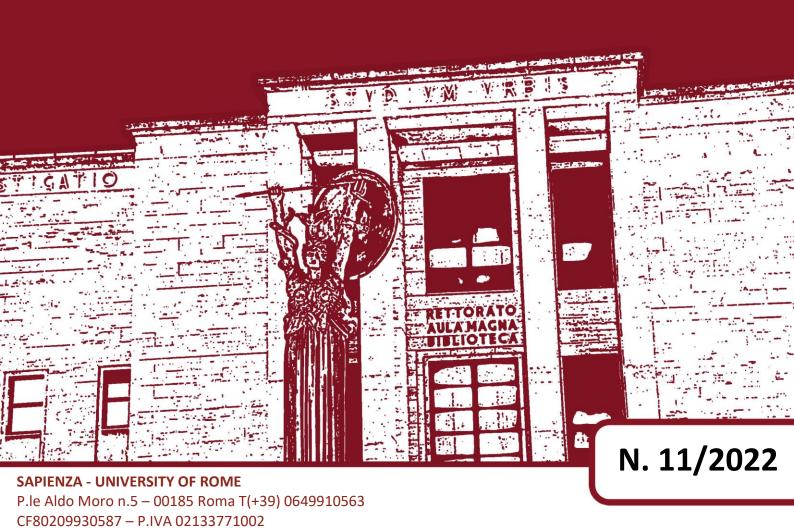


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Oligolopy Pricing: the Role of Firm Size and Number

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Oligopoly Pricing: The Role of Firm Size and Number*

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Abstract

This paper examines a homogeneous-good Bertrand-Edgeworth oligopoly model to explore the role of firm size and number in pricing. We consider the price impact of merger, breakup, investment, divestment, entry, and exit. A merger leads to higher prices only when it increases the size of the largest seller and industry capacity is neither too big nor too small post-merger. Similarly, breaking-up a firm only leads to lower prices when it concerns the biggest producer and aggregate capacity is within an intermediate range. Investment and entry (weakly) reduce prices, whereas divestment and exit yield (weakly) higher prices. Taken together, these findings suggest that size matters more than number in the determination of oligopoly prices.

Keywords: Bertrand-Edgeworth Competition; Edgeworth Price Cycle; Firm Size Distribution; Oligopoly Pricing; Price Dispersion.

JEL codes: D43; L11; L13.

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1 Introduction

There are many reasons for why market competition may not result in a competitive outcome, *i.e.*, a situation where firms price at marginal cost. One of these reasons was provided by the Irish scholar Francis Y. Edgeworth (1925), namely the presence of capacity constraints.¹ Indeed, when demand is sufficiently large in comparison to production capacity, firms may well be capable of selling all of their supplies at supracompetitive prices; that is, at prices above marginal cost.

For a given level of demand, however, there are at least two distinct factors that affect available capacity, the number of firms and their size. For instance, capacity constraints tighten both when an incumbent divests part of its business and when it redeploys its assets to leave the industry all together. In case of exit, there is a reduction in the number of competitors, whereas this need not be true in case of divestment. Though one would expect both events to induce an upward pressure on prices, after all there is less production capacity available, the precise impact on prices is a priori unclear. In particular, one may wonder whether firm size and number have a comparable effect on oligopoly pricing and, if not, what drives the difference.

The purpose of this paper is to shed light on the role of firm size and number in capacity-constrained oligopoly pricing. Toward that end, we analyze an n-firm homogeneous-good Bertrand-Edgeworth oligopoly.² As is well-known, this model captures three different types of market situation, each of which can be characterized by the ratio of industry capacity to market demand.

When production capacity is 'small' relative to market demand, there is a symmetric pure-strategy Nash equilibrium which equates aggregate capacity and demand. When capacity is 'large' relative to market demand, there also is a pure-strategy solution, which has firms pricing at marginal cost. In what follows, we refer to the first as the *monopolistic zone* and the second as the *competitive zone*. Finally, if industry capacity is 'intermediate', then there is no pure-strategy equilibrium. As pointed out by Edgeworth (1925), prices are indeterminate in this case and oscillate indefinitely between some lower and upper bound. We refer to this as the *Edgeworth zone*.

To examine the role of firm size and number in pricing in a capacity-constrained oligopoly model, we consider the impact of changes in (the allocation of) production capacity. Specifically, we explore the price effects of six different capacity-altering events: merger, breakup, investment, divestment, entry, and exit. In case of merger or breakup, there is a change in the number of sellers, but aggregate capacity is not affected. With investment and divestment, the number of firms remains constant, but capacity changes. Finally, with entry and exit, both capacity and the number of firms change.

¹For a general discussion, see Tirole (1988) and Vives (1999).

²Price competition under capacity constraints has been studied by Levitan and Shubik (1972), Kreps and Scheinkman (1983), Osborne and Pitchik (1986), Vives (1986), Allen and Hellwig (1986), Maskin and Tirole (1988), Deneckere and Kovenock (1992), Tasnádi (1999), amongst many others.

These events can have two distinct types of effect, both of which may affect pricing. On the one hand, they can have an impact on the span of each of the three zones. This implies that the events can cause the industry to switch zones. For instance, when capacities are large and the market is in the competitive zone, divestment or exit might reduce production capacity to such an extent that the industry moves into the Edgeworth zone. On the other hand, given that the industry is in the Edgeworth zone, the events may also affect the degree of price dispersion. We refer to this as the *price fluctuation range*, *i.e.*, the range of prices that are set with positive probability.

We find that a merger either has no effect on price or elevates price. There are two necessary conditions for prices to rise as a result of merger: (i) the merger increases the size of the largest seller (including the possibility of a merger between smaller producers that becomes the new leading firm post-merger), and (ii) the industry is in the Edgeworth zone or the merger brings the industry into the Edgeworth zone. Mergers that have no effect on the market position or size of the biggest industry player do not affect price. Moreover, if the industry is in the monopolistic or the competitive zone, then a merger only affects prices when it causes the industry to move into the Edgeworth zone. As with mergers, breaking-up a firm in smaller, separate entities has an impact on price only when it concerns the biggest seller. Moreover, it is only effective, in the sense of leading to lower prices, when the industry is in the Edgeworth zone. In all other cases, splitting-up suppliers in smaller, independent units does not affect price.

