The Unlevered Systematic Risk Analysis: A New Bottom Up Approach

Antonio Renzi *, Giuseppe Sancetta **, and Beatrice Orlando ***

The aim of the current paper is to propose a bottom-up approach as a complement in risk return analyses, particularly suitable for private firms and divisional evaluation. In those countries where private firms are more common than public firms, firm's economic fundamentals are more significant than market data. In this case, the use of a bottom up approach for the beta estimation is more suitable than a top down method, because it allows to consider both environmental and business drivers of risk and to study the systematic risk as a function of firm fundamentals. The paper is structured in three sections: in the first section, there is the literature review on the CAPM and the beta estimation; in the second section, there is the explanation of the model - a bottom up approach to unlevered risk - considering the role of the intrinsic business risk and of the operating leverage; finally, it is presented the analysis of the unlevered beta, according the bottom up model, both in stationary and the dynamic states.

JEL Codes: G3; G31; G32

1. Introduction

According to CAPM, the expected return of a share depends on three variables: the risk free rate, the average market premium, and the beta value (Lintner, 1965; Mossin, 1966; Sharpe, 1964). The beta value is the amount of market risk absorbed by an individual firm. The CAPM is founded on the idea that a rational investor tends to optimize the portfolio diversification, for eliminating the idiosyncratic risk. Consequently, the expected return can be influenced only by the systematic risk. The CAPM considers both investors’ expectations and investors attitude to optimize the risk-return relationship, thanks to the possibility of arbitraging between the risk free debts and the risk assets. In a perfect equilibrium market, with perfect competition, homogeneous expectations and information efficiency, the arbitraging continuously balances assets’ demand and supply. But real markets are far from perfect competition equilibrium. An alternative to the CAPM could be the use of a bottom up method to measure systematic risk. In the bottom up logic, the systematic risk is partially dependent on firm endogenous variables. Therefore, two kinds of systematic risk drivers should be considered: external and internal ones. Internal drivers of systematic risk absorption are linked to a firm’s industrial and capital structure. Environmental factors, such as market area competing conditions, act as drivers of intrinsic business risk. According to this perspective, the systematic risk is the result of the interaction between firm’s structural

* Professor Antonio Renzi, Department of Management, Sapienza University of Rome, Italy. Email: antonio.renzi@uniroma1.it, contact number (0)39
** Professor Giuseppe Sancetta, Department of Management, Sapienza University of Rome, Italy. Email: giuseppe.sancetta@uniroma1.it, (0)39
*** Dr. Beatrice Orlando, Department of Management, Sapienza University of Rome, Italy. Email: beatrice.orlando@uniroma1.it; (0)39
characteristics and the degree of volatility common to the business area. The adoption of a bottom up approach is useful both for firms and for financial analysts, since it allows the study of the systematic risk as a function of firm fundamentals. The risk breakdown gives some helpful information on the firm possibility to maneuver the beta through managerial decisions and it increases the effectiveness of the unlevered beta estimation. Usually, practitioners estimate the unlevered beta as the average market beta. The adoption of a bottom up approach allows to overcome this strong limit and to differentiate company beta according to firm' specific characteristics. The aim of the paper is to introduce an alternative bottom up approach to traditional risk-return models. The paper is structured in three parts: in the first section, there is the literature review on the CAPM and the beta estimation; in the second section there is the explanation of the model - a bottom up approach to unlevered risk - considering the role both of intrinsic business risk and of operating leverage; finally, it is presented the analysis of the unlevered beta, according the bottom up model, both in stationary and the dynamic states. Finally, authors suggest some conclusions.


