a forward-looking sticky price in Calvo's economy. ■

Proposition 3. The price hazard affects welfare independently of the wage hazard and vice versa. The rationale of the result is that price and wage dispersion are independent.

Proof. The proposition directly derives from the mixed derivatives of the welfare loss, w_t, i.e., for any inflation and output path ($\{\pi_i^p, \pi_i^w, x_i\}_{i=0}^T$), $\frac{\partial w_t^2}{\partial \alpha_p \partial \alpha_w} = \frac{\partial w_t^2}{\partial \alpha_p \partial \varphi_w} = \frac{\partial w_t^2}{\partial \alpha_w \partial \varphi_p} = \frac{\partial w_t^2}{\partial \varphi_p \partial \varphi_w} = 0$. ■

Appendix D

This appendix shows how keeping constant the average duration, increases in φ_i for $i \in \{p, w\}$ reduce the stickiness of prices/wages. Assume the probability of resetting a price in the Calvo model is α_i^C (with an average price duration $1/\alpha_i^C$), and then it is easy to check that to keep the duration constant independently of the slope hazard we need to fix $\alpha_i = (1 - \varphi_i)\alpha_i^C - \varphi_i$. This implies $\alpha_i < \alpha_i^C$ when $\varphi_i > 0$ and vice versa. It follows that, keeping constant the average price duration, the unconditional probability of resetting the price ($\alpha_i + \varphi_i = (1 - \varphi_i)\alpha_i^C$) is smaller than α_i^C when the hazard is positively sloped ($\varphi_i > 0$), and vice versa. Assuming the same duration, prices are more flexible when $\varphi_i > 0$ than $\varphi_i < 0$.

Appendix E

The algorithm is a three-step optimization procedure to find the optimal policy for a theoretical macro model. The algorithm is based on Soderlind (1999) and requires the estimation results from the model as inputs.

1. The first step of the algorithm is to calibrate the deep parameters of the model to the estimated values, which involves adjusting the model parameters to match the estimated time series. This step is solved by Bayesian estimation of the model.

2. The second step is to extract the historical-shock series from the estimation results, which involves identifying the patterns of shocks that have occurred in the past and using this information to inform the model.
3. The third step of the algorithm is to run the Soderlind (1999) algorithm to compute the optimal policy in response to the dynamics of the observed shocks and the subsequent economic reaction. Note that the algorithm considers the monetary authority's past promises when commitment is simulated.

Overall, the algorithm is a way to optimize the policy response of a theoretical macro model based on the estimation results from the data. The algorithm seeks to find the optimal policy given the historical patterns of shocks.

Technical Appendix – A DSGE model with vintage-duration price and wage adjustments

A1. Duration-dependent-price adjustments

We consider a rich price/wage setting mechanism, assuming that the probability of changing prices[39] may not be independent of the time elapsed since the last reset. Formally, this implies considering the probabilities of posting a new price conditional on the length of a price spell and the probability that a price fixed in the past survives in the future. The hazard and survival functions connect these probabilities.

The hazard function represents the distribution of the length of time that has elapsed from the last price reset, i.e., the hazard function denotes the probability of a price change conditional on the event that a price has been unchanged for the previous l periods. Its output is called the hazard rate, α_l.

Formally, the hazard function is defined by the sequence of probabilities $\{\alpha_l\}_{l=1}^{\infty}$ and can be parameterized by making use of a set of $n+1$ parameters:

$$\alpha_l = \alpha + \Sigma_{j=1}^{\min(l-1,n)} \varphi_j \left[\Pi_{k=l-j}^{l-1}(1-\alpha_k)\right]^{-1}, \tag{1}$$

where α is the initial value of the hazard function, while φ_j are terms that determine the hazard slope (n is the number of parameters that control the slope, i.e., the sequence of parameters $\{\varphi_l\}_{l=1}^{n}$ affecting the gradient of the hazard function). For the sake of simplicity, and coherently with some macro empirical works,[40] we assume that only one parameter controls the slope of the hazard. Then, for $n = 1$ the hazard defined in (1) becomes:

$$\alpha_l = \alpha + \frac{\varphi}{1-\alpha_{l-1}}, \quad \text{for } l > 1 \tag{2}$$

with $\alpha_1 = \alpha$. According to (2), the hazard can be then

1. flat (when $\varphi = 0$);
2. upward-sloping (when $\varphi > 0$);
3. downward-sloping (when $\varphi < 0$).