Investment and entry increase production capacity and overall have a comparable price reducing effect. If the industry is in the monopolistic zone, then both these events may bring the industry into the Edgeworth zone and, ultimately, into the competitive zone. Roughly speaking, the opposite occurs with divestment and exit, both of which reduce available production capacity. Starting in the competitive zone, these events may lead the industry into the Edgeworth zone and, eventually, into the monopolistic zone. The fact that entry and investment as well as divestment and exit work in the same direction, indicates that production capacity is a more important price determinant than the number of competitors in these types of industry.

As to price dispersion, we find that a merger (break-up) may yield a wider (narrower) price fluctuation range, but only when it affects the size of the industry leader. Investment by the largest industry player also leads to more price dispersion. Taken together, our findings thus suggest that size matters more than number in the determination of oligopoly prices.

The next section introduces the model and provides a detailed specification of the price fluctuation range. Section 3 explores the price effects of merger, breakup, investment, divestment, entry, and exit. Section 4 offers a brief discussion of our findings and Section 5 concludes. All proofs are relegated to the Appendix.

2 Methods

2.1 Oligopoly Model

We examine a Bertrand-Edgeworth oligopoly comprising a given set of profit-maximizing firms: $N = \{1, 2, ..., n\}$. Each firm $i \in N$ has a production capacity $k_i > 0$ and produces a homogeneous good on demand at a common marginal cost $c \geq 0$. Without loss of generality, it is assumed that $k_1 \geq k_2 \geq ... \geq k_n$. Total industry capacity is denoted by $K = \sum_{i=1}^n k_i$ and K_{-i} is the combined production capacity of all firms other than i.

The market demand function is $D: \mathbb{R}_+ \mapsto \mathbb{R}_+$. It is assumed that D(c) > 0 and that D(p) is twice continuously differentiable, with $D'(\cdot) < 0$ and $D''(\cdot) \le 0$ at any market price $p \ge 0$. Sellers select prices simultaneously from $[0, \infty)$ so that $[0, \infty)^n$ is the set of all possible price profiles. As products are perfect substitutes, consumers prefer to purchase from a lowest-priced seller. Since these suppliers may be capacity-constrained, however, it is possible that only part of them is served in which case higher-priced sellers might still face demand.

To specify individual (residual) demand, let $\Omega(p_i) = \{j \in N | p_j = p_i\}$ and $\Delta(p_i) = \{j \in N | p_j < p_i\}$ denote the set of firms that price at and below p_i , respectively.³ Furthermore, let $p_{-i} = (p_1, ., p_{i-1}, p_{i+1}, ., p_n)$ indicate the prices of all firms other than i. Demand for firm i's product is then given by $D_i(p_i, p_{-i}) = D(p_i)$ when all its competitors charge a strictly higher price. If there is at least one other seller setting the same price, then its demand is:

$$D_i(p_i, p_{-i}) = \max \left\{ \frac{k_i}{\sum_{j \in \Omega(p_i)} k_i} \left(D(p_i) - \sum_{j \in \Delta(p_i)} k_j \right), 0 \right\}.$$

Finally, if firm i sets the strictly highest price in the industry, then its demand is:

$$D_i(p_i, p_{-i}) = \max \{D(p_i) - K_{-i}, 0\}.$$

Thus, (i) customers first visit the cheapest seller(s) at the set prices, (ii) at equal prices, demand is allocated in proportion to production capacity, and (iii) rationing is efficient.⁴

A firm's profit is then given by:

$$\pi_i(p_i, p_{-i}) = (p_i - c) \cdot \min\{k_i, D_i(p_i, p_{-i})\}, \ \forall i \in N.$$

³Formally, it is $\Omega(p_i, p_{-i})$ and $\Delta(p_i, p_{-i})$, where p_{-i} is the vector of prices of all firms other than i. We use this shorthand notation to emphasize on which value of p_i these sets are based. In the ensuing analysis, the effect of the complete price vector on demand will be clear from the context.

⁴Such a surplus maximizing scheme is also used by Levitan and Shubik (1972), Kreps and Scheinkman (1983), Osborne and Pitchik (1986), Bos, Marini and Saulle (2022), Edwards and Routledge (2022), amongst many others.

Hence, if firm i sets the strictly highest price in the industry, then its profits are given by $\pi_i(p_i, p_{-i}) = (p_i - c) \cdot \max \{D(p_i, p_{-i}) - K_{-i}, 0\}$. To facilitate our analysis, we write:

$$p_i^* = \arg\max_{p_i} (p_i - c) \cdot \max \{D(p_i, p_{-i}) - K_{-i}, 0\}$$

to indicate the corresponding residual profit-maximizing price. Note that, by concavity of the residual profit-function, there is a unique p_i^* provided that there is a p_i for which $D(p_i, p_{-i}) > K_{-i}$.

The next two results establish some properties that will prove useful in the ensuing analysis. The first states that the residual profit-maximizing price is increasing with firm size. This is so because a firm's residual demand, *ceteris paribus*, depends positively on its production capacity. The second sheds light on the relation between a firm's production capacity and market demand at the residual profit-maximizing price.

Lemma 1 Fix K and suppose that $D(c) > K_{-n}$. For all $i, j \in N$ and $i \neq j$, if $k_i > k_j$, then $p_i^* > p_j^*$.

Lemma 2 For all $i, j \in N$ and $i \neq j$, $D(p_i^*)$ does not depend on k_i . Moreover,

$$0 < \frac{\partial D\left(p_i^*\left(k_j\right)\right)}{\partial k_i} < 1.$$

Finally, let \underline{p} be the price for which total industry capacity equals market demand, i.e., $K = D(\underline{p})$. We refer to p as the market-clearing price.

