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Abstract

Cyclical inequality and idiosyncratic risk imply additional channels that amplify the transmission of persistent balance-sheet policies, through their effects on private sector's expectations and consumption risk. Through these channels, unconventional monetary policy improves the central bank's ability to anchor expectations and rule out endogenous instability. Moreover, they allow the central bank to optimally complement interest-rate policy in particular in response to financial shocks that expose the economy to the effective-lower-bound on the policy rate, and can promote a swifter exit from the liquidity trap.

JEL codes: E21, E32, E44, E58

Keywords: Cyclical inequality; idiosyncratic risk; optimal monetary policy; HANK; THANK, ELB.

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1 Introduction

Unconventional monetary policy and inequality are arguably two of the most debated topics in macroeconomics in recent decades. Since the Great Financial Crisis of 2007-2009, most central banks in advanced economies have progressively increased the use of unconventional balance-sheet policies to overcome the limitations of conventional policy implied by the effective lower bound (ELB) on the nominal interest rate. The heavy use of such policies, in a time where economic inequality was already in the spotlight of the public attention for its increasing levels, stimulated a lively policy debate about the distributional implications of monetary policy, along both the conventional and unconventional dimensions.[1] On the academic side, while the empirical research has scrutinised both dimensions of policy,[2] the theoretical literature so far has mostly focused on the distributional implications of interest-rate policies.[3]

This paper aims at contributing in this dimension and provides a theoretical analysis of the interplay between unconventional balance-sheet policies and inequality in a small-scale New Keynesian model with heterogeneous households, idiosyncratic risk and credit frictions. We introduce heterogeneity and idiosyncratic uncertainty in a tractable framework *à la* Bilbiie (2018), and use insights from Benigno and Nisticò (2017), Benigno et al. (2020) and Sims et al. (2021).

In our model economy, households are either savers or borrowers. The savers are patient and have a low marginal propensity to consume (MPC), while the borrowers are relatively more impatient and have a high MPC. The savers smooth consumption over time investing in short-term deposits, work in the production sector and own the financial and non-financial firms. The borrowers work in the production sector and issue long-term bonds to finance their consumption. In equilibrium, they have access to a lower level of disposable labor income with respect to the savers. Importantly, households only learn at the beginning of the period whether they are savers or borrowers, and cannot insure against the ensuing idiosyncratic income and consumption risk. An intermediation sector engages in maturity transformation by issuing short-term deposits to the savers and buying reserves from the central bank and long-term bonds from the borrowers. In purchasing the latter, they face a leverage constraint that makes room for unconventional policy. The central bank issues reserves and purchases long-term bonds, besides controlling the short-term nominal interest rate, while the fiscal authority collects taxes and makes transfers on a balanced budget.

We show that the dynamics of inequality play a key role in shaping the transmission of central bank's balance-sheet policies to aggregate demand, in particular by mitigating the propagation and amplification of negative shocks through their effect on idiosyncratic consumption risk.

In our economy, unconventional monetary policy affects current aggregate spending through three channels. The first is the familiar one working through the relaxation of the leverage constraint of financial intermediaries, which reduces borrowing costs and stimulates borrowers' con-

[1] See Yellen (2014), Bernanke (2015, 2017), and Schnabel (2021), among others.

[2] See Colciago et al. (2019) for a recent survey of the main empirical literature, discussing the lack of a general consensus on the effect of monetary policy – particularly along the unconventional dimension – on inequality.

[3] See Gornemann et al (2016), Kaplan et al (2018), Bilbiie (2018), Auclert (2019), and Acharya and Dogra (2020), among others. Two exceptions are Cui and Sterk (2021) and Sims et al (2022).

sumption. Idiosyncratic risk and cyclical inequality imply two additional and novel channels. The "idiosyncratic-risk channel" in particular is key for the transmission of persistent balance-sheet policies: improving consumption expectations for the borrowers reduces consumption risk for the savers which, in turn, find it optimal to reduce their precautionary savings and expand their demand. This substantially amplifies the expansionary effect of an unconventional monetary policy shock that initially only affects the borrowers. Through the "cyclical-inequality channel", moreover, a persistent increase in central bank's reserves further stimulates current aggregate demand if consumption inequality is structurally countercyclical (as the empirical evidence suggests): the fall in expected inequality is amplified in general equilibrium by the expected boom in output, which reinforces the fall in consumption risk for the savers and their incentive to increase spending.

We show that, through the "idiosyncratic-risk channel", unconventional monetary policy improves the ability of the central bank to anchor the private sector's expectations and rule out endogenous instability. Endogenous unconventional policy rules in this economy can perfectly substitute for conventional interest-rate feedback rules in the implementation of a (locally) unique rational-expectations equilibrium. Appropriately specified balance-sheet policy rules allow the equilibrium to be determinate even in the case of an interest-rate peg, or a permanent liquidity trap.

This result is particularly meaningful because it implies that unconventional monetary policy allows the central bank to fully stabilise inflation and the output gap even in the face of shocks that the conventional dimension of policy would find impossible to sterilise due to the ELB on nominal interest rates. We show, however, that (unconventional) strict inflation targeting is not necessarily an optimal policy regime from a welfare perspective, as it may require strong and persistent effects on consumption inequality that are detrimental for social welfare. Nevertheless, optimal unconventional monetary policy within a "new-style" regime improves the ability of the central bank to reduce fluctuations in inflation and the output gap during a liquidity trap, and may promote a swifter exit from zero-interest rate policies.

Our analysis therefore rationalises the benefits of the recent evolution of central banking in advanced economies towards the "new-style" regime, where both the conventional and unconventional tools are endogenously activated in response to the state of the economy.

This paper contributes mainly to two strands of the theoretical New-Keynesian literature.

The first one is the Heterogeneous Agents New Keynesian (HANK) literature, and in particular the *analytical* HANK literature.[4] With respect to this strand of the literature, and in particular with respect to Bilbiie (2018) which we follow in the tractable specification of heterogeneity and idiosyncratic risk, we contribute by introducing borrowing agents and credit frictions and by focusing on the unconventional dimension of monetary policy, whereas virtually all of the literature has so

[4] The acronym is due to Kaplan et al (2018), initiating the *quantitative* HANK literature using frameworks with a rich household heterogeneity due to market incompleteness and generally require computationally complex numerical methods to be solved. A non-exhaustive list of contributions in the *analytical* HANK literature, which instead uses tractable versions of the HANK model to scrutinise the theoretical channels and implications, includes Acharya and Dogra (2020), Acharya et al (2022), Bilbiie (2018, 2020), Challe (2020), Debortoli and Galí (2018, 2021), Ravn and Sterk (2018), Werning (2015). For a review of this literature, see Galí (2018); for a discussion of the relation between the HANK model and the corresponding Representative-Agent New-Keynesian (RANK) counterpart, see Kaplan and Violante (2018).

far focused on the conventional interest-rate policy.[5] Notable exceptions are Cui and Sterk (2021) and Sims et al (2022), which study the implications for unconventional policy of a prototypical *quantitative* HANK model. The former focus on the implications of different MPCs out of liquid versus illiquid assets, and find that, while balance-sheet policies can be highly stimulative, they also bear the welfare cost of potentially increasing inequality. The latter focus on credit frictions in an economy where the borrowing agents are wholesale firms financing purchases of fiscal capital by issuing long-term bonds, and find instead that the response of the economy to unconventional policy is essentially the same as in the RANK model. With respect to these papers, we exploit the tractability of our model to analytically characterise the additional theoretical transmission channels that idiosyncratic risk and cyclical inequality imply for balance-sheet policies.