The slope of the hazard function controls our vintage-price-dependent adjustment. Dealing with a flat hazard implies that the probability of posting a new price is independent of the time elapsed since the last reset: this happens in the Calvo lottery, where the probability of being selected for a

[39] Here, we use the terms price and wage interchangeably.

[40] See, e.g., Sheedy (2007, 2010) and Di Bartolomeo and Di Pietro (2017).

price change is random. A positive slope shifts upward the hazard rate, then older prices becomes more likely to be reset than newer prices. Similarly, a negative slope implies that older prices are less likely to be reset than newer prices.

This kind of pricing mechanism gives rise to a “selection effect.” Economics agents (firms or households) aim to adjust prices to close the wedge between the actual relative price and the desired price. Due to the presence of nominal rigidities, when a temporary inflationary shock hits the economy, some agents cannot adjust their prices, but the average price level has increased because some other agents were able to update prices. In the next period, as the shock has dissipated, the agents that adjusted are charging a higher price than the desired one, whereas agents that did not adjust meet lower prices because price inflation occurred. It follows that the latter will attempt to raise their prices to maintain the desired relative price (“catch-up” effect) and the former to reduce them because now their relative price is too high (“roll-back” effect). If both groups have the same probability of readjusting prices, as in Calvo, the two effects offset one another, but this does occur under non-constant hazard.

Under positively (negatively) sloped hazard functions, prices that remained unvaried for longer (shorter) periods are more (less) likely to be updated, involving a positive (negative) selection effect. Positive hazard functions can generate intrinsic inflation persistence because the “catch-up” effect prevails over the “roll-back” effect and, thus, inflation remains positive even after the shock dampens.

The hazard function is related to a survival function, which expresses the probability that a price remains fixed for l periods. A sequence of probabilities also defines the survival function: $\{\varsigma_l\}_{l=0}^{\infty}$, where ς_l represents the probability that a price last updated at time t will remain in use at time $t+l$ (survival rate). Formally, the survival function is defined as:

$$\varsigma_l = \Pi_{h=1}^{l}(1-\alpha_h) \tag{3}$$

where $\varsigma_0 = 1$.

By using the definition of the survival function provided in (3), we can transform the non-linear recursion (2) for the price adjustment probabilities into a linear recursion for the corresponding survival function:

$$\varsigma_l = (1-\alpha)\varsigma_{l-1} - \varphi\varsigma_{l-2}, \quad \text{for } l > 1 \tag{4}$$

where $\varsigma_1 = (1-\alpha)$ for $l = 1$.

Denoting with θ_{lt} the fraction of agents (e.g., monopoly firms in the case of price reset or trade unions in the case of wage reset) that at time t adopt a price updated $t-l$ periods ago, the sequence $\{\theta_{lt}\}_{l=0}^{\infty}$ then indicates the distribution of the duration of price stickiness at time t. If both the hazard function and the evolution over the time of the distribution of price duration satisfy some regularity conditions,[41] the following relation holds:

$$\theta_l = (\alpha + \varphi)\varsigma_l \quad (5)$$

where θ_l represents the unique stationary distribution to which the economy always converges. Moreover, in such a case, the unconditional probability of a price reset is $\alpha + \varphi$ and the unconditional expected duration of price stickiness is $(1-\varphi)/(\alpha+\varphi)$.

The pricing mechanism with non-constant hazard functions encompasses the Calvo pricing mechanism as a special case: as explained before, when the hazard is flat, i.e., for $\varphi = 0$, the probability of posting a new price is time independent of time, and the expected duration of a price spell reduces to $1/\alpha$ as in the Calvo lottery (where α, in this case, denotes the likelihood to update a price).