As Fama says, the Capital Asset Pricing Model, CAPM, and its descending versions answer to the question "What is the appropriate measure of the risk of a capital asset?" (Fama, 1968). Originally, the Markowitz (1959) model of the efficient frontier states the efficient portfolio is that one giving the highest expected return for each given level of risk, considering that investors are risk averse. So, basing on the efficient frontier and its criteria of risk diversification, each investor can make an optimal choice in line with individual risk characteristics. The development of the CAPM is based on this essential concept. In particular, the main assumption is the firm specific risk \( \alpha \), also called idiosyncratic risk, is completely diversifiable; thus, the return depend on the systematic risk \( \beta \), that is intended completely not diversifiable, common to all securities and due to the market shocks. The general model is based on the logic of the perfect efficiency of market (Fama, 1970). The model was initially introduced by Treynor (1961), but quite contemporary other authors worked on the Markowitz theory (Sharpe, 1964; Lintner, 1965; Mossin, 1966), and it relies on the linear assumption that the return of each risky asset is related to the return of the market portfolio. Lintner’ version of the model, criticizing Sharpe, includes the disturbance effect (Lintner, 1965), where Sharpe do not takes in account errors (Sharpe, 1964), but Fama proves both versions lead to equal results (Fama, 1968). The model considers two hypotheses: the complete agreement of investors; and the borrowing and lending at a free-rate assumption (Fama and French, 2004). The Black model adds that the expected return on assets uncorrelated with the market must be less than the expected market return (Black, 1972). A later version of the model is the ICAM, inter-temporal capital asset pricing model, proposed by Merton, where investors are also concerned to consume or invest the end-of-the- period payoff (Merton, 1973). The relation between the return and the \( \beta \) is not clear: there could be a negative of a flat relationship between the two variables (Jensen et al, 1972). To the end of this article, it is important to remark the capital asset pricing models is conceived for asset traded on a public market. The problem for private companies is they are assumed substantially similar to public comparables, without considering their specificity and their private nature. In particular, the CAPM presumes that investors decide without analyzing firm fundamental. Moreover, private companies don’t have a well defined historical variance for their assets, since they are not publically traded, while the CAPM use historical variance as a measure of risk. So, standing on the general assumption for what securities
market efficiently reflects information regarding individual and overall stock market (Fama, 1970), for private firms it is needed to adjust this measure considering the lack of information for non traded assets.

Lev (1972) warns that risk measures based merely on historical analysis are inappropriate. Beaver, Kettler and Scholes (1970) showed that the use of fundamentals as instrumental variables allows a better prediction of future returns. The Bowman’s risk-return paradox (1980) and the prospect theory of Khaneman and Tversky (1979), show the risk assumptions underlying the CAPM model are in contrast with later empirical tests, but their findings are partially mitigated by other research evidences: including in the sample firms outside the public market, as delisted firms, may moderate the negative relationship (Chou et al, 2009).

3. Bottom up approach antecedents

The first attempt of linking firm specific characteristic to the systematic risk was the Hamada (1972) analysis of the firm’s capital structure on the β. Lev (1974) later investigation tied the operating leverage to the market value of the firm, confirming that a variation in the degree of the operating leverage induces a variation of the same sign in the overall and systematic volatility of the stock return. Conine (1980), extending Hamada’ model, incorporate risky debts into the computation of levered beta. Banz (1981) remarks the existence of a size effect, depending on operating leverage. Fama and French (1992) find the beta measure is linked to firm size and to the book-to-market equity ratio. Hawawini and Viallet (1999) suggest an interpretation of business risk, the risk faced by all firms, as a combination of economic and operating risk. Many later studies connect the systematic risk to fundamental analysis (Gahlon and Gentry, 1982; Rhee, 1986; Mandelker and Rhee, 1984; Chung, 1989). Benea (2003) suggests to measure the unlevered beta starting from the selection of a group of comparables. Other scholars connect the beta calculation to firm’s size and operating leverage (Carlson et al., 2004). Latter studies split the beta risk into different economics and business components (Roodposhti et al, 2009) and confirm the validity of fundamental measures for asset beta calculation (Mseddi and Amid, 2010).