We also contribute to the theoretical literature on unconventional monetary policy.[6] With the exception of the two papers above, most of this literature has focused on the aggregate dimensions of balance-sheet policies, regardless of whether the economy is populated by a representative household (as e.g. in Gertler and Karadi, 2011 and 2013, Benigno and Nisticò, 2020, Sims and Wu, 2021, Karadi and Nakov, 2021, Benigno and Benigno, 2022, Bhattarai et al, 2022) or two types of heterogeneous households (as for example in Chen et al, 2012, Benigno and Nisticò, 2017, Del Negro et al, 2017, Sims et al, 2021, Bonciani and Oh, 2021, Wu and Xie, 2022). With respect to this literature, and in particular with respect to Sims et al. (2021) and Wu and Xie (2022), with which we share the tractable specification of the credit friction, we contribute by studying the implications of idiosyncratic uncertainty that implies a stochastic transition between savers and borrowers, which allows us to identify additional transmission channels of balance-sheet policies related to consumption inequality. Cúrdia and Woodford (2011, 2016) similarly study an economy where agents stochastically cycle between the borrowing and saving type, focusing on how this transition affects the dynamics and policy implications of credit spreads. With respect to these papers, we focus instead on the distributional dimension of balance-sheet policies and how this dimension shapes the transmission mechanism to aggregate variables as well, and makes consumption risk a major channel.

The paper is organised as follows. Section 2 describes our model economy with heterogeneous households, idiosyncratic risk and credit frictions. Section 3 derives the linearised model and discusses the relevant welfare criterion for our economy. Section 4 draws the policy implications of our framework, under both a positive and normative perspective. Section 5 concludes.

[5] Bilbiie et al (2022) develop an empirical version of Bilbiie (2018) and estimate it on US data to evaluate the role of cyclical inequality and idiosyncratic risk for business-cycle fluctuations.

[6] A non-exhaustive list of contributions in this strand includes Cúrdia and Woodford (2011), Gertler and Karadi (2011, 2013), Chen et al (2012), Benigno and Nisticò (2017, 2020), Del Negro et al (2017), Cui and Sterk (2021), Karadi and Nakov (2021), Sims and Wu (2021), Sims et al (2021, 2022), Bonciani and Oh (2021), Bhattarai et al (2022), Benigno and Benigno (2022), Wu and Xie (2022).

2 The Model Economy

The economy is populated by infinitely-lived households consuming a bundle of differentiated goods and supplying labor for their production. We introduce heterogeneous households in a tractable framework *à la* Bilbiie (2018), and using insights from Cúrdia and Woodford (2016), Benigno and Nisticò (2017), Benigno et al. (2020) and Sims et al. (2021). There are two types of households: low-MPC savers, who accumulate financial wealth in short-term deposits and can smooth consumption intertemporally, and high-MPC borrowers, who are relatively more impatient and in equilibrium only consume out of their current debt and disposable labor income. Stochastic transition between these two types introduces a layer of idiosyncratic uncertainty that makes this heterogeneity relevant for business-cycle fluctuations and the transmission of balance-sheet policies in equilibrium.

An intermediation sector engages in maturity transformation by supplying short-term deposits to savers and buying long-term bonds from borrowers, and has access to central-bank reserves through open-market operations. In purchasing long-term bonds, the financial intermediaries face a leverage constraint – in the spirit of Gertler and Karadi (2011, 2013), and Sims et al. (2021) – that is going to make room for unconventional monetary policy.

A continuum of monopolistic firms produces the differentiated goods using labor services and technology, and subject to nominal price rigidities. The public sector includes a fiscal authority which imposes taxes and provides transfers within a balanced budget, and a central bank in charge of monetary policy.

2.1 Households

We consider a closed-economy model with a continuum of households belonging to either one of two types: savers (denoted with an index "s") and borrowers (denoted with an index "b"). The "saver" and "borrower" types include a mass $1-z$ and z of agents, respectively. Each saver faces a probability $1-p_s$ of becoming a borrower as the next period begins, and each borrower a probability $1-p_b$ of becoming a saver. To keep the relative mass of the two agent types constant over time, we impose the restriction $(1-z)(1-p_s) = z(1-p_b)$. Savers and borrowers share the same period-utility function, $u_j \equiv \xi[U(C_j) - V(L_j)]$, with $j = s, b$ and where ξ is an intertemporal disturbance. Moreover, they are endowed with discount factors β_s and β_b, respectively, with $0 < \beta_b \leq \beta_s \equiv \beta < 1$, which makes borrowers relatively more impatient than savers, as in Benigno et al (2020), among others. To facilitate aggregation, we assume that the period-utility function is exponential in consumption C and isoelastic in hours worked L, as in Benigno and Nisticò (2017), among others:

$$U(C_{j,t}) \equiv 1 - \exp(-vC_{j,t}) \qquad\qquad V(L_{j,t}) \equiv \frac{L_{j,t}^{1+\varphi}}{1+\varphi} \tag{1}$$

for some positive parameter v and any $j = s, b$, and where consumption is the usual Dixit-Stiglitz bundle

$$C \equiv \left[\int_0^1 C(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

with $C(i)$ denoting the consumption of the differentiated good of brand i, and $\epsilon > 1$ the elasticity of substitution between any two brands in the continuum indexed by $i \in [0, 1]$. As in Bilbiie (2018), we assume that all agents within each type pool their resources and obligations to share the same level of consumption in equilibrium.[7]

2.1.1 Savers

Savers maximize their expected discounted lifetime utility

$$\mathcal{U}_{s,t} = u_{s,t} + \beta E_t\{p_s \mathcal{U}_{s,t+1} + (1-p_s)\mathcal{U}_{b,t+1}\} \tag{3}$$

taking thus into account the probability of becoming a borrower in the future when evaluating expected future utility flows, and subject to the following flow-budget constraint

$$P_t C_{s,t} + (1-p_s)(1+i_t^B)Q_{t-1}B_{t-1} + D_t + N_t = W_{s,t}L_{s,t} + p_s(1+i_{t-1}^D)D_{t-1} + \Pi_t - T_{s,t}. \tag{4}$$

The nominal resources available to savers at the beginning of each period t therefore include the labor income $W_{s,t}L_{s,t}$, the payoff on the deposits from the previous period $(1+i_{t-1}^D)D_{t-1}$ held by the share p_s of savers that did not turn borrowers, the per-capita nominal profits $\Pi_t \equiv (1-z)^{-1}(\Pi_t^p + \Pi_t^f)$ remitted by the monopolistic producers and the financial intermediaries – both owned by the savers – net of taxes/transfers $T_{s,t}$, with P_t the general consumption price level. The savers use these resources to purchase a bundle of consumption goods $C_{s,t}$, save in one-period nominal deposits D_t, transfer nominal equity N_t to financial intermediaries, and share *pro quota* the burden of paying off the long-term debt B_{t-1} brought by the mass $z(1-p_b)$ of borrowers that have turned savers at the beginning of period t, where we used the restriction $(1-z)(1-p_s) = z(1-p_b)$. These are long-term securities that pay a coupon decaying geometrically at rate k, and sell at price Q_t, implying a rate of return $1 + i_t^B \equiv (1 + kQ_t)/Q_{t-1}$.

The optimal choice of consumption, hours worked and deposits implies the Euler equation

$$\xi_t U_c(C_{s,t}) = \beta E_t \left\{ \frac{1+i_t^D}{1+\pi_{t+1}} \xi_{t+1} \Big[p_s U_c(C_{s,t+1}) + (1-p_s) U_c(C_{b,t+1}) \Big] \right\} \tag{5}$$

where π_{t+1} is the net inflation rate between period t and $t+1$, and the labor supply

$$\frac{V_l(L_{s,t})}{U_c(C_{s,t})} = \frac{W_{s,t}}{P_t}. \tag{6}$$

Equation (5) captures the main implication of this class of heterogeneous-agents models: a positive probability p_s of changing type in the future activates precautionary-saving motives that affect current spending decisions anytime there is expected inequality in future consumption across types. If the borrowers are expected to consume less than the savers, then a positive probability of turning borrower tomorrow makes a current saver want to hedge against the possible future drop

[7] Analogous implications would follow from an imperfect-insurance scheme as in Cúrdia and Woodford (2016).

in consumption by saving more today.