A2. The DSGE model

We consider a simple small-scale New Keynesian DSGE model characterized by monopolistic competition in the goods and labor markets. Due to monopolistic competition, firms and households are price and wage-setters, respectively. We assume the presence of nominal price and wage rigidities modeled according to the vintage-dependent mechanism described above. On the demand side, the economy is populated by many identical households. Each household comprises a continuum of members that supply different labor services and consume goods. On the supply side, we have a continuum of firms that produce a differentiated good using household labor.

Households

We assume a representative infinitely lived household j that seeks to maximize the expected sum of period utility from consuming goods ($C_t(j)$) and supply a specific type of labor ($N_t(j)$) over all future periods, discounted by the time preferences, β, which is defined as:

$$\mathcal{W}_t = E_0\left\{\Sigma_{t=0}^{\infty}\beta^t\left[\exp\left(z_t\right)\frac{(C_t(j) - hC_{t-1}(j))^{1-\sigma}}{1-\sigma} - \frac{N_t(j)^{1+\gamma}}{1+\gamma}\right]\right\} \quad (6)$$

[41] See Sheedy (2007).

where E_0 is the expectation operator conditional on time $t = 0$ information, σ is the relative-risk-aversion coefficient, γ is the inverse of the Frisch elasticity, h is an internal habit on consumption, and z_t is a stochastic preference disturbance evolving as a stationary $AR(1)$ process. The consumption index $C_t(j)$ is given by:

$$C_t(j) = \left[\int_0^1 C_t(i,j)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \tag{7}$$

where $C_t(i,j)$ indicates the quantity of good i consumed by the household j in period t and ε_p denotes the elasticity of substitution between goods and is a measure of the degree of monopoly of the representative firm.

Assuming complete financial markets, the household faces a standard budget constraint specified in nominal terms as follows:

$$P_t C_t(j) + Q_t B_t(j) \leq B_{t-1}(j) + W_t(j) N_t(j) + T_t(j) \tag{8}$$

where B_t denotes the holdings of one-period nominally riskless state-contingent bonds purchased in period t and maturing in period $t+1$, Q_t is the bond price, T_t represents a lump-sum government nominal transfer and $W_t(j)$ is the nominal wage paid to household j. The term P_t is the aggregate price index and evolves as

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon_p} di\right]^{\frac{1}{1-\varepsilon_p}} \tag{9}$$

The household chooses the level of consumption and bonds holding that maximize (6) subject to (8). The resulting first-order conditions are:[42]

$$U_{c,t} = \exp(z_t)(C_t - hC_{t-1})^{-\sigma} - \beta h E_t \exp(z_{t+1})(C_{t+1} - hC_t)^{-\sigma} \tag{10}$$

$$E_t \Lambda_{t,t+1} \frac{R_t}{\Pi_{t+1}^p} = 1 \tag{11}$$

where $U_{c,t}$ denotes the marginal utility of consumption; $\Lambda_{t,t+1} = \beta E_t U_{c,t+1}/U_{c,t}$ is the stochastic discount rate; $\Pi_t^p = P_t/P_{t-1}$ is the gross price inflation rate. The first equation represents the marginal utility of consumption for agents who can access credit markets; the second is the Euler equation.

[42] The optimal condition for the labor supply will be discussed later.

Moreover, the representative household must also decide how to allocate its consumption expenditure among the differentiated goods. This involves maximizing the consumption index (7) for any given level of expenditure. Accordingly, the demand for consumption is

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_p} C_t \tag{12}$$

Combining (10) and (11), we obtain:

$$\beta E_t\left[\frac{\exp\,(z_{t+1})(C_{t+1} - hC_t)^{-\sigma} - \beta h\exp(z_{t+2})\,(C_{t+2} - hC_{t+1})^{-\sigma}}{\exp\,(z_t)(C_t - hC_{t-1})^{-\sigma} - \beta h\exp\,(z_{t+1})(C_{t+1} - hC_t)^{-\sigma}}\right]\frac{R_t}{\Pi_{t+1}^p} = 1 \tag{13}$$

where $\Pi_{t+1}^p = P_{t+1}/P_t$ and $R_t = 1/Q_t$, this latter expresses the well-known inverse relation between asset price and asset return.