As is well-known, the existence of a pure-strategy Nash equilibrium in the above game critically depends on the relation between demand and available production capacity.⁵ Specifically, there is a pure-strategy solution when total industry capacity is sufficiently small (i.e., $K \leq D(p_1^*)$) and when it is sufficiently large (i.e., $K \geq D(c) + k_1$). In the following, we refer to the first case as the monopolistic zone and to the second as the competitive zone. In the monopolistic zone, there is a symmetric pure-strategy equilibrium in which all firms set the market-clearing price $\underline{p} > 0$. In the competitive zone, there too is a symmetric pure-strategy solution which has all firms price at marginal cost.⁶

If joint industry capacity is intermediate (i.e., $D(p_1^*) < K < D(c) + k_1$), then there is no purestrategy equilibrium. As envisaged by Edgeworth (1925), price stability is unlikely in this case and prices are expected to oscillate indefinitely between some lower and upper bound. We refer to this as the *Edgeworth zone*.

⁵See, *e.g.*, Tasnádi (2004).

⁶There are also many asymmetric equilibria with a subset of sellers pricing above costs.

2.2 Price Fluctuation Range

Before examining the price effect of changes in firm size and number, it is useful to distinguish two types of impact. First, a change in capacity allocation may affect the size of the Edgeworth zone $D(p_1^*) < K < D(c) + k_1$ (and therefore the size of the monopolistic and competitive zones). Second, given that joint capacity K is such that the industry is in the Edgeworth zone, it may impact what we refer to as the *price fluctuation range*, *i.e.*, the difference between the maximum and minimum prices that are set with positive probability.

In order to characterize the price fluctuation range, define:

$$\widehat{p}_{i} = \frac{(p_{i}^{*} - c) \cdot (D(p_{i}^{*}, p_{-i}) - K_{-i})}{k_{i}} + c,$$

as firm i's iso-profit price. That is, \widehat{p}_i is the price below p_i^* at which firm i sells at capacity and obtains the same profit as when it is the single highest-priced firm pricing at p_i^* . Figure 1 graphically illustrates the concept for the industry leader (i.e., firm 1) with marginal cost normalized to zero.

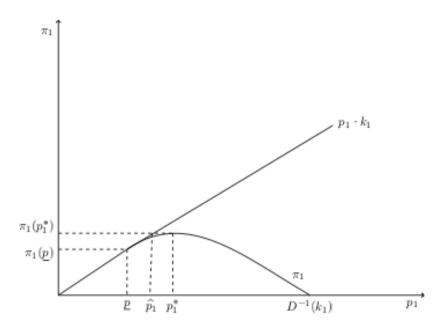


Figure 1 - Firm 1's residual profit maximizing and iso-profit price for c=0.

The next result relates the iso-profit price to production capacity. It shows that the iso-profit price positively depends on firm size when industry capacity is given. Roughly speaking, larger firms have a higher residual profit per unit of capacity. Since unit costs are the same, this implies that a firm's iso-profit price is increasing in its capacity, all else unchanged.

Proposition 1 Suppose that $D(p_1^*) < K < D(c) + k_1$ and fix K. For all $i, j \in N$ and $i \neq j$, if $k_i > k_j$, then $\widehat{p}_i > \widehat{p}_j$.

This finding implicitly defines the lower bound of the price fluctuation range to be given by \hat{p}_1 . To see this, first note that none of the firms has an incentive to put mass on prices in excess of their residual profit-maximizing price. Moreover, one can easily construct cases where charging the residual profit-maximizing price is a best-response. Taken together, and following Lemma 1 above, this implies that p_1^* constitutes the upper bound of the price fluctuation range.

Suppose then that firm 1 sets p_1^* . Since this is optimal only when firm 1 has residual demand at that price, it holds that $D(p_1^*, p_{-1}) > K_{-1}$. That is, all its rivals set a lower price and are capacity-constrained. Yet, in that case, each of its rivals can raise their price arbitrarily close to p_1^* and still sell at capacity. If that happens, then it is a best-response of the largest seller to undercut the price of its competitors by the smallest possible amount. This holds true, since it goes from producing below capacity to producing at capacity while selling at (approximately) the same price. In turn, this induces its rivals to cut their prices slightly. This hypothetical downward spiral stops when a firm prefers hiking its price to cutting it further. By Proposition 1, this is the largest seller.

Yet, if the largest seller does not put mass on prices below \hat{p}_1 , then neither will any of its rivals. Because firm 1 has no incentive to cut price further, there is nothing to gain by pricing below \hat{p}_1 . In fact, following the above logic, they have an incentive to follow the price hike by firm 1, which then induces a new Edgeworth price cycle. Combining the above then leads to the following conclusion.

Corollary Suppose that $D\left(p_{1}^{*}\right) < K < D\left(c\right) + k_{1}$. The price fluctuation range is given by: $P = \left[\widehat{p}_{1}, p_{1}^{*}\right]$.

3 Results

We now proceed by studying three scenarios that may alter the size distribution of firms: (i) Merger and Breakup, (ii) Investment and Divestment, and (iii) Entry and Exit. For all these cases, we examine the impact on the Edgeworth zone and price fluctuation range, respectively.

3.1 Merger and Breakup

The first possibility that we consider is a change in the size distribution of firms that results from merger or breakup. In the first case, the number of firms is reduced, whereas it increases in the second case. Importantly, in both cases it is assumed that total industry capacity K remains constant.

3.1.1 Edgeworth Zone

As pointed out above, the Edgeworth zone is characterized by the following condition: $D(p_1^*) < K < D(c) + k_1$. This condition captures the range of industry capacity for which prices may expected to be unstable. Notice that both the lower bound $D(p_1^*)$ and the upper bound $D(c) + k_1$ critically depend on

the size of the largest seller. In fact, these boundaries are affected only when there is a change in the production capacity of the industry leader. This implies that the Edgeworth zone remains unaltered as long as k_1 is largest and unaffected by merger or breakup.

Suppose then that there is a merger that becomes the new industry leader in terms of production capacity. Since k_1 has increased post-merger, the upper bound $D(c) + k_1$ increases. Moreover, since K is constant, the (new) largest firm faces more residual demand. Following Lemma 1, this implies that p_1^* increases, so that the lower bound $D(p_1^*)$ decreases. Taken together, this implies an expansion of the Edgeworth zone when a merger leads to an increase of the largest supplier's size. It is worth highlighting that a merger will never induce a shift from the Edgeworth zone to the monopolistic or competitive zone, whereas the opposite can occur.⁷ That is, if the industry is in the monopolistic or competitive zone, then a merger may bring the industry into the Edgeworth zone, thereby leading to more price dispersion.