Moreover, the following no-arbitrage condition determines the interest-rate on short-term deposits

$$1 = (1 + i_t^D) E_t \left\{ \Lambda_{t,t+1}^s \right\}, \tag{7}$$

where $\Lambda_{t,t+1}^s$ denotes the nominal stochastic discount factor used by savers, defined as

$$\Lambda_{t,t+1}^s \equiv \beta \frac{\xi_{t+1}}{\xi_t} \frac{p_s U_c(C_{s,t+1}) + (1-p_s) U_c(C_{b,t+1})}{(1+\pi_{t+1}) U_c(C_{s,t})}. \tag{8}$$

2.1.2 Borrowers

Borrowers also maximize their expected discounted lifetime utility

$$\mathcal{U}_{b,t} = u_{b,t} + \beta_b E_t \{ p_b \mathcal{U}_{b,t+1} + (1-p_b) \mathcal{U}_{s,t+1} \} \tag{9}$$

thus taking into account the probability of becoming a saver in the future when evaluating expected future utility flows, and subject to the following flow-budget constraint

$$P_t C_{b,t} + p_b (1 + i_t^B) Q_{t-1} B_{t-1} = W_{b,t} L_{b,t} + (1-p_b)(1 + i_{t-1}^D) D_{t-1} + Q_t B_t + T_{b,t}. \tag{10}$$

The nominal resources available to borrowers at the beginning of each period t therefore include the labor income $W_{b,t} L_{b,t}$, the per-capita share of payoff on the portfolio of deposits from the previous period $(1 + i_{t-1}^D) D_{t-1}$ brought by the mass $(1-z)(1-p_s)$ of savers that have turned borrowers at the beginning of the period – where we used the restriction $(1-z)(1-p_s) = z(1-p_b)$ – the resources borrowed selling long-term debt B_t at price Q_t and the transfers $T_{b,t}$ received by the fiscal authority. The borrowers use these resources to purchase a bundle of consumption goods $C_{b,t}$, and pay off the long-term debt B_{t-1} accumulated in the previous period by the share p_b of borrowers that have not turned savers at the beginning of period t.

The optimal choice of consumption, hours worked and long-term debt implies the Euler equation

$$\xi_t U_c(C_{b,t}) = \beta_b E_t \left\{ \frac{1 + i_{t+1}^B}{1 + \pi_{t+1}} \xi_{t+1} \Big[p_b U_c(C_{b,t+1}) + (1-p_b) U_c(C_{s,t+1}) \Big] \right\} \tag{11}$$

and the labor supply

$$\frac{V_l(L_{b,t})}{U_c(C_{b,t})} = \frac{W_{b,t}}{P_t}. \tag{12}$$

Moreover, the following no-arbitrage condition determines the interest-rate on long-term bonds

$$1 = E_t \left\{ \Lambda_{t,t+1}^b (1 + i_{t+1}^B) \right\}, \tag{13}$$

where $\Lambda^b_{t,t+1}$ denotes the nominal stochastic discount factor used by borrowers, defined as

$$\Lambda^b_{t,t+1} \equiv \beta_b \frac{\xi_{t+1}}{\xi_t} \frac{p_b U_c(C_{b,t+1}) + (1-p_b)U_c(C_{s,t+1})}{(1+\pi_{t+1})U_c(C_{b,t})}. \tag{14}$$

Equation (11) emphasizes an additional implication of idiosyncratic risk in our economy: a positive probability p_b of changing type in the future activates "anticipative-borrowing" motives that affect current spending decisions of borrowers anytime there is expected inequality in future consumption across types. If the savers are expected to consume more than the borrowers, then a positive probability of turning saver tomorrow makes a current borrower want to anticipate the possible future rise in consumption by borrowing more and consume more also today.

As we are going to show shortly, the anticipative motives arising in our economy – both "precautionary-saving" and "anticipative-borrowing" – are going to play a key role for the transmission mechanism of policy and non-policy shocks in our economy.

2.2 Financial Intermediaries

We model the intermediation sector in the same spirit as Sims et al. (2021), which simplify the case in Gertler and Karadi (2011, 2013) and using insights from Benigno and Benigno (2022). At the beginning of each period, a large number of banks enters the economy and lives for two periods. The banks collect from the set of savers the stock of one-period nominal deposits D^f, and the stock of nominal net worth N^f_t. They allocate these resources in a portfolio of nominal assets including central-bank reserves R^f and long-term bonds issued by private borrowers, B^f. Therefore, the balance sheet of the banking sector in period t reads

$$Q_t B^f_t + R^f_t = D^f_t + N^f_t. \tag{15}$$

The banks remunerate nominal deposits at the gross rate $1+i^D$, which they take as given. On the other side of the balance sheet, they collect the gross return on the long-term assets $1+i^B$ and on central bank reserves $1+i^R \geq 1$, both of which they take as given.[8] As a consequence, the intermediary's profits at the beginning of time $t+1$ are

$$\Pi^f_{t+1} \equiv (1+i^B_{t+1})Q_t B^f_t + (1+i^R_t)R^f_t - (1+i^D_t)D^f_t. \tag{16}$$

The financial intermediaries choose the amount of deposits D^f to collect and the portfolio allocation between long-term bonds B^f and reserves R^f in order to maximize rents from intermediation while satisfying two constraints. The first is the balance-sheet constraint (15). The second is a

[8] As discussed in Benigno and Benigno (2022), the zero-lower bound on the interest rate on reserves $1+i^R \geq 1$ needs not be an assumption, but rather an equilibrium outcome if the central bank issues also cash as an alternative store of value that the financial intermediaries can use to substitute for reserves in case they paid a negative interest rate. The economy remains cashless in equilibrium, but the existence of such an alternative store of value makes the zero-lower bound on i^R effective.

leverage constraint limiting the amount of long-term bonds the bank can purchase:

$$Q_t B_t^f \leq P_t \theta_t. \tag{17}$$

with θ_t following an exogenous stochastic process.

The financial intermediaries entering in period t then solve the problem

$$\max_{B_t^f, R_t^f, D_t^f} E_t \left\{ \Lambda_{t,t+1}^s \Pi_{t+1}^f \right\} - N_t^f \tag{18}$$

subject to (15), (16) and (17), and where the financial intermediaries use the savers' stochastic discount factor $\Lambda_{t,t+1}^s$ to discount next-period nominal profits, since banks are owned by savers.

Optimality conditions of the above problem imply, by no-arbitrage, that the interest rate on deposits equals at all times the interest rate on central-bank reserves, $i_t^D = i_t^R \geq 0$, while private long-term bonds pay a term premium in case the leverage constraint binds

$$E_t \left\{ \Lambda_{t,t+1}^s \left(i_{t+1}^B - i_t^D \right) \right\} = \zeta_t, \tag{19}$$

where $\zeta_t \geq 0$ is the Lagrange multiplier associated to constraint (17).

2.3 Firms

The production sector is standard New Keynesian, and mostly follows Benigno and Nisticò (2017). A continuum of firms of measure one produces each one brand of differentiated goods using the linear technology

$$Y_t(i) = A_t L_t(i) \tag{20}$$

for all brands $i \in [0, 1]$. The labor input combines the hours worked of savers and borrowers through the Cobb-Douglas bundle

$$L_t(i) = \left[L_{s,t}(i) \right]^{1-z} \left[L_{b,t}(i) \right]^z, \tag{21}$$

which implies that the wage bills for each type of labor is the same as the average wage bill, $W_{s,t} L_{s,t} = W_{b,t} L_{b,t} = W_t L_t$ where $W_t = W_{s,t}^{1-z} W_{b,t}^z$.