4. The bottom up analysis of unlevered risk

Focusing on unlevered risk, many of the scholars have modeled the volatility of operating performances as a function of the intrinsic business risk and of the operating leverage. The intrinsic business risk is related to revenue volatility. This kind of dispersion depends both on internal decisions and external factors. According to Chung (1989), each firm is subject to the intrinsic business risk because of the business area the firm belongs to. In this perspective, the intrinsic business risk is assumed as exogenous to firm’s decisions. Ross, Westerfield, and Jaffe (2002) remark how revenue volatility could depend on endogenous factors as a discontinuity in firm’s sales volume and price, due to innovation activities or other transformations in course. The intrinsic business risk can be measured as the standard deviation of expected revenues:

\[
IBR = \sqrt{\sum_{i=1}^{n} p_i \left(REV_i - REV'\right)^2} 
\]

with \( REV' = \sum_{i=1}^{n} p_i \left(REV_i \right) \)
In the formula above, $IBR$ stands for intrinsic business risk, $p_i$ for probability of the $i$-th scenario, $REV_i$ are the revenues of the $i$-th scenario, and $e \cdot REV'$ are the average expected revenues.

The degree of intensity of $IBR$ effects on operating earning volatility depends on the rigidness of operating costs, and therefore, on a firm’s structural characteristics. As the level of fixed costs grows, the elasticity of EBIT increases more than fluctuation in demand. Thus, the EBIT elasticity corresponds to the degree of operating leverage, since it acts as an endogenous driver of unlevered risk:

$$DOL = \frac{EBIT}{EBIT_{t0}} \cdot \frac{REV_{t0}}{REV} \quad (2)$$

Where, $DOL$ stands for the degree of operating leverage.

Assuming the level of fixed and cost per sold unit are constant during the temporal range comprised between $t_0$ and $t_1$, the operating leverage can be reformulated as:

$$DOL = 1 + \frac{FC_{t0}}{EBIT_{t0}} \quad (3)$$

Where $FC$ stands for operating fixed costs.

Recombining $IBR$ and $DOL$ equations, we obtain the unlevered risk measured as the standard deviation of ROI (Return on Investments):

$$\sigma_u = \sqrt{\sum_{i=1}^{n} p_i (ROI_i - ROI')^2} \quad (4)$$

with $ROI = \frac{EBIT}{NI}$; $ROI' = \sum_{i=1}^{n} p_i (ROI_i)$

Where $NI$ stands for net investments and $\sigma_u$ is the unlevered risk in terms of ROI volatility.

Assuming for the period a constant level of fixed costs, net investments, and the contribution margin per sale (price per sale unit less variable cost per sale unit), we obtain:

$$\sigma_u = \frac{IBR}{REV_{t0}} \cdot \sqrt{(DOL_{t0} \cdot ROI_{t0})^2} \quad (5)$$

The operating leverage generates a volatility of operating results greater than the intrinsic business risk. Therefore, the operating leverage has, at least, a twofold importance for firm evolution: on one hand, it expresses the possibility to increase profitability through production volume adjustments; on the other hand, it is a signal of the impossibility of further adjustments to structural costs in facing negative business cycles (Brigham, 1995).
From equation (5) it is possible to infer that the unlevered risk has an impact on the initial level of ROI equal to the operating leverage multiplied for the ratio between the IBR and the initial revenues:

\[
\sigma_u' = \frac{\sigma_u}{\sqrt{\text{ROI}_{t0}}} = \frac{\text{IBR}_{t0}}{\text{REV}_{t0}} \cdot \sqrt{\text{DOL}_{t0}} = \frac{\text{IBR}_{t0}}{\text{REV}_{t0}} \cdot \left(1 + \frac{\text{FC}_{t0}}{\text{EBIT}_{t0}}\right)^2
\] (6)

Where \(\sigma_u'\) stands for the relative degree of operating leverage.

Therefore, passing from equation (5) to equation (6) can be useful to compare firms across time and space and to build clusters of firms that are homogenous in terms of risk and return.

5. A bottom up approach to unlevered beta

In this section there is the explanation of the model. The main assumptions are: each business area absorbs an amount of systematic risk of the capital markets; each firm takes in an amount of systematic risk from the business area in which it operates.