This conclusion is reversed in case of a breakup of firms in smaller, separate, entities. As with merger, both the lower bound $D(p_1^*)$ and the upper bound $D(c) + k_1$ remain unaffected when the split-up does not involve the largest seller. If the breakup concerns the biggest industry player, however, then the upper bound decreases. The decline in capacity moreover means that the residual profit-maximizing price of the biggest firm decreases, which implies an increase of the lower bound $D(p_1^*)$. Therefore, the Edgeworth zone contracts if, and only if, the leading firm is split-up in separate units. Notice that, in that case, there may be a shift from the Edgeworth zone into the monopolistic or competitive zones. In other words, if the industry is in the Edgeworth zone, then breakup of the biggest industry player might lead to lower prices and potentially to more price stability.

3.1.2 Price Fluctuation Range

Let us now examine the effect of a merger or breakup on the price fluctuation range. Starting with a merger, the next result shows that when the merger is the industry leader in terms of production capacity, the price fluctuation range shifts upward. In stating this result, let the merger be indicated with the subscript 's'.

Proposition 2 Suppose that $D(p_1^*) < K < D(c) + k_1$ and that there is a subset $S \subset N$ of firms that merge. If this merger is the biggest firm post-merger, then $p_s^* > p_1^*$ and $\hat{p}_s > \hat{p}_1$. That is, the price fluctuation range shifts upward.

The intuition behind this result is as follows. Since industry capacity remains unaltered, the (new) largest firm faces more residual demand, which means it has a higher residual profit-maximizing price (Lemma 1). Since the industry leader's residual profits are higher post-merger, this implies an increase of the highest iso-profit price (Proposition 1). Such a merger, therefore, leads to an upward shift of the

⁷A comparable finding in a three-firm Bertrand-Edgeworth model can be found in Chen and Li (2018).

price fluctuation range.

Since both the upper bound and the lower bound increase when the merger is the biggest seller, it is *a priori* unclear whether the fluctuation range expands or contracts. Assuming linear demand, the next example shows that it may expand:

Example 1 Assume that market demand is given by D(p) = a - bp and that a - bc > 0. Assume further that $D(p_1^*) < K < D(c) + k_1 \iff a - bp_1^* < K < a - bc + k_1$. Now suppose that firm i is the highest priced seller facing residual demand:

$$D_i^*(p_i^*, p_{-i}) - K_{-i} = a - K_{-i} - bp_i$$

with $a > K_{-i}$ and b > 0. Its residual profit-maximizing price p_i^* is then given by:

$$p_i^* = \frac{a + bc - K + k_i}{2b},$$

with corresponding residual demand and profits:

$$D_i^*(p_i^*, p_{-i}) - K_{-i} = \frac{a - bc - K + k_i}{2} \text{ and } \pi_i^*(p_i^*, p_{-i}) = \frac{(a - bc - K + k_i)^2}{4b}.$$

Firm i's iso-profit price is given by:

$$\widehat{p}_i = \frac{(p_i^* - c) \cdot (D(p_i^*, p_{-i}) - K_{-i})}{k_i} + c = \frac{(a - bc - K + k_i)^2}{4bk_i}.$$

Now consider the fluctuation range:

$$p_{1}^{*} - \widehat{p}_{1} = \frac{a + bc - K + k_{1}}{2b} - \frac{(a - bc - K + k_{1})^{2}}{4bk_{1}} = \frac{(a - K + k_{1})(K + k_{1} - a)}{4bk_{1}}$$

$$= \frac{2k_{1} \cdot (a + bc - K + k_{1}) - (a - bc - K + k_{1})^{2}}{4bk_{1}}$$

$$= \frac{k_{1}(k_{1} + 4bc) - (a - K)^{2} + bc(2(a - K) - bc)}{4bk_{1}}.$$

Taking the derivative with respect to k_1 gives:

$$\frac{d(p_1^* - \widehat{p}_1)}{dk_1} = \frac{4bk_1 \cdot (2k_1 + 4bc) - 4b\left(k_1(k_1 + 4bc) - (a - K)^2 + bc\left(2(a - K) - bc\right)\right)}{(4bk_1)^2}$$

$$= \frac{k_1^2 + (K - a + bc)^2}{4bk_1^2} > 0.$$

We conclude that the fluctuation range is expanding with firm 1's capacity.

If the merger is not the biggest firm post-merger, then the residual profit-maximizing price p_1^* remains unaffected (since K_{-1} does not change). As firm 1's residual profits are unaffected by the merger, the

iso-profit $(\hat{p}_1 - c) \cdot k_1$ should also stay the same. Since k_1 does not change in this case, this implies that \hat{p}_1 does not change either. Hence, the price fluctuation range is not affected by a merger when the merger is not the biggest firm in the industry.

Finally, as to the impact of a breakup, suppose that firm 1 is split-up in smaller, separate entities. This implies that there is a new industry leader that is smaller than before the breakup. That firm faces less residual demand when setting the highest price so that the upper bound of the price fluctuation range decreases. By Proposition 1, the same holds for the iso-profit price. Hence, a breakup of the largest seller leads to a downward shift of the price fluctuation range. Moreover, the price fluctuation range remains unaffected when the breakup concerns a smaller supplier.

3.2 Investment and Divestment

In the preceding subsection, we have studied changes in the size distribution of suppliers under the assumption that aggregate capacity K is fixed. However, the size allocation may also change when there are modifications in industry capacity. In this subsection, we consider production capacity investments and divestments by incumbents. That is, we examine the impact of changes in K assuming that the number of producers remains constant. The question of what happens when the number of firms changes too is taken up in the next subsection.

3.2.1 Edgeworth Zone

Recall that the Edgeworth zone is defined by $D(p_1^*) < K < D(c) + k_1$. Starting with the upper bound (i.e., $D(c) + k_1$), it is clear that it increases when the biggest industry player invests in additional production capacity and decreases when this firm divests. The upper bound may also increase when there is a smaller supplier that invests to such a degree that it becomes the new industry leader in terms of capacity.