Firms set their price according to the Calvo mechanism, whereby each period a share $\alpha \in [0, 1]$ of firms keeps last period's price while the remaining share $1 - \alpha$ optimally sets the price at level P_t^*. Given this structure, the equilibrium inflation rate then satisfies

$$1 = (1 - \alpha) \left(\frac{P_t^*}{P_t} \right)^{1-\epsilon} + \alpha \left(1 + \pi_t \right)^{\epsilon - 1}. \tag{22}$$

A common optimal price level P_t^* is chosen by all firms that are able to reset their price at t, as it maximizes the expected discounted stream of future profits

$$E_{t_0}\left\{\sum_{t=t_0}^{\infty}\alpha^{t-t_0}\Lambda^s_{t_0,t}Y_t(i)\left[\frac{P_t(i)}{P_t}-MC_t\right]\right\}, \tag{23}$$

subject to the demand for brand i, $Y_t(i)=(P_t(i)/P_t)^{-\epsilon}Y_t$, in which aggregate output satisfies the resource constraint

$$Y_t=(1-z)C_{s,t}+zC_{b,t}. \tag{24}$$

In the objective of the firm (23), the stochastic discount factor used is that of savers, which own the firms, and real marginal costs are given by

$$MC_t=(1-\tau)\frac{W_t}{P_tA_t}, \tag{25}$$

where τ is an employment subsidy.

The solution to the firms' problem implies, also using (22):

$$\left(\frac{1-\alpha(1+\pi_t)^{\epsilon-1}}{1-\alpha}\right)^{\frac{1}{1-\epsilon}}=\frac{\epsilon}{\epsilon-1}\frac{F_t}{K_t}, \tag{26}$$

with

$$F_t\equiv E_t\left\{\sum_{T=t}^{\infty}\alpha^{T-t}\Lambda^s_{t,T}Y_T\left(\frac{P_T}{P_t}\right)^{\epsilon}MC_t\right\}=Y_tMC_t+\alpha E_t\left\{\Lambda^s_{t,t+1}\left(1+\pi_{t+1}\right)^{\epsilon}F_{t+1}\right\} \tag{27}$$

$$K_t\equiv E_t\left\{\sum_{T=t}^{\infty}\alpha^{T-t}\Lambda^s_{t,T}Y_T\left(\frac{P_T}{P_t}\right)^{1-\epsilon}\right\}=Y_t+\alpha E_t\left\{\Lambda^s_{t,t+1}\left(1+\pi_{t+1}\right)^{1-\epsilon}K_{t+1}\right\}. \tag{28}$$

In equilibrium, firms' real marginal costs follow from aggregation of the labor supply equations of savers and borrowers, which the specification of preferences (1) and technology (21) keep tractable:

$$MC_t=(1-\tau)\frac{W_t}{P_tA_t}=(1-\tau)\frac{(Y_t\Delta^p_t)^{\varphi}}{\upsilon\exp(-\upsilon Y_t)A_t^{1+\varphi}}, \tag{29}$$

where we have also used the production function (20), the aggregator (2) and the resource constraint (24), and where Δ^p_t is an index of relative-price dispersion across firms

$$\Delta^p_t=\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}di, \tag{30}$$

which evolves according to

$$\Delta^p_t=\alpha(1+\pi_t)^{\epsilon}\Delta^p_{t-1}+(1-\alpha)\left(\frac{1-\alpha(1+\pi_t)^{\epsilon-1}}{1-\alpha}\right)^{\frac{\epsilon}{\epsilon-1}}. \tag{31}$$

2.4 The Government and the Aggregate Equilibrium

The government consists of a fiscal authority, running a balanced budget every period, and a central bank in charge of monetary policy.

The fiscal authority provides transfers and charges taxes. The (nominal) transfers include the employment subsidy to firms $\tau W_t L_t$ and a lump-sum transfer to borrowers $T_{b,t}$ that partially limits their liability position and simplifies their equilibrium.[9] These transfers are financed using lump-sum taxes on the savers and the remittances T_t^c received by the central bank. The budget constraint of the fiscal authority, in nominal terms, is therefore:

$$\tau W_t L_t + z T_{b,t} = T_t^c + (1-z) T_{s,t}. \tag{32}$$

When it comes to the borrowers, we assume that in order to repay and service their outstanding debt, borrowers must pledge the whole payoff from the deposits brought by the savers that switched type, and a fraction $(1-\varpi)$ of their labor income, with $0 \leq \varpi \leq 1$. Whatever is left is instead covered by the subsidy, whose net per-capita level is therefore

$$T_{b,t} = p_b(1+i_t^B)Q_{t-1}B_{t-1} - (1-\varpi)W_{b,t}L_{b,t} - (1-p_b)(1+i_{t-1}^D)D_{t-1}. \tag{33}$$

The appealing feature of this transfer scheme, analogously to Sims et al. (2021), is to make the intertemporal dynamics of private debt irrelevant for the consumption of borrowers and reduce the equilibrium budget constraint of borrowers to a static relationship. In our set up, however, differently from Sims et al. (2021), the equilibrium consumption of borrowers not only depends on the long-term debt contracted in period t, but also on the fraction ϖ of their current labor income that is available for consumption:

$$P_t C_{b,t} = Q_t B_t + \varpi W_{b,t} L_{b,t}. \tag{34}$$

Therefore, the additional appealing feature of this transfer scheme is to provide a simple (reduced-form) characterization of income inequality between savers and borrowers in equilibrium: given $W_{s,t}L_{s,t} = W_{b,t}L_{b,t}$, indeed, lower values of ϖ imply higher disposable income inequality in equilibrium.[10] Moreover, this transfer makes the "borrower" type behave very similarly to a hand-to-mouth agent in the benchmark THANK environment of Bilbiie (2018) and Bilbiie et al. (2022), allowing us to relate our results to that literature as well.

The central bank purchases long-term bonds B^c from the banking sector through open-market operations, using internal resources. The latter are equal to interest-bearing short-term nominal reserves that the central bank can issue at will, plus any retained financial profit from the past.

[9] A similar scheme is assumed in Sims et al. (2021).

[10] An alternative but equivalent way to introduce disposable labor income inequality would be to assume that borrowers face some unemployment risk that exclude from the labor market a fraction $1-\varpi$ of agents.

The central bank therefore faces the following flow-budget constraint

$$Q_t B_t^c = R_t + (1 + i_t^B) Q_{t-1} B_{t-1}^c - (1 + i_{t-1}^R) R_{t-1} - T_t^c. \tag{35}$$

The nominal reserves of the central bank define the unit of account in the economy. This implies that the central bank is not subject to any solvency constraint, as its liabilities are nominally risk-free regardless of its policy.[11] This further implies that the central bank can freely choose three instruments of policy: the interest rate on reserves i^R, the amount of reserves R, and the remittances T^c to transfer to the private sector through the treasury.

We assume that the central bank remits in each period t its entire financial income

$$T_t^c = (1 + i_t^B) Q_{t-1} B_{t-1}^c - (1 + i_{t-1}^R) R_{t-1}, \tag{36}$$

implying that the central bank has a constant level of nominal net worth, which we normalize to zero, so that the central bank's balance sheet is simply[12]

$$Q_t B_t^c = R_t. \tag{37}$$

The remaining two policy tools are the main objects of interest of our analysis, with the interest rate on reserves i^R capturing the *conventional* dimension of monetary policy, and the amount of reserves R – i.e. the size of the central bank's balance sheet – the *unconventional* one.