According to these assumptions, the beta of the j-th firm can be measured as a function of the sector beta, adjusted for a corrective factor that indicates the amount of unlevered systematic risk which the firm actually takes in. The adjustment factor is equal to the ratio between the two covariant \(\sigma_{j,m}\) and \(\sigma_{s,m}\).

\[
\phi_j = \frac{\sigma_{j,m}}{\sigma_{s,m}} = \frac{\beta_{j(u)}}{\beta_{s(u)}}
\] (7)

Where \(\beta_{j(u)}\) stands for the unlevered beta of j-th firm j, \(\beta_{s(u)}\) is the unlevered beta of the sector s, \(\sigma_{j,m}\) is the covariant between the ROI of the j-th firm and the unlevered return of the market, \(\sigma_{s,m}\) is the covariant between the ROI of the sector s and the unlevered return of the market.

From this equation (7) results:

\[
\beta_{j(u)} = \beta_{s(u)} \cdot \phi_j
\] (8)

In the equation (8) above, the unlevered beta is measured as that amount of systematic risk the firm j absorbs from the s sector. According to a bottom up logic, the result of (8) can be studied on the basis of two hypotheses:

a) Invariance of fixed costs, net investments, and the difference between price per sold unit and the variable cost per unit (during a given temporal range)

b) Variance of fixed costs, net investments and the difference between price per sold unit and the variable cost per unit, during a given temporal range.

With regard to a) hypothesis, the adjustment factor \(\phi_j\) is a function of the impact of the firm’s operating leverage and its ROI on the s sector operating leverage and ROI. In particular:
\[ \phi_j = \sqrt{\lambda_j \cdot \gamma_j} = \sqrt{\frac{DOL_j^2(t_{0(j)}) \cdot ROI_j^2(t_{0(j)})}{DOL_s^2(t_{0(j)}) \cdot ROI_s^2(t_{0(j)})}} \quad (9) \]

Where, \( DOL_j \) is the operating leverage of the \( j \) firm, \( DOL_s \) is the operating leverage of the \( s \) sector, \( ROI_j \) is the ROI of the \( j \) firm, and \( ROI_s \) is the ROI of the \( s \) sector.

Thus, it is possible to derive the following expression:

\[ \beta_{j(a)} = \beta_{s(a)} \cdot \sqrt{\lambda_j \cdot \gamma_j} \quad (10) \]

The assumption underlying the equation (10) is that the market share is constant as well. So, (10) is a simulation tool that helps studying the \( j \)-th equity cost sensitivity to the average \( s \) sector equity cost as a function of cost rigidity.

In the same sector, the \( \lambda_j \) factor is positively related to the fixed costs dimension and, then, to net investments. In this perspective it is relevant to consider the following ratio:

\[ j = \frac{FC_j}{NI_j} = \frac{EBIT_j(t_{0(j)}) \cdot (DOL_j(t_{0(j)}) - 1)}{NI_j} = ROI_j(t_{0(j)}) \cdot (DOL_j(t_{0(j)}) - 1) \quad (11) \]

with constant \( NI_j \)

Where \( \mu_j \) is the ratio between the firm fixed costs and net investments. From this ratio, it follows that:

\[ \lambda_j = \left( \frac{\mu_j}{ROI_j(t_{0(j)})} + 1 \right) \left( \frac{\mu_s}{ROI_s(t_{0(j)})} + 1 \right) \quad (12) \]

Where \( \mu_s \) is the ratio between fixed costs and net investments with regard to the \( s \) sector.