Since investments and divestments directly affect total industry capacity K, there may be a shift from the Edgeworth zone to the competitive zone. This can occur when a smaller seller invests, for example. The reverse cannot happen, because divestment by the biggest seller (or any other firm) reduces total industry capacity by the same amount. That is to say, the inequality $K < D(c) + k_1$ is immune to divestments.

As to the lower bound (i.e., $D(p_1^*)$), it is affected only when the residual profit-maximizing price of the largest producer changes. This price is effectively determined by the combined capacity of this firm's rivals (i.e., K_{-1}). Notice that K_{-1} is unaffected by firm 1's investments or divestments as long as it maintains its leading position. The lower bound does change in response to investment or divestment by a smaller supplier, however, since this directly affects K_{-1} . For instance, divestment by a smaller producer yields a reduction of K_{-1} . This implies an increase of p_1^* and, therefore, a decrease of the lower bound $D(p_1^*)$. Similarly, investment by a smaller supplier leads to an increase of K_{-1} . This induces a lower p_1^* and, consequently, an increase of the lower bound $D(p_1^*)$.

Since total industry capacity is affected, it is then possible that the industry shifts from the monopolistic zone to the Edgeworth zone (e.g., when the leading industry player invests) and vice versa (e.g., when the leading industry player divests).

3.2.2 Price Fluctuation Range

Let us now direct our attention to the potential impact of investment or divestment on the price fluctuation range. The following result considers the case where the largest seller invests in extra capacity.

Proposition 3 Suppose that $D(p_1^*) < K < D(c) + k_1$. The price fluctuation range $p_1^* - \widehat{p}_1$ is increasing in the size of the largest industry player k_1 .

As an increase in k_1 does not affect K_{-1} , the residual profit-maximizing price p_1^* remains unaffected. Since residual demand also stays the same, residual profits do not change either. By the very definition of the iso-profit price, this implies that $(\hat{p}_1 - c) \cdot k_1$ should also stay the same. However, since k_1 increases, this requires a reduction of the iso-profit price \hat{p}_1 . Taken together, this means that the price fluctuation range $p_1^* - \hat{p}_1$ expands downward when the industry leader invests in additional production capacity, all else equal.

The reverse holds when the largest supplier divests as long as it remains the industry leader. Like with investment, the residual profit-maximizing price p_1^* does not change. Yet, to keep $(\hat{p}_1 - c) \cdot k_1$ constant, the iso-profit price rises to compensate for the decline in k_1 . In case divestment is severe so that firm 1 looses its leading position, the new leader (firm 2) faces more residual demand than firm 1 when setting the highest price. This induces an increase of the price fluctuation range upper bound. Since firm 2's capacity remains unaltered, this implies that its iso-profit price increases too. Hence, in this case, the fluctuation range shifts upward.

Let us now consider the impact of growth by a firm that is not the industry leader. When a smaller firm invests without taking over the leading role, it reduces firm 1's residual demand and residual profit-maximizing price p_1^* . This implies a reduction of firm 1's residual profits. Since k_1 does not change, it follows that the iso-profit price \hat{p}_1 reduces too. Notice that the effect is similar when a smaller firm invests to such a degree that it becomes the new industry leader. We conclude that the price fluctuation range shifts downward whenever a smaller supplier invests.

As to divestment by a smaller producer, this has a direct negative effect on K_{-1} . Since this increases firm 1's residual demand, the residual profit-maximizing price p_1^* rises. The resulting rise in residual profits implies that the iso-profit $(\hat{p}_1 - c) \cdot k_1$ should increase as well. Since k_1 has not changed in this case, the iso-profit price \hat{p}_1 increases too. Hence, divestment by a smaller supplier leads to an upward

shift of the price fluctuation range.

3.3 Entry and Exit

Finally, let us consider the possibility of a simultaneous change in aggregate capacity and the number of firms. This may happen in case of entry or exit. As before, we first discuss the impact on the Edgeworth zone and then examine how the price fluctuation range is affected.

3.3.1 Edgeworth Zone

Suppose that industry capacity expands as a result of entry. If the newcomer is the largest firm in the industry, then the upper bound of the Edgeworth zone $(i.e., D(c) + k_1)$ increases. This is so since there is a new 'number one' post-entry, which implies an increase of k_1 . At the same time, however, the lower bound $(i.e., D(p_1^*))$ increases too. Indeed, the new industry leader faces less residual demand than its predecessor so that p_1^* is lower and $D(p_1^*)$ is higher post-entry.

If the newcomer is smaller than the largest firm in the industry, then this has no effect on the upper bound of the Edgeworth zone since k_1 does not change. Yet, also in this case, entry leads to a reduction of residual demand for the biggest firm and, consequently, to a reduction of p_1^* . Hence, entry always leads to an increase of the lower bound of the Edgeworth zone $D(p_1^*)$.

Exit by the industry leader reduces the upper bound $D(c)+k_1$. It also implies that the new industry leader (i.e., firm 2 pre-exit) faces more residual demand when setting its residual profit-maximizing price, which therefore increases. As a result, $D(p_1^*)$ decreases so that the Edgeworth zone shifts leftward. That is, the monopolistic zone becomes smaller, whereas the competitive zone expands. If the exiting firm is not the industry leader, then the upper bound remains unaltered. Yet, the leading firm faces more residual demand post-exit so that p_1^* increases and, therefore, $D(p_1^*)$ decreases. Consequently, the Edgeworth zone expands in this case.

3.3.2 Price Fluctuation Range

Entry and exit also affect the price fluctuation range. The next two results show that entry leads to a downward shift of the price fluctuation range, whereas exit induces an upward shift. In stating these results, let the prime indicate the situation post-entry.

Proposition 4 Suppose that $D(p_1^*) < K < D(c) + k_1$. If a new firm enters the industry, then $p_1^{*'} < p_1^*$ and $\widehat{p}_1' < \widehat{p}_1$. That is, the price fluctuation range shifts downward.

Proposition 5 Suppose that $D(p_1^*) < K < D(c) + k_1$. If an incumbent exits the industry, then $p_1^{*'} > p_1^*$ and $\widehat{p}_1' > \widehat{p}_1$. That is, the price fluctuation range shifts upward.