2.4.1 Aggregate Equilibrium

Equilibrium in the asset markets requires, for each period t

$$(1 - z) D = D^f \tag{38}$$
$$(1 - z) N = N^f \tag{39}$$
$$zB = B^f + B^c \tag{40}$$
$$R = R^f \tag{41}$$

Now define $u_t \equiv \frac{R_t}{P_t}$, $b_t \equiv \frac{Q_t B_t}{P_t}$, $b_t^f \equiv \frac{Q_t B_t^f}{P_t}$, $b_t^c \equiv \frac{Q_t B_t^c}{P_t}$, $d_t \equiv \frac{D_t}{P_t}$, $d_t^f \equiv \frac{D_t^f}{P_t}$, $n_t \equiv \frac{N_t}{P_t}$, $n_t^f \equiv \frac{N_t^f}{P_t}$, and $w_t \equiv \frac{W_t}{P_t}$.

An equilibrium is therefore a vector $\{\mathbf{Y}_t\}_{t=t_0}^{\infty}$ collecting seventeen stochastic processes

$$\mathbf{Y}_t \equiv (Y_t,\, \pi_t,\, C_{s,t},\, C_{b,t},\, i_t^D,\, i_t^B,\, i_t^R,\, b_t,\, u_t,\, \Lambda_{t,t+1}^s,\, \zeta_t,\, MC_t,\, \Delta_t^p,\, F_t,\, K_t,\, w_t,\, L_t)$$

[11] See Benigno (2020) and Benigno and Nisticò (2020) for a discussion.

[12] Since our analysis will be conducted in a first-order approximation of the model, we choose to disregard the implications of this assumption for the determination of the initial price level, which we take as predetermined. For a discussion, see Benigno (2020), Benigno and Nisticò (2020) and Benigno and Benigno (2022).

that satisfy the following fifteen restrictions, expressed in real terms

$$\xi_t U_c(C_{s,t}) = \beta E_t \left\{ \frac{1+i_t^D}{1+\pi_{t+1}} \xi_{t+1} \Big[p_s U_c(C_{s,t+1}) + (1-p_s) U_c(C_{b,t+1}) \Big] \right\} \tag{42}$$

$$\xi_t U_c(C_{b,t}) = \beta_b E_t \left\{ \frac{1+i_{t+1}^B}{1+\pi_{t+1}} \xi_{t+1} \Big[p_b U_c(C_{b,t+1}) + (1-p_b) U_c(C_{s,t+1}) \Big] \right\} \tag{43}$$

$$\theta_t + u_t = z b_t \tag{44}$$

$$i_t^D = i_t^R \geq 0 \tag{45}$$

$$E_t \left\{ \Lambda_{t,t+1}^s \left(i_{t+1}^B - i_t^D \right) \right\} = \zeta_t \tag{46}$$

$$Y_t = (1-z) C_{s,t} + z C_{b,t} \tag{47}$$

$$MC_t = (1-\tau) \frac{(Y_t \Delta_t^p)^{\varphi}}{v \exp(-v Y_t) A_t^{1+\varphi}} \tag{48}$$

$$\left(\frac{1-\alpha(1+\pi_t)^{\epsilon-1}}{1-\alpha} \right)^{\frac{1}{1-\epsilon}} = \frac{\epsilon}{\epsilon-1} \frac{F_t}{K_t}, \tag{49}$$

$$F_t = Y_t MC_t + \alpha E_t \left\{ \Lambda_{t,t+1}^s \left(1+\pi_{t+1}\right)^{\epsilon} F_{t+1} \right\} \tag{50}$$

$$K_t = Y_t + \alpha E_t \left\{ \Lambda_{t,t+1}^s \left(1+\pi_{t+1}\right)^{1-\epsilon} K_{t+1} \right\} \tag{51}$$

$$w_t = \frac{(Y_t \Delta_t^p)^{\varphi}}{v \exp(-v Y_t) A_t^{\varphi}} \tag{52}$$

$$\Delta_t^p = \alpha(1+\pi_t)^{\epsilon} \Delta_{t-1}^p + (1-\alpha) \left(\frac{1-\alpha(1+\pi_t)^{\epsilon-1}}{1-\alpha} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{53}$$

$$Y_t \Delta_t^p = A_t L_t \tag{54}$$

$$C_{b,t} = b_t + \varpi w_t L_t \tag{55}$$

$$\Lambda_{t,t+1}^s \equiv \frac{p_s U_c(C_{s,t+1}) + (1-p_s) U_c(C_{b,t+1})}{(1+\pi_{t+1}) U_c(C_{s,t})}, \tag{56}$$

for a given vector of exogenous processes $\{\mathbf{X}_t\}_{t=t_0}^{\infty}$ with $\mathbf{X}_t \equiv (\xi_t,\, A_t,\, \theta_t)$, and where we focus on equilibria where the banks' leverage constraint is always binding, implying $b_t^f = \theta_t$. For future reference, define the vector $\bar{\mathbf{Y}}_t$ as including $\mathbf{Y}_t$, its own lags and its own expected leads. With fifteen restrictions to determine seventeen processes, we have two degrees of freedom that we can exploit to specify the two dimensions of monetary policy.

Definition 1 *A* **conventional monetary policy** *– or interest-rate policy – specifies the stochastic process for the interest rate on reserves* $\{i_t^R\}_{t=t_0}^{\infty}$*, possibly as a function of endogenous and exogenous processes,* $i_t^R = \mathcal{I}(\bar{\mathbf{Y}}_t,\, \mathbf{X}_t)$*, where the function* $\mathcal{I}(\cdot)$ *is non-negative for any value of its arguments.*

Definition 2 *An* **unconventional monetary policy** *– or balance-sheet policy – specifies the* *stochastic process for the central bank's asset holdings (equal to reserves)* $\{u_t\}_{t=t_0}^{\infty}$*, possibly as a function of endogenous and exogenous processes,* $u_t = \mathcal{B}(\bar{\mathbf{Y}}_t,\, \mathbf{X}_t)$*, where the function* $\mathcal{B}(\cdot)$ *is non-negative for any value of its arguments.*

3 The Linear Model and the Welfare Criterion

We want to study the distributional implications of unconventional monetary policy in our economy, as well as the positive and normative implications of inequality for conventional and unconventional monetary policy, using a linear-quadratic framework. We proceed by approximating the model around an efficient zero-inflation steady state.

3.1 The Steady State

We start by assuming an optimal level of employment subsidy, i.e. such that the long-run monopolistic distortions are completely offset: $\tau^* = 1/\epsilon$, with ϵ the steady-state price-elasticity of demand. Under this employment subsidy, the long-run level of output is determined by the steady-state version of equation (48), and satisfies

$$\frac{\bar{Y}^{\varphi}}{\upsilon \exp(-\upsilon \bar{Y})\bar{A}^{1+\varphi}} = 1. \tag{57}$$

Notice that equations (48) and (57) do not involve any term related to the banking sector. This means that in our economy aggregate output in the long-run can reach its efficient level even if the steady state features credit frictions – i.e. if $\bar{\theta} < \infty$ – without any role for either conventional or unconventional monetary policy. This however does not mean that unconventional monetary policy has no role whatsoever in the long-run, as we are going to show shortly.

Equation (57), together with the steady-state version of (52) and (54), also implies $\bar{w}\bar{L} = \bar{Y}$ and the following cross-sectional distribution of consumption in the steady state:

$$\bar{C}_s = \frac{1 - z\varpi}{1 - z}\bar{Y} - \frac{z}{1 - z}\bar{b} \tag{58}$$

$$\bar{C}_b = \varpi\bar{Y} + \bar{b}, \tag{59}$$

which we can summarize with the following index of long-run consumption inequality

$$\Gamma \equiv \frac{\bar{C}_s}{\bar{C}_b} = \frac{1 - z(\varpi + \bar{b}_Y)}{(1 - z)(\varpi + \bar{b}_Y)}, \tag{60}$$

where we defined $\bar{b}_Y \equiv \bar{b}/\bar{Y}$.