Firms

The supply side of the economy is made up of a continuum of monopolistically competitive firms. The production function of the representative firm i is described by a Cobb-Douglas without capital:

$$Y_t(i) = A_t N_t(i)^{1-\phi}, \tag{14}$$

where $Y_t(i)$ is the output of good i at time t, A_t represents the state of technology, $N_t(i)$ is the quantity of labor employed by i-firm and $1-\phi$ measures the elasticity of output with respect to labor.[43] The quantity of labor used by firm i is defined by:

$$N_t(i) = \left[\int_0^1 N_t(i,j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \tag{15}$$

where $N_t(i,j)$ is the quantity of j-type labor employed by firm i in period t, whereas ε_w measures the elasticity of substitution between workers.

Cost minimization for the quantity of labor employed yields to the labor demand schedule:

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t(i) \tag{16}$$

where $W_t(j)$ is the nominal wage paid to j-type worker and W_t is the aggregate wage index defined as:

[43] As there is no capital, the aggregate resource constraint involves that consumption is equal to output.

$$W_t = \left[\int_0^1 W_t(j)^{1-\varepsilon_w} dj\right]^{\frac{1}{1-\varepsilon_w}} \tag{17}$$

Given its demand (16), the representative firm j chooses the optimal price P_t^* that maximizes the discounted expected future stream of profits:

$$\max_{P_t^*} \sum_{\tau=t}^{\infty} \left(\beta^{\tau-t} \varsigma_{p,\tau-t}\right) E_t \left\{ \frac{U_{c,\tau}}{U_{c,t}} \left[\frac{P_t^*}{P_t} Y_{\tau|t} - \mathcal{C}_t\left(Y_{\tau|t}\right) \right] \right\} \tag{18}$$

where β is the discount factor; $U_{c,\tau}$ denotes the marginal utility of consumption for the firms whose last price reset was in period τ; $\mathcal{C}_t\left(Y_{\tau|t}\right)$ is the real cost function. The key parameter in (18) is the price survival rate ($\varsigma_{p,\tau-t}$), which is the probability that a price last updated at time t remains in use at time $t+\tau$. The evolution of the survival rate has been already defined.

The first-order condition of this problem is given by:

$$\sum_{\tau=t}^{\infty} \left(\beta^{\tau-t} \varsigma_{p,\tau-t}\right) E_t \left\{ \frac{U_{c,\tau}}{U_{c,t}} Y_{\tau|t} \left[\frac{P_t^*}{P_t} - \mu_p MC_{\tau|t} \right] \right\} = 0 \tag{19}$$

where μ_p is the average price markup and $MC_{\tau|t}$ is the real marginal cost. Taking in account that the price index is given by (9), we can write it in terms of a time-invariant weighted average of past reset prices:

$$P_t = \left(\Sigma_{l=0}^{\infty} \theta_{p,l} {P_{t-l}^*}^{1-\varepsilon_p}\right)^{\frac{1}{1-\varepsilon_p}} \tag{20}$$

where the term $\theta_{p,l}$ indicates the stationary fraction of firms using a price last posted l periods ago. To get an expression for the price Phillips curve, we log-linearize (19) and (20) around a deterministic steady state characterized by no trend inflation.[44] In this way, we obtain the linearized version of the equations describing the price adjustment process. Using the definition of survival function provided in (4) and the evolution of the unique stationary distribution (5) to which the economy converges, we get

$$\begin{aligned} p_t^* = \beta\left(1-\alpha_p\right) E_t p_{t+1}^* - \beta^2 \varphi_p E_t p_{t+2}^* + [1 - \beta\left(1-\alpha_p\right) \\ + \beta^2 \varphi_p](p_t - \Xi_p MC_t) \end{aligned} \tag{21}$$

and

[44] Small-caps letters denote deviations from the steady state.