Therefore, the greater the value of \( \mu_j \), the greater is the amount of IBR absorbed by the \( j \) firm. This factor is mainly influenced by: the characteristics of the sector’s industrial process, the existence of operating inefficiency, and the firm’s growth propensity. This means, when \( \lambda_j > 1 \), a high rate of systematic risk of firm \( j \) operating in the \( s \) sector primarily depends on the existence of slack resource. In its turn, firm slack could depend on previous investment activities or on the firm’s inability to optimize resources dimension and input utilization. In case of slack due to strategic growth investments, the high rate of systematic risk should be balanced thanks to over-time increasing expected returns. When \( \lambda_j > 1 \) due to internal inefficiency, firm \( j \) tends to have suboptimal behavior in terms of risk-return relationship. With regard to the \( b \) hypothesis (variance of fixed costs, net investments and the difference between price per sale unit and variable cost per unit, during a given temporal range), on one hand, the unlevered risk depends on the variability of the following factors:

\[ \theta_j = \frac{CM_j}{NI_j} ; \quad j = \frac{FC_j}{NI_j} \]

Where, \( CM \) is the contribution margin (the difference between revenues and variable costs).
On the other hand, the impact of above coefficients volatility on $\sigma_{j(u)}$ value is negatively influenced by the covariance between them. Specifically, the impact of those coefficients on unlevered risk can be expressed as:

$$\sigma_{j(u)} = \sqrt{\sigma_{\theta j}^2 + \sigma_{\mu j}^2 - 2 \cdot \sigma_{\theta j, \mu j}}$$

(13)

Where, $\sigma_{\theta j}$ is the standard deviation of $\theta_j$; $\sigma_{\mu j}$ is the standard deviation of $\mu_j$; $\sigma_{\theta j, \mu j}$ is the covariance between $\theta_j$ and $\mu_j$.

Equation (13) takes in account the positive and the negative components of ROI, respectively $\theta_j$ and $\mu_j$ and, in formal terms, this explains why the covariance $\sigma_{\theta j, \mu j}$ has a negative value when measuring $\sigma_{j(u)}$. More in depth, it is necessary to analyze the volatility of the contribution margin in comparison to the structural volatility to interpret the significance of the negative value of the covariance $\sigma_{\theta j, \mu j}$ on unlevered risk. Therefore, there is a balance between risks $\sigma_{\theta j}$ and $\sigma_{\mu j}$ when there is also a positive relation between structural instability, due to decisions to invest/disinvest, and the ratio $CM/NI$. Specifically, the maximum balance between the two types of risk is achieved when the following values are realized: $\rho_{\theta j, \mu j} = +1$ and $\sigma_{\theta j} = \sigma_{\mu j}$. In this matter, it is of interest to analyze the following coefficient:

$$\xi_j = \rho_{\theta j, \mu j} \cdot \frac{\min(\sigma_{\theta j}, \sigma_{\mu j})}{\max(\sigma_{\theta j}, \sigma_{\mu j})}$$

(14)

This coefficient value ranges between upper and lower bounds:

$$-1 \leq \xi_j \leq +1$$

Recombining (13) with (14) we have:

$$\xi_j = +1 \rightarrow (\sigma_{j(u)} = 0; \ \beta_{j(u)} = 0)$$

Starting with the equation (13), the adjustment factor $\phi_j$, with regard to the hypothesis b), can be measured as:

$$b) \ \phi_j = \frac{\sigma_{j,m}}{\sigma_{s,m}} = \frac{\rho_{j,m}}{\rho_{s,m}} \cdot \frac{\sqrt{\sigma_{\theta j}^2 + \sigma_{\mu j}^2 - 2 \cdot \sigma_{\theta j, \mu j} \cdot \rho_{\theta j, \mu j}}}{\sqrt{\sigma_{\theta s}^2 + \sigma_{\mu s}^2 - 2 \cdot \sigma_{\theta s, \mu s} \cdot \rho_{\theta s, \mu s}}}$$

(15)

Where, $\rho_{j,m}$ is the correlation coefficient between the average return of capital markets and the average $ROI$ of firm $j$; $\rho_{s,m}$ is the correlation coefficient between the average return of capital markets and the average $ROI$ of the sector $s$.