The steady-state versions of the Euler equations (42)–(43) then imply

$$1 = \beta\Gamma_s(1 + \bar{\imath}^R) \tag{61}$$

$$1 = \beta_b\Gamma_b(1 + \bar{\imath}^B) \tag{62}$$

where we have used the steady state version of (45) and we defined

$$\Gamma_s \equiv p_s + (1 - p_s) \exp\left[(\Gamma - 1)\,\sigma_b\right] \tag{63}$$

$$\Gamma_b \equiv p_b + (1 - p_b) \exp\left[(1 - \Gamma)\,\sigma_b\right], \tag{64}$$

with $\sigma_b \equiv \sigma \frac{\bar{C}_b}{\bar{Y}}$ and $\sigma \equiv v\bar{Y}$. Accordingly, we can write

$$\frac{1+\bar{\imath}^B}{1+\bar{\imath}^R} = \frac{\beta\Gamma_s}{\beta_b\Gamma_b} \geq 1, \tag{65}$$

where the inequality holds if the value of Γ implies $\beta\Gamma_s \geq \beta_b\Gamma_b$.

Remark 1 *In our economy with credit frictions and idiosyncratic uncertainty, the steady-state credit spread depends on an additional component, compared to existing literature, triggered by the anticipative motives that affect both savers and borrowers when the steady state is unequal.*

Indeed, the credit spread in equation (65) is determined by two components: *i*) the familiar one related to the difference between the time-discount factors β and β_b (as in Benigno et al., 2020, among others), and *ii*) an additional component related to the anticipative motives implied by the idiosyncratic uncertainty ($0 < p_s$, $p_b < 1$) that drive a wedge between Γ_s and Γ_b. In fact, Γ_s and Γ_b capture the *ex-ante* gross rate of growth in the marginal utility of consumption respectively for savers and borrowers. Thereby, $\Gamma > 1$ implies a positive consumption risk for savers, who therefore want to buy more deposits for precautionary reasons, compared to an equal steady state with $\Gamma = 1$. This implies $\Gamma_s > 1$ and a downward pressure on the return on deposits, by equation (61): $\beta(1+\bar{\imath}^R) < 1$. Moreover, $\Gamma > 1$ also implies better prospects for the consumption of the borrowers, who therefore want to borrow more for anticipative reasons, compared to an equal steady state. This implies $\Gamma_b < 1$ and an upward pressure on the return on long-term bonds, by equation (62): $\beta_b(1+\bar{\imath}^B) > 1$. As a result, if $\Gamma > 1$ the economy displays a positive credit spread in the steady state (and thus a binding leverage constraint for the banking sector) even in the limiting case $\beta_b \to \beta$.

Finally, notice that the steady-state version of equation (44),

$$\bar{b}_Y = z^{-1}(\bar{\theta}_Y + \bar{u}_Y), \tag{66}$$

with $\bar{\theta}_Y \equiv \bar{\theta}/\bar{Y}$ and $\bar{u}_Y \equiv \bar{u}/\bar{Y}$, implies a first result about the distributional role of unconventional monetary policy.

Remark 2 *Assume the steady state is not frictionless – i.e. $\bar{\theta}_Y < \infty$ – and the leverage constraint is binding for any $\Gamma \geq 1$ – i.e. $\beta_b < \beta$. Then, for a given level of structural credit frictions $\bar{\theta}_Y$, the size of the central bank's balance sheet $\bar{u}$ determines the debt position of the private sector, and thereby affects the level of consumption inequality.*

By setting an appropriate size for its balance sheet, and in particular $\bar{u}_Y = z(1-\varpi) - \bar{\theta}_Y$, the central bank can completely offset consumption inequality in the long run and imply $\Gamma = 1$.

Moreover, the higher the amount of credit frictions in the long-run (i.e. the lower $\bar{\theta}_Y$) the larger the central bank's balance sheet needs to be in order to offset steady-state consumption inequality.

As we are going to show later, we do not need to impose a steady state with no consumption inequality in order to characterize a simple second-order approximation of social welfare for our normative analysis. Therefore, and in order to account for the possible cyclical implications of an unequal steady-state, in the rest of the analysis we will assume that the central bank does not completely offset steady-state inequality, and focus on the case $\varpi + \bar{b}_Y < 1$, implying $\Gamma > 1$.

3.2 The Linear Model

Now let a small-case variable denote the percentage deviation of the corresponding upper-case from either its own steady-state level or steady-state output.[13] Accordingly, a first-order approximation of equations (42)–(43) reads

$$c_{s,t} = \gamma_s E_t c_{s,t+1} + (1-\gamma_s) E_t c_{b,t+1} - \sigma^{-1}(\hat{\imath}_t^R - E_t \pi_{t+1} + E_t \Delta \hat{\xi}_{t+1}) \tag{67}$$

$$c_{b,t} = \gamma_b E_t c_{b,t+1} + (1-\gamma_b) E_t c_{s,t+1} - \sigma^{-1} E_t \{\hat{\imath}_{t+1}^B - \pi_{t+1} + \Delta \hat{\xi}_{t+1}\} \tag{68}$$

where $\gamma_s \equiv p_s/\Gamma_s$ and $\gamma_b \equiv p_b/\Gamma_b$, with $0 < \gamma_s, \gamma_b < 1$ and we have used $\hat{\imath}_t^R = \hat{\imath}_t^D$, as implied by a first-order approximation of (45).

Using the approximation of the resource constraint (47) and equations (67)–(68) yields

$$\begin{aligned} y_t = E_t y_{t+1} &- [(1-z)(1-\gamma_s) - z(1-\gamma_b)] E_t \omega_{t+1} \\ &- \sigma^{-1}(1-z)(\hat{\imath}_t^R - E_t \pi_{t+1}) - \sigma^{-1} z E_t \{\hat{\imath}_{t+1}^B - \pi_{t+1}\} - \sigma^{-1} E_t \Delta \hat{\xi}_{t+1}, \end{aligned} \tag{69}$$

where $\omega_t \equiv c_{s,t} - c_{b,t}$ defines consumption inequality. Note that – unlike in the case of an equal steady-state, where $\gamma_s = p_s$ and $\gamma_b = p_b$ and the coefficient in square brackets in the first line goes to zero by the restriction $(1-z)(1-p_s) = z(1-p_b)$ – when the steady state is unequal, the anticipative motives characterising also the long-run imply an additional and direct effect of expected consumption inequality on the dynamics of aggregate demand, beyond the indirect one implied by the long-term interest rate. A higher expected inequality, indeed, on the one hand induces a contractionary effect through higher precautionary savings, and on the other hand it stimulates an expansionary effect through higher anticipative borrowing.[14] If $\Gamma > 1$, the former effect dominates and the net effect is a fall in aggregate demand.

A first-order approximation of the borrower's budget constraint, using the labor supply equation

[13] In particular, we define $y_t \equiv (Y_t - \bar{Y})/\bar{Y}$, $c_{s,t} \equiv (C_{s,t} - \bar{C}_s)/\bar{Y}$, $c_{b,t} \equiv (C_{b,t} - \bar{C}_b)/\bar{Y}$, $a_t \equiv (A_t - \bar{A})/\bar{A}$, $\hat{b}_t \equiv (b_t - \bar{b})/\bar{Y}$, $\hat{\theta}_t \equiv (\theta_t - \bar{\theta})/\bar{Y}$, $\hat{u}_t \equiv (u_t - \bar{u})/\bar{Y}$, $\hat{\xi}_t \equiv (\xi_t - \bar{\xi})/\bar{\xi}$, and $\hat{\imath}_t^j \equiv \frac{i_t^j - \bar{\imath}^j}{1+\bar{\imath}^j}$, for $j = B, D, R$.