The logic behind these results is as follows. If a newcomer enters the industry that is smaller than the market leader, then this causes a drop in residual demand for the largest firm. Consequently, the residual profit-maximizing price p_1^* is lower post-entry. Since the largest seller's production capacity remains unaltered, this implies a drop of the iso-profit price \hat{p}_1 . The same holds when the entrant is the new industry leader. First, its residual demand is lower than the residual demand of the preceding leader. This has the implication that the upper bound of the price fluctuation range (p_1^*) decreases. Since residual profits are lower and the industry leader's capacity is larger, the corresponding iso-profit price (\hat{p}_1) is lower post-entry. As a result, entry induces a downward shift of the price fluctuation range.

Exit mirrors the situation with entry. If a firm leaves the industry, then there is more residual demand for the (new) industry leader, *i.e.*, p_1^* increases. Since residual profits are higher and the largest firm is weakly smaller post-exit, the iso-profit price (\hat{p}_1) increases too. Consequently, exit creates an upward shift of the price fluctuation range.

3.4 Recapitulation

The preceding analysis presents many different situations in which there may or may not be a price effect. In the following, we present three tables (one for each zone) that provide an overview of the impact of the six events on pricing. In these tables, the '\tau' and '\tau' mean that prices go up or go down, respectively. The '-' sign means that there is no impact on price and the 'x' indicates that the situation cannot occur. Each of the three tables distinguishes between 'same zone' and 'switch zone'. 'Same zone' means that the industry remains in the respective zone after the event, whereas 'switch zone' means that the event causes a shift to an adjacent zone. Lastly, 'regular' means that the industry leader (in terms of production capacity) is not involved, whereas 'leader' means that the largest firm takes part in the event.

| | Small Capacity | | | |
|------------|----------------|--------------|--------------|--------------|
| | Same Zone | | Switch Zone | |
| | Regular | Leader | Regular | Leader |
| Merger | - | - | 1 | 1 |
| Breakup | - | - | X | X |
| Investment | \downarrow | \downarrow | \downarrow | \downarrow |
| Divestment | 1 | 1 | X | X |
| Entry | \downarrow | 1 | <u> </u> | \downarrow |
| Exit | 1 | 1 | X | X |

Table 1 - Price Effects in the Monopolistic Zone.

Starting in the monopolistic zone (Table 1), a merger has a price effect only when it induces a switch to the Edgeworth zone. In that case, there is a 'jump' from a single market clearing price to a higher range of prices. This requires an increase of the production capacity of the largest player. If the biggest firm increases in size, then a merger may result in higher prices and create price dispersion. Breakup has no effect and cannot lead to a shift in zones. The same holds for divestment and exit, *i.e.*, these events cannot move the industry from the monopolistic zone to the Edgeworth zone. Yet, since they lead to a reduction of total industry capacity, they positively affect price levels. The opposite holds for investment and entry, which increase industry capacity and lead to a reduction of prices. This holds true independent of whether these events trigger a switch to the Edgeworth zone.⁸

| | Large Capacity | | | |
|------------|----------------|--------|-------------|------------|
| | Same Zone | | Switch Zone | |
| | Regular | Leader | Regular | Leader |
| Merger | - | - | 1 | 1 |
| Breakup | - | - | X | X |
| Investment | - | - | X | X |
| Divestment | _ | - | ↑ | <u> </u> |
| Entry | - | - | X | X |
| Exit | - | - | ↑ | \uparrow |

Table 2 - Price Effects in the Competitive Zone.

Turning to the competitive zone (Table 2), the effect of a merger is comparable to when the industry is in the monopolistic zone. It affects price only when it results in a switch in zones, which requires an increase in size of the industry leader. When this happens, prices rise. Breakup cannot induce a shift in zones and neither can capacity expanding events like investment or entry. Divestment and exit may bring the industry into the Edgeworth zone and, therefore, induce a price increase. Taken together, there may be a price increasing effect, but only when (1) there is a reduction in the number of firms or production capacity, and (2) the industry leaves the competitive zone.

| | Intermediate Capacity | | | | |
|------------|-----------------------|---------------|---------------|--------------|--|
| | Same Zone | | Switch Zone | | |
| | Regular | Leader | Regular | Leader | |
| Merger | -/↑ | 1 | = | - | |
| Breakup | - | \downarrow | - | \downarrow | |
| Investment | \ | \rightarrow | \rightarrow | X | |
| Divestment | 1 | ↑ | ↑ | 1 | |
| Entry | ↓ ↓ | \downarrow | \downarrow | \downarrow | |
| Exit | 1 | ↑ | ↑ | 1 | |

Table 3 - Price Effects in the Edgeworth Zone.

⁸It is worth noting that there is a boundary case where $K = D(\underline{p}) = D(p_1^*)$ prior to investment or entry. In this situation, an increase in the leader's production capacity induces a shift from the monopolistic to the Edgeworth zone, but its residual profit-maximizing price p_1^* does not change. Notice, however, that there is still a price decrease in the sense that the price fluctuation range $[\widehat{p}_1, p_1^*]$ expands downward (i.e., \widehat{p}_1 decreases when moving into the Edgeworth zone).

Finally, let us summarize the potential price effects when the industry is in the Edgeworth zone. Starting with the events that affect available industry capacity, divestment and exit always drive prices upward independent of whether there is a switch in zones. The reverse holds for investment and entry. Breaking-up firms that are not the industry leader has no effect on price, whereas breaking-up the largest firm leads to lower prices. A merger does not induce a shift to the competitive or monopolistic zone, but can have a price increasing effect when the largest industry player has grown post-merger.

4 Discussion

Recent years have seen a renowned interest in oligopoly pricing under capacity constraints (see, e.g., Bos and Vermeulen (2021), Montez and Schutz (2021) and Edwards and Routledge (2022)) and, in particular, in providing rationales for Edgeworth price cycles in markets with perfect and less than perfect substitutes (see, e.g., Myatt and Ronayne (2019) and Gabszewicz, Marini and Zanaj (2022)).