$$p_t = (1-\alpha_p)p_{t-1} - \varphi_p p_{t-2} + (\alpha_p + \varphi_p)p_t^* \quad (22)$$

where $\Xi_p = (1-\phi)/(1-\phi+\phi\varepsilon_p)$, is the elasticity of real marginal cost for the firm's own output, while α_p and φ_p are the parameters affecting the initial value and the slope of the price hazard, respectively.[45]

The real marginal cost is given by

$$MC_t = \frac{(1-\phi)\Omega_t N_t}{Y_t} \quad (23)$$

where $\Omega_t = W_t/P_t$ is the real wage, which can be written as

$$\Omega_t = \frac{\Pi_t^W}{\Pi_t^P}\Omega_{t-1} \quad (24)$$

where $\Pi_t^W = W_t/W_{t-1}$ defines the gross wage inflation rate.

Labor market

Households are wage setters. In setting wages, each maximizes (6) taking account of (15) and the labor demand (16). The optimization problem of a household consists of choosing the optimal reset wage that maximizes:

$$\max_{W_t^*} \sum_{\tau=t}^{\infty} \left(\beta^{\tau-t}\varsigma_{w,\tau-t}\right) E_t \left[U_{c,\tau}\frac{W_t^*}{P_\tau}N_{\tau|t} - \frac{{N_{\tau|t}}^{1+\gamma}}{1+\gamma}\right] \quad (25)$$

where $U_{c,\tau}$ indicates the marginal utility of consumption and $N_{\tau|t}$ denotes the level of employment in period τ among workers whose last wage reset was in period t. As for the optimal price choice problem in (16), the parameter $(\varsigma_{w,\tau-t})$, denoting the wage survival rate, is used to discount. Again, the evolution of the survival rate is that previously defined.

The first order condition coming from this maximization is given by:

$$\sum_{\tau=t}^{\infty} \left(\beta^{\tau-t}\varsigma_{w,\tau-t}\right) E_t \left\{N_{\tau|t}U_{c,\tau}\left[\frac{W_t^*}{P_\tau} - \mu_w MRS_{\tau|t}\right]\right\} = 0 \quad (26)$$

where $N_{\tau|t}$ denotes the labor, μ_w represents the desired average wage markup, $MRS_{\tau|t}$ is the marginal rate of substitution between consumption and labor in period τ, and W_t^*/P_τ is the real wage earned

[45] For a detailed derivation of the Phillips curve under non-constant hazard function see Sheedy (2007, 2010).

by a household at time $\tau \geq t$, which last wage reset was at time t. The marginal rate of substitution is defined as:

$$MRS_t = -\frac{U_{n,t}}{U_{c,t}} \tag{27}$$

where the term $U_{n,t}$ denotes the marginal utility of labor.

The aggregate wage index reported in (17) can be expressed as a weighted average of past reset wages:

$$W_t = \left(\Sigma_{l=0}^{\infty} \theta_{w,l} {W_{t-l}^{*}}^{1-\varepsilon_w}\right)^{\frac{1}{1-\varepsilon_w}} \tag{28}$$

where $\theta_{w,l}$ represents the stationary fraction of households earning a wage last updated l periods ago.

Taking a first-order Taylor expansion of (26) and (28) around the non-stochastic steady state, we can obtain the linearized version of the equations describing the wage adjustment mechanism. As for the price adjustment mechanism described before, we use the definition of survival function given by (4) and the evolution of the stationary distribution provided by (5) and, then, get

$$\begin{aligned} w_t^* = \beta(1-\alpha_w)E_t w_{t+1}^* - \beta^2\varphi_w E_t w_{t+2}^* + [1 - \beta(1-\alpha_w) \\ + \beta^2\varphi_w](w_t - \Xi_w \mu_t^w) \end{aligned} \tag{29}$$

and

$$w_t = (1-\alpha_w)w_{t-1} - \varphi_w w_{t-2} + (\alpha_w + \varphi_w)w_t^* \tag{30}$$

where $\Xi_w = 1/(1+\varepsilon_w\gamma)$, μ_t^w denotes the deviations of the economy's average wage markup from its desired level, while α_w and φ_w are the parameters affecting the initial value and the hazard slope for the wages, respectively.[46]