[14] As we are going to show shortly, the expansionary pressure through anticipative borrowing is however always offset once we net out the indirect effects through the long-term interest rate.

and the production function reads

$$c_{b,t} = \varpi(1+\varphi+\sigma)y_t - \varpi(1+\varphi)a_t + \hat{b}_t$$
$$= \chi x_t + \varpi y_t^* + z^{-1}(\hat{\theta}_t + \hat{u}_t) \tag{70}$$

where in the second line $y_t^* \equiv \frac{1+\varphi}{\sigma+\varphi}a_t$ denotes the potential level of output arising under flexible prices (we call it potential, and not simply natural, because here it is also efficient, given the optimal employment subsidy) and $x_t \equiv y_t - y_t^*$ defines the output gap. Moreover, in the second line we used a first-order approximation of (44) and we defined $\chi \equiv \varpi(1+\varphi+\sigma)$, the borrower's MPC out of aggregate income, that here depends on the amount of disposable labor-income inequality ϖ. In the benchmark THANK model of Bilbiie (2018), this parameter captures the MPC out of aggregate income of hand-to-mouth agents, and is key in the analysis of the role of inequality for the transmission of conventional interest-rate policy. We are going to show that this parameter plays an important role for monetary policy also along the unconventional balance-sheet dimension.

Notice that, as in the steady-state equilibrium, the potential level of output is determined by equation (48) only, evaluated in the flexible-price equilibrium, with no role for either credit frictions, consumption inequality and therefore unconventional monetary policy. Later, we will evaluate whether the welfare-maximizing social planner allocation is sufficiently summarized by such efficient level of aggregate output, and what is the role of unconventional monetary policy in pursuing such allocation.

Equation (70), a first-order approximation of the resource constraint (47) and the definition of consumption inequality, imply

$$\omega_t = (1-z)^{-1}\left[(1-\chi)\,x_t + (1-\varpi)y_t^* - z^{-1}(\hat{\theta}_t + \hat{u}_t)\right], \tag{71}$$

which shows that the consumption inequality responds to any shock relevant for potential output, in addition to financial shocks $\hat{\theta}_t$ and, importantly, unconventional monetary-policy shocks $\hat{u}_t$.

We can use equations (68) and (70) to express the expected real return on long-term bonds as

$$\sigma^{-1}E_t\{\hat{i}_{t+1}^B - \pi_{t+1} + \Delta\hat{\xi}_{t+1}\} = (1-\gamma_b)E_t\omega_{t+1}$$
$$+ \chi E_t \Delta x_{t+1} + \varpi E_t \Delta y_{t+1}^* + z^{-1}E_t\{\Delta\hat{\theta}_{t+1} + \Delta\hat{u}_{t+1}\}, \tag{72}$$

which allows us to substitute out the expected return on long-term bonds from equation (69)

$$y_t = E_t y_{t+1} - \frac{1-z}{\sigma}(\hat{i}_t^R - E_t\pi_{t+1} + E_t\Delta\hat{\xi}_{t+1}) - (1-z)(1-\gamma_s)E_t\omega_{t+1}$$
$$- z\chi E_t\Delta x_{t+1} - z\varpi E_t \Delta y_{t+1}^* - E_t\{\Delta\hat{\theta}_{t+1} + \Delta\hat{u}_{t+1}\}. \tag{73}$$

Note that the first term in equation (72) captures the effect of anticipative borrowing on the long-term interest rate, whereby a higher expected inequality stimulates anticipative borrowing and implies upward pressures on borrowing costs. Equation (73) then shows that, once we use equation (72) to substitute out the long-term rate from equation (69), the indirect contractionary effect of anticipative borrowing through the long-term rate exactly offsets the direct expansionary effect in (69), leaving precautionary saving as the only relevant anticipative motive for aggregate demand.

Using the definition of output gap finally yields, after some algebra, the IS schedule:

$$x_t = E_t x_{t+1} - \delta \frac{1-z}{\sigma}(\hat{\imath}_t^R - E_t \pi_{t+1} + E_t \Delta \hat{\xi}_{t+1}) - \delta(1-z)(1-\gamma_s) E_t \omega_{t+1} \\ + \delta(1 - z\varpi) E_t \Delta y^*_{t+1} - \delta E_t \{\Delta \hat{\theta}_{t+1} + \Delta \hat{u}_{t+1}\}, \quad (74)$$

where we defined $\delta \equiv [1 - z\chi]^{-1}$. Notice that for a large enough share of borrowers z the parameter δ turns negative, implying that the model is potentially exposed to the same "inverted aggregate demand logic" (IADL) discussed in Bilbiie (2008), since the borrowers have a unitary MPC out of their disposable labor income in equilibrium. The severity of this exposure here also depends on the share ϖ of labor income available to borrowers once paid off the outstanding debt. The larger this share, the more severe the exposure to the IADL. If the transfer only covers what remains after the entire labor income is pledged (i.e. $\varpi = 0$), the interest-rate elasticity is unambiguously negative, as in the standard model. This points to an interesting difference with respect to Bilbiie (2008, 2018): while in those cases reducing the exposure to the IADL requires a larger fiscal transfer to the constrained agents to reduce the procyclicality of their consumption, here the opposite is true, as the procyclicality of the borrowers' consumption is already dampened by the need to use their labor income to service the outstanding debt. Henceforth, we will restrict attention to the case $\delta > 0$.

Equation (74), moreover, clarifies the key difference of our economy with respect to analogous environments with savers and borrowers but no idiosyncratic uncertainty, such as Sims et al. (2021). In our economy, the stochastic transition between agent types implies that the aggregate demand is directly affected by expected inequality, capturing consumption risk for the savers, with upward revisions in expected inequality acting as a negative demand shock on aggregate output.

We are going to show that this margin implies two additional transmission channels of unconventional monetary policy, compared to similar economies with borrowers and savers.

Finally, a first-order approximation of the aggregate-supply block delivers the familiar New Keynesian Phillips Curve

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \quad (75)$$

with $\kappa \equiv \alpha^{-1}(1-\alpha)(1-\alpha\beta)(\sigma+\varphi)$.

3.3 The Welfare Criterion

To study the normative implications of inequality for (conventional and unconventional) monetary policy in our baseline economy with heterogeneous households, we are interested in the Ramsey policy that maximizes the expected social welfare

$$\mathcal{W}_{t_0} \equiv E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left[(1-\tilde{z})\Big(U(C_{s,t})-V(L_{s,t})\Big)+\tilde{z}\Big(U(C_{b,t})-V(L_{b,t})\Big)\right]\right\}, \tag{76}$$

for some Pareto-weight $\tilde{z}$, with $0 < \tilde{z} < 1$.[15] To evaluate the implied tradeoffs and derive the optimal monetary policy, we can use a purely quadratic loss function deriving from a second-order approximation of (76) around a socially optimal allocation. Such allocation is consistent with the solution of the Ramsey problem that maximizes (76) subject to the resource and technological constraint

$$A_t L_{s,t}^{1-z} L_{b,t}^{z} = Y_t = (1-z)C_{s,t} + zC_{b,t}. \tag{77}$$

The solution of this problem requires an appropriate cross-sectional distribution of steady-state consumption, given the Pareto-weight $\tilde{z}$:

$$\frac{1-\tilde{z}}{\tilde{z}} = \frac{1-z}{z}\frac{U_c(\bar{C}_b)}{U_c(\bar{C}_s)} \tag{78}$$

and an intratemporal efficiency condition for each type of agent:

$$\frac{V_L(\bar{L}_s)\bar{L}_s}{U_c(\bar{C}_s)} = \frac{V_L(\bar{L}_b)\bar{L}_b}{U_c(\bar{C}_b)} = \bar{Y}. \tag{79}$$

For a given long-run consumption inequality in the decentralized allocation of our economy, therefore, an appropriately chosen Pareto-weight $\tilde{z}$ makes sure that the steady state satisfies condition (78), and the optimal employment subsidy τ^* makes sure it satisfies condition (79). Under these two restrictions, the long-run equilibrium of our economy is indeed consistent with the socially optimal allocation, and a quadratic Taylor expansion of (76) is a valid second-order approximation of expected social welfare that can be evaluated using only first-order-approximated equilibrium conditions. Such approximation leads to the following quadratic loss function:[16]

$$\mathcal{L}_{t_0} = \frac{\sigma+\varphi}{2}E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(x_t^2+\lambda_\pi\pi_t^2+\lambda_\omega\omega_t^2\right)\right\}, \tag{80}$$

where the last term $\lambda_\omega\omega_t^2$ captures the welfare loss coming from variations in consumption inequality,

[15] For tractability reasons, here we are considering the limiting case where $\beta_b \to \beta$, as in Benigno et al. (2020). Note, however, that, as discussed in Section 3.1, idiosyncratic uncertainty in our economy implies that in an unequal steady state this limiting case is still consistent with a binding leverage constraint, implying a positive spread between borrowing and saving interest rates, unlike in Benigno et al. (2020).