Such a revamped interest comes from several sources, perhaps the most important one being the existing empirical evidence. Price dispersion is shown to be persistent in both physical and online shops (see, e.g., Aas, Wulfsberg and Moen (2018) and Gorodnichenko, Sheremirov and Talavera (2018)). Edgeworth price cycles are also well documented. For example, Eckert (2003) and Noel (2007a, 2007b) provide evidence of sawtooth pricing paths in Canadian retail gasoline market. Wang (2008) explores empirically the existence of collusive price cycles in the Australian retail gasoline market. Similarly, Foros and Steen (2013) and Linder (2018) describe Edgeworth-like price cycles in European retail gasoline markets. Furthermore, such pricing dynamics has also been documented for pharmaceutical markets and search-engine advertising (see, e.g., Hauschultz and Munk-Nielsen (2020) and Zhang and Feng (2011)). Finally, it is noteworthy that similar pricing patterns have been observed in controlled laboratory experiments (see, e.g., Kruse, Rassenti, Reynolds, and Smith (1994) and Fonseca and Normann (2013)).

With few exceptions, these and similar studies take the size distribution of firms as given. Our analysis highlights the price impact of changes in firm size and number. In particular, we have shown how changes in market structure may affect the degree of price dispersion by making an industry more or less competitive. With an eye on the potential welfare implications, this warrants more (empirical) research on oligopoly pricing under capacity constraints.

5 Concluding Remarks

We have analyzed an *n*-firm homogeneous-good Bertrand-Edgeworth model to examine the impact of firm size and number on oligopoly pricing. We considered six different events that have an effect on the number of firms (merger and breakup), available production capacity (investment and divestment), or both (entry and exit). We showed that a merger leads to higher prices only when it increases the size of

the largest seller and industry capacity is neither too big nor too small post-merger. Similarly, breakingup a firm only leads to lower prices when it concerns the biggest producer and aggregate capacity is within an intermediate range. Investment and entry (weakly) reduce prices, whereas divestment and exit yield (weakly) higher prices. The overall takeaway from our explorations is, therefore, that size matters more than number in the determination of oligopoly prices.

We see several avenues for future research. One is to conduct a similar study assuming that products are less than perfect substitutes. Another is to consider cost (in)efficiencies that may result from merger. This may well create an (additional) downward or upward pressure on prices. As to the latter, we exclusively considered unilateral effects and ignored potential coordinated effects. Clearly, the upward price effect of a merger may well be stronger in case it facilitates a 'mutual meeting of the minds'.

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6 Appendix: Proofs

Proof of Lemma 1. Consider some firm $i \in N$. Its residual profit-maximizing price, p_i^* , is the price that solves:

$$D(p_i) - K_{-i} + (p_i - c) \frac{\partial D_i(p_i, p_{-i})}{\partial p_i} = 0 \Longleftrightarrow D(p_i) + (p_i - c) \frac{\partial D_i(p_i, p_{-i})}{\partial p_i} = K - k_i.$$

Notice that, for a given industry capacity K, the LHS is decreasing in p_i and the RHS is decreasing in k_i . Hence, $\forall i, j \in N$ and $i \neq j$, if $k_i > k_j$, then $p_i^* > p_j^*$.

Proof of Lemma 2. Following the proof of Lemma 1, the residual profit-maximizing price of firm i is implicitly defined by:

$$D(p_i^*) - K_{-i} + (p_i^* - c) \frac{\partial D(p_i^*, p_{-i})}{\partial p_i^*} = 0.$$

Rearranging gives:

$$D(p_i^*) = K_{-i} - (p_i^* - c) \frac{\partial D(p_i^*, p_{-i})}{\partial p_i^*}.$$

Observe that the RHS does not depend on k_i .

Next, note that:

$$\frac{\partial D\left(p_{i}^{*}\left(k_{j}\right)\right)}{\partial k_{i}}=1-\frac{\partial p_{i}^{*}}{\partial k_{i}}\cdot\left(D'\left(p_{i}^{*}\right)+\left(p_{i}^{*}-c\right)D''\left(p_{i}^{*}\right)\right),$$

where:

$$\frac{\partial p_i^*}{\partial k_j} \cdot (D'(p_i^*) + (p_i^* - c) D''(p_i^*)) > 0,$$

since $\frac{\partial p_i^*}{\partial k_j} < 0$, $D'(p_i^*) < 0$, and $D''(p_i^*) \le 0$. This implies $\frac{\partial D\left(p_i^*(k_j)\right)}{\partial k_j} < 1$. Moreover, $\frac{\partial D\left(p_i^*(k_j)\right)}{\partial k_j} > 0$, because $\frac{\partial p_i^*}{\partial k_j} < 0$ and $D'(p_i^*) < 0$.

Proof of Proposition 1. Consider some firm $i \in N$. Its iso-profit price is given by:

$$\widehat{p}_{i} = \frac{(p_{i}^{*} - c) (D (p_{i}^{*}, p_{-i}) - K_{-i})}{k_{i:}} + c.$$

Holding total industry capacity K fixed, we now show that \hat{p}_i increases with k_i . Taking the first-derivative with respect to k_i :

$$\frac{d\widehat{p}_{i}}{dk_{i}} = \frac{k_{i} \cdot \left[\frac{dp_{i}^{*}}{dk_{i}} \left(D\left(p_{i}^{*}, p_{-i}\right) - K + k_{i}\right) + \left(p_{i}^{*} - c\right) \left(\frac{\partial D\left(p_{i}^{*}, p_{-i}\right)}{\partial p_{i}^{*}} \frac{dp_{i}^{*}}{dk_{i}} + 1\right)\right] - \left(p_{i}^{*} - c\right) \left(D\left(p_{i}^{*}, p_{-i}\right) - K_{-i}\right)}{k_{i}^{2}}.$$

Rearranging gives:

$$\frac{d\widehat{p}_{i}}{dk_{i}} = \frac{\frac{dp_{i}^{*}}{dk_{i}} \cdot k_{i} \cdot \left[\left(D\left(p_{i}^{*}, p_{-i}\right) - K + k_{i} \right) + \left(p_{i}^{*} - c \right) \frac{\partial D\left(p_{i}^{*}, p_{-i}\right)}{\partial p_{i}^{*}} \right] - \left(p_{i}^{*} - c \right) \left(D\left(p_{i}^{*}, p_{-i}\right) - K \right)}{k_{i}^{2}}.$$
(1)

Notice that, by the definition of the residual profit-maximizing price p_i^* , it holds that:

$$(D(p_i^*, p_{-i}) - K + k_i) + (p_i^* - c) \frac{\partial D(p_i^*, p_{-i})}{\partial p_i^*} = 0.$$

Hence, the derivative (1) reduces to:

$$\frac{d\widehat{p}_{i}}{dk_{i}} = \frac{-(p_{i}^{*} - c)(D(p_{i}^{*}, p_{-i}) - K)}{k_{i}^{2}},$$

which is positive since $K = D(\underline{p})$ and $p_i^* > \underline{p}$. We conclude that if $k_i > k_j$, then $\widehat{p}_i > \widehat{p}_j$, for all $i, j \in N$ and $i \neq j$.