Monetary policy

The nominal interest rate R_t is set according to a simple Taylor rule

$$\frac{R_t}{\overline{R}} = \left(\frac{R_{t-1}}{\overline{R}}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\delta_\pi} \left(\frac{Y_t}{\bar{Y}_t}\right)^{\delta_y}\right]^{1-\rho_R} \exp\left(\varepsilon_t^R\right) \tag{31}$$

where ρ_R denotes the degree of interest rate smoothing, δ_π measures the response of the monetary authority to inflation, δ_y is the response of the interest rate to the output gap and ε_t^R is a i.i.d. monetary

[46] Further details about the wage Phillips derivation under the hazard function mechanism see Di Bartolomeo and Di Pietro (2017).

policy shock. The term $\overline{R}$ denotes the steady state value of the nominal interest rate and is equal to β^{-1}.

A3. Derivation of the price- and wage-adjustment equations

We start from the log-linearized duration-dependent price Phillips curve, which is derived as follows. The duration dependent price adjustment mechanism is described by equations (19) and (20). Taking their first-order Taylor approximation, we obtain

$$p_t^* = \sum_{\tau=t}^{\infty} \left(\frac{\beta^{\tau-t} \varsigma_{p,\tau-t}}{\sum_{j=0}^{\infty} \beta^j \varsigma_{p,j}} \right) E_t \left[p_t - \Xi_p MC_t \right] \tag{32}$$

$$p_t = \sum_{l=0}^{\infty} \theta_{p,l} p_{t-l}^* \tag{33}$$

By exploiting the recursive parametrization of the survival functions given by equation (4), equation (32) can be rewritten as:

$$\begin{aligned} p_t^* = \beta(1-\alpha_p) E_t p_{t+1}^* - \beta^2 \varphi_p E_t p_{t+2}^* + [1 - \beta(1-\alpha_p) \\ + \beta^2 \varphi_p](p_t - \Xi_p MC_t) \end{aligned} \tag{34}$$

The recursive parametrization of the hazard function implies a recursion for the stationary distribution of the duration of wage stickiness given by (5). Thus, we can rewrite (33) in recursive form as:

$$p_t = (1-\alpha_p) p_{t-1} - \varphi_p p_{t-2} + (\alpha_p + \varphi_p) p_t^* \tag{35}$$

Finally combining equations (34) and (35), we get an expression for the duration-dependent price Phillips curve:

$$\pi_t^p = \psi_p \pi_{t-1}^p + \beta[1 + (1-\beta)\psi_p] E_t \pi_{t+1}^p - \beta^2 \psi_p E_t \pi_{t+2}^p + k_p (mc_t + \zeta_t) \tag{36}$$

Where the terms ψ_p and k_p are functions of the parameters governing the hazard functions and have the following form:

$$\psi_p = \frac{\varphi_p}{(1-\alpha_p) - \varphi_p[1 - \beta(1-\alpha_p)]}$$

$$k_p = \frac{(\alpha_p + \varphi_p)[1 - \beta(1-\alpha_p) + \beta^2 \varphi_p]}{(1-\alpha_p) - \varphi_p[1 - \beta(1-\alpha_p)]} \frac{1-\phi}{1-\phi+\phi\varepsilon_p}$$