The breakdown of adjustment factor $\phi_j$ in the two proposed versions is relevant for studying the endogenous maneuverability margins of unlevered systematic risk and controlling for top down beta. First, identifying the adjustment factor $\phi_j$ allows the comparison between risk factors emerging from the sector and firm-specific risk drivers. Second, when the two
methods, top down and bottom up, give results that show a huge difference; the capital markets are not properly considering operating risk factors for listed firms: when top down beta and bottom up beta are not aligned, this means there is a gap between the market value (due to demand and supply) and the intrinsic value (which is estimated in relation to firm and market fundamentals). In theory, if there is an efficient information system, perfect rationality, and market equilibrium between capital demand and supply, then, the bottom up analysis of beta should give the same result as the top down estimation. The combination of the hypotheses a) and b) gives some interesting indications on the relation between operating leverage and unlevered risk in the long run. The operating leverage amplifies $\phi_j$ value, and, so, changes in firm structure and volatility of operating performance play a significant role in unlevered risk definition. That means two different firms, operating in the same business, with similar level of $\lambda_j$ and $\gamma_j$, and with the same exposition to IBR, could have a different exposure to systematic risk in the medium and in the long run, due to different investment/divestment decisions and their correlation with the contribution margin dynamic. In this perspective, it is critical to look in depth at the following ratio:

$$\chi_j = \frac{\xi_j}{\xi_s} = \frac{\min(\sigma_{\theta j}, \sigma_{\mu j})}{\max(\sigma_{\theta s}, \sigma_{\mu s})} \frac{\max(\sigma_{\theta s}, \sigma_{\mu s})}{\min(\sigma_{\theta s}, \sigma_{\mu s})}$$

The analysis of the $\chi_j$ coefficient allows an in depth relation between the unlevered systematic risk and the structural dynamic of the firm. In a time series analysis, a greater value $\xi_j$ indicates that the firm is more likely to balance operating leverage volatility with investing/divesting activity than other players operating in the same sector. However, that does not necessary imply that systematic risk decreases when $\chi_j$ assumes greater values. It actually means investment/divestment decisions impact in a diversified manner on the adjustment factor $\phi_j$ and are a reverse function of the $\chi_j$ dimension. As an example, two companies of the same size, operating in the same business and implementing the same development project with a certain level of contribution margin volatility, could have a different adjustment factor. The impact of $\phi_j$ will be greater for the company having the smaller $\chi_j$. As a general rule, even if we consider projects that are equal, a value of $\chi_j > 1$ means that the volatilities $\sigma_{\theta j}$ and $\sigma_{\mu j}$ of the j-th company have a lower effect on $\phi_j$ than for competitors. Conversely, when $\chi_j < 1$, there is a greater correlation among $\phi_j$ and the volatilities $\sigma_{\theta j}$ and $\sigma_{\mu j}$ than for competitors. Greater levels of firm dynamism increase the impact of $\chi_j$ (positive or negative) on the generation of unlevered systematic risk. For instance, a growing company has to monitor $\chi_j$ more than a company in its maturity phase and operating in a market with less price volatility.

6. Conclusion

The proposed approach for determining the unlevered beta of firms overcomes some underlying simplifications as the assumption that the unlevered beta of a company is equal to the sector beta. The model is based on the idea that the unlevered systematic risk is somewhat diversifiable trough managing the operating leverage and the correlation (negative or positive) between investment and disinvestment decisions, and volatility of operating performance. In this perspective, the unlevered beta value depends on the exposure to the business cycle due to the level of fixed costs, and on the correlation between contribution margin and fixed costs dynamics. This approach allows a structure-specific evaluation of the
firm, taking into account its fixed costs, net investments and contribution margin, adjusted for a factor that rectifies for the unlevered market risk actually absorbed by the firm with a good level of approximation. The model has a double practical utility. On one hand, it illustrates which endogenous factors to manage for reducing the risk absorption due to the operating business area. On the other hand, it may be helpful for financial analysts to adjust expectations on firm returns, taking into account internal variables that reduce the absorption of sector beta. An estimation based on a bottom up approach could be more representative of actual returns of the firm, considering the exposure to exogenous factors. Moreover, the proposed bottom up model is helpful to study the risk return relationship with respect to entrepreneurial objectives.

References


Hawawini G., Viallet C., 1999. Finance for executives, South-Western College Publishing, Cincinnati, OH.