Proof of Proposition 2. Consider a merger $S \subset N$ that involves firm 1. The claims that $p_s^* > p_1^*$ and $\hat{p}_s > \hat{p}_1$ follow directly from Lemma 1 and Proposition 3.

This leaves the case where a subset of smaller suppliers merge and becomes the biggest firm in the industry. In that case, this new industry leader faces more residual demand when setting the highest price so that the upper bound of the fluctuation range increases (i.e., $p_s^* > p_1^*$). Moreover, by Proposition 3, the iso-profit price is increasing with capacity when total industry capacity is constant. Since $k_s > k_1$, this implies that also in this case it holds that $\hat{p}_s > \hat{p}_1$ post-merger.

Proof of Proposition 3. Consider the price fluctuation range $[\hat{p}_1, p_1^*]$ for a given industry capacity K:

$$p_{1}^{*} - \widehat{p}_{1} = p_{1}^{*} - \frac{(p_{1}^{*} - c)(D(p_{1}^{*}, p_{-1}) - K_{-1})}{k_{1}} - c = \frac{(p_{1}^{*} - c) \cdot k_{1} - (p_{1}^{*} - c)(D(p_{1}^{*}, p_{-1}) - K + k_{1})}{k_{1}}$$

$$= \frac{(p_{1}^{*} - c)(K - D(p_{1}^{*}, p_{-1}))}{k_{1}}.$$

Now suppose there is an increase in k_1 . To assess the impact on the fluctuation range, notice that:

$$\frac{d\left(p_{1}^{*}-\widehat{p_{1}}\right)}{dk_{1}} = \frac{d\left(\frac{\left(p_{1}^{*}-c\right)\left(K-D(p_{1}^{*},p_{-1})\right)}{k_{1}}\right)}{dk_{1}}$$

$$= \frac{k_{1} \cdot \left[\frac{dp_{1}^{*}}{dk_{1}}\left(K-D(p_{1}^{*},p_{-1})\right)+\left(p_{1}^{*}-c\right)\left(1-\frac{\partial D(p_{1}^{*},p_{-1})}{\partial p_{1}^{*}}\frac{dp_{1}^{*}}{dk_{1}}\right)\right]-\left(p_{1}^{*}-c\right)\left(K-D(p_{1}^{*},p_{-1})\right)}{k_{1}^{2}}.$$

In this case, it holds that $\frac{dp_1^*}{dk_1} = 0$ since K_{-1} remains unaltered with an expansion of k_1 . Hence, the preceding equality simplifies to:

$$\frac{(p_1^* - c) (D(p_1^*, p_{-1}) - K_{-1})}{k_1^2},$$

which is positive. We conclude that the price fluctuation range is increasing in the size of the largest industry player. ■

Proof of Proposition 4. Suppose there is an entrant with capacity $k' > k_1$, then this newcomer faces less residual demand than its predecessor (firm 1) when it is the highest-priced firm in the industry: $D(\cdot) - K_{-1} > D(\cdot) - K$. It follows that $p_1^{*'} < p_1^*$. This implies that residual profits of the industry leader are smaller post-entry. By definition of the iso-profit price, this means that $(\widehat{p}_1' - c) \cdot k_1'$ must decrease too. Yet, $k' > k_1$, which implies that $\widehat{p}_1' < \widehat{p}_1$ to ensure that $(\widehat{p}_1' - c) k' < (\widehat{p}_1 - c) k_1$.

Now suppose there is an entrant with capacity $k' < k_1$, then firm 1 faces less residual demand when it is the highest-priced firm in the industry: $D(\cdot) - K_{-1} > D(\cdot) - K_{-1} - k'$. It follows that $p_1^{*'} < p_1^*$. This implies that residual profits of the industry leader are smaller post-entry. By definition of the iso-profit price, this means that $(\widehat{p}_1' > \widehat{p}_1 - c) k_1'$ must decrease too. Yet, $k_1' = k_1$, which implies that $\widehat{p}_1' < \widehat{p}_1$ to ensure that $(\widehat{p}_1' - c) k_1' < (\widehat{p}_1 - c) k_1$.

Proof of Proposition 5. Suppose that the largest seller (firm 1) leaves the industry. This means that firm 2 is the new industry leader that now has a higher residual profit-maximizing price, because $K_{-1} > K_{-1} - k_2$. Hence, its residual demand is higher. Consequently, firm 2's residual profit exceeds firm 1's residual profit post-exit. This implies that firm 2's residual profit post-exit exceeds $(\widehat{p}_1 - c) \cdot k_1$ and, therefore, also $(\widehat{p}_1 - c) \cdot k_2$, because $k_2 \leq k_1$. By the definition of the iso-profit price this means that $\widehat{p}'_1 > \widehat{p}_1$ to ensure that the iso-profit post-exit equals the new leader's residual profit.

Now suppose that a smaller supplier leaves the industry. This directly reduces K_{-1} , which increases the residual demand for firm 1. Consequently, $p_1^{*'} > p_1^*$. Since firm 1's residual demand and price go up, its residual profits will also increase. In turn, this implies that its iso-profit $(\widehat{p}_1 - c) k_1$ must increase too. Yet, k_1 does not change so that its iso-profit price \widehat{p}_1 increases. We conclude that $\widehat{p}_1' > \widehat{p}_1$.