Now we show the derivation of the log-linearized duration-dependent wage Phillips curve. We proceed as follows. The key equations describing the wage adjustment mechanism are (26) and (28). We take the first-order Taylor approximation of both and get

$$w_t^* = \sum_{\tau=t}^{\infty} \left(\frac{\beta^{\tau-t} \varsigma_{w,\tau-t}}{\sum_{j=0}^{\infty} \beta^j \varsigma_{w,j}} \right) E_t \left[w_\tau - \Xi_w \mu_t^w \right] \tag{37}$$

$$w_t = \sum_{l=0}^{\infty} \theta_{w,l} w_{t-l}^* \tag{38}$$

Given the recursive parametrization of the survival functions as expressed by equation (4), equation (37) can be rearranged as

$$\begin{aligned} w_t^* = \beta(1-\alpha_w) E_t w_{t+1}^* - \beta^2 \varphi_w E_t w_{t+2}^* + [1 - \beta(1-\alpha_w) \\ + \beta^2 \varphi_w](w_t - \Xi_w \mu_t^w) \end{aligned} \tag{39}$$

We then exploit the recursion for the stationary distribution of the duration of wage stickiness given by (5) to rewrite (38) in recursive form as:

$$w_t = (1-\alpha_w) w_{t-1} - \varphi_w w_{t-2} + (\alpha_w + \varphi_w) w_t^* \tag{40}$$

Finally combining equations (39) and (40), we get an expression for a duration-dependent wage Phillips curve:

$$\pi_t^w = \psi_w \pi_{t-1}^w + \beta[1 + (1-\beta)\psi_w] E_t \pi_{t+1}^w - \beta^2 \psi_w E_t \pi_{t+2}^w - k_w(\omega_t - mrs_t) \tag{41}$$

Where the terms ψ_w and k_w are again functions of the parameters governing the hazard functions and have the following form:

$$\psi_w = \frac{\varphi_w}{(1-\alpha_w) - \varphi_w[1 - \beta(1-\alpha_w)]}$$

$$k_w = \frac{(\alpha_w + \varphi_w)[1 - \beta(1-\alpha_w) + \beta^2 \varphi_w]}{(1-\alpha_w) - \varphi_w[1 - \beta(1-\alpha_w)]} \frac{1}{1 + \varepsilon_w \gamma}$$

A4. The log-linearized economy

Log-linearizing around the steady state the model described in A2 (i.e., (13), (14), (23), (27), (24), and (31)), we obtain:[47]

[47] Small-caps letters denote deviations from the steady state. Log-linearizations are trivial, apart from those regarding the Phillips curves that are described in A3.

$$y_t = \frac{1}{1+h}E_t y_{t+1} + \frac{h}{1+h}y_{t-1} - \frac{1-h}{\sigma(1+h)}\left(r_t - E_t\pi_{t+1}^p + E_t z_{t+1} - z_t\right) \quad (42)$$

$$y_t = a_t + (1-\phi)n_t \quad (43)$$

$$mc_t = \omega_t + n_t - y_t \quad (44)$$

$$mrs_t = \frac{\sigma}{1-h}(y_t - hy_{t-1}) + \gamma n_t - z_t \quad (45)$$

$$\omega_t = \pi_t^w - \pi_t^p + \omega_{t-1} \quad (46)$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left(\delta_\pi \pi_t^p + \delta_y y_t\right) + \varepsilon_t^r \quad (47)$$

$$\pi_t^p = \psi_p \pi_{t-1}^p + \beta\left[1 + (1-\beta)\psi_p\right]E_t\pi_{t+1}^p - \beta^2\psi_p E_t\pi_{t+2}^p + k_p(mc_t + \zeta_t) \quad (48)$$

$$\pi_t^w = \psi_w \pi_{t-1}^w + \beta[1 + (1-\beta)\psi_w]E_t\pi_{t+1}^w - \beta^2\psi_w E_t\pi_{t+2}^w - k_w(\omega_t - mrs_t) \quad (49)$$

where ζ_t is an additive price-mark-up shock; the structure of the stochastic shocks is as follows:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \quad (50)$$

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z \quad (51)$$

$$\zeta_t = \rho_\zeta \zeta_{t-1} + \varepsilon_t^\zeta \quad (52)$$

where $\varepsilon_t^j \sim N\left(0, \sigma_j^2\right)$ are white noise shocks uncorrelated among them and ρ_j are the parameters measuring the degree of autocorrelation (or monetary policy inertia), for $j = \{a, z, \zeta, r\}$.