[16] Please refer to the Appendix for details.

relative to the optimal steady state, and the relative welfare weights are defined as

$$\lambda_\pi \equiv \frac{\epsilon}{\kappa} \tag{81}$$

$$\lambda_\omega \equiv \sigma \frac{z(1-z)(1+\varphi+\sigma)}{(\sigma+\varphi)(1+\varphi)}. \tag{82}$$

The immediate implication of the loss function (80) is that consumption inequality arises as an additional target for welfare-maximizing monetary policy. In the next Section, we are going to show that such an additional target implies for the *conventional* monetary policy an endogenous trade-off with inflation/output stability, regardless of whether or not the supply block of our economy satisfies the "divine coincidence". Moreover, this endogenous trade-off can only be resolved – and the socially optimal allocation implied by (80) potentially achieved – if the *unconventional* dimension of monetary policy is appropriately specified.

Analogous terms capturing the welfare costs of consumption dispersion arising from some kind of reduced-form households' heterogeneity appear in several other contributions in the literature.[17] Notice that, as in most of these contributions, the relative welfare weight on consumption dispersion reflects the heterogeneity *between* the two agent types, and is therefore independent of the idiosyncratic uncertainty that in our economy generates stochastic transition between the types.[18] Importantly, however, in our economy the credit friction implies a time-varying wedge between inequality and the output gap that responds to both real and financial shocks, as implied by equation (71). Regardless of the shocks hitting the economy, therefore, the role of consumption inequality for welfare cannot simply be reflected in a larger weight on output stabilization (as instead in the benchmark THANK model of Bilbiie, 2018) and it always implies an additional and independent target with respect to inflation and output stabilization.[19]

Moreover, since idiosyncratic uncertainty in our economy affects the dynamics of this wedge, it will matter for the transmission of conventional and unconventional monetary-policy shocks, and for the way in which monetary policy optimally deals with the potential endogenous and exogenous tradeoffs in the face of financial and real shocks. In this transmission process, an important role will be played by the amount of disposable labor-income inequality, which shapes the cyclical properties of consumption inequality, as reflected by the value of composite parameter $\chi \equiv \varpi(1+\varphi+\sigma)$: large amounts of income inequality (low values of ϖ), indeed, imply $\chi < 1$, which makes consumption inequality procyclical, while the opposite is true for levels of ϖ large enough to imply $\chi > 1$.

We turn to this discussion in the next Section.

[17] A non exhaustive list includes Benigno and Nisticò (2017), Benigno et al. (2020), Bilbiie (2018), Bilbiie and Ragot (2021), Bonchi and Nisticò (2022), Nisticò (2016) and, more indirectly, also Cùrdia and Woodford (2016). Notice also that an analogous welfare-based loss function also arises in the economy analyzed in Sims et al. (2021), although the latter choose to endow the central bank with a more familiar "dual mandate".

[18] A notable exception, in the list above, are Nisticò (2016) and Bonchi and Nisticò (2022), where a different insurance mechanism makes this additional term actually reflect the heterogeneity *within* the "saver" type, with the relative welfare weight critically depending on the transition probabilities.

[19] This is in general true also in other models where this kind of credit frictions gives rise to financial intermediation, like Benigno and Nisticò (2017), Benigno et al. (2020), Cùrdia and Woodford (2016), Sims et al. (2021).

4 Policy Implications

In this Section we evaluate the monetary-policy implications of our model along several dimensions.

4.1 Two Benchmark Allocations

To understand the policy implications and the tradeoffs that are relevant in our economy, notice first that the welfare-based loss function (80) implies a first-best allocation that features not only inflation and output-gap stability, but also stability of consumption inequality at the (possibly non-zero) steady-state level: $\pi_t = x_t = \omega_t = 0$ for all t.

To study the role of conventional and unconventional monetary policy in pursuing this allocation, let us focus first on the benchmark, "natural" allocation arising in an equilibrium where prices are perfectly flexible – i.e. $\alpha = 0$ – and denote variables in such an equilibrium with an apex ^{n}. Moreover, let us focus for the moment on conventional monetary policy only, and therefore assume that the size of the central bank's balance sheet is constant: $\hat{u}_t = 0$ for all t.

From the aggregate supply block in our economy we can verify that, in the absence of cost-push shocks, such an allocation implies that both inflation and the output gap are at their targets: $\pi_t = x_t = 0$ for all t. In the benchmark New Keynesian model, imposing this allocation on the dynamic IS schedule allows to derive the dynamics of the natural rate of interest that is consistent with the natural equilibrium, $\hat{r}_t^n$.

To evaluate the natural rate of interest (on deposits and central-bank reserves) in our economy, first impose $x_t = \hat{u}_t = 0$ for all t on the consumption-inequality equation (71), which yields

$$\omega_t^n = \frac{1-\varpi}{1-z} y_t^* - \frac{1}{z(1-z)} \hat{\theta}_t. \tag{83}$$

Using the above on the IS schedule (74), and imposing $\pi_t = x_t = \hat{u}_t = 0$ for all t then implies

$$\hat{r}_t^n = \sigma \frac{1-z\varpi}{1-z} E_t \Delta y_{t+1}^* - \sigma \frac{(1-\gamma_s)(1-\varpi)}{1-z} E_t y_{t+1}^* - E_t \Delta \hat{\xi}_{t+1} - \frac{\sigma}{1-z} E_t \Delta \hat{\theta}_{t+1} + \sigma \frac{1-\gamma_s}{z(1-z)} E_t \hat{\theta}_{t+1}. \tag{84}$$

There are several insights we can draw from the above.

First. The natural equilibrium, despite being efficient in terms of the aggregate output level y_t^* it implies, is not consistent with the socially optimal allocation, as it implies fluctuations in consumption inequality ω_t^n with respect to the optimal steady state, that follow from changes in both the potential output and the financial leverage constraint. In particular, for given fluctuations in potential output, fluctuations in consumption inequality are larger, the larger is the disposable labor-income inequality in equilibrium – i.e. the lower ϖ. In the absence of income inequality – i.e. $\varpi = 1$ – potential output becomes irrelevant for consumption inequality but the natural allocation is still socially suboptimal, as consumption inequality still responds to the financial shocks hitting the banking system. If $\varpi = 1$ and $\hat{\theta}_t = 0$ for all t, the natural allocation would instead be also socially optimal, as $\omega_t^n = 0$ and the natural interest rate collapses to the one arising in the benchmark New Keynesian model with no household heterogeneity: $r_t^n = \sigma E_t \Delta y_{t+1}^* - E_t \Delta \hat{\hat{\xi}}_{t+1}$.

Second. The natural rate of interest falls in response to deleveraging shocks (i.e. negative shocks to $\hat{\theta}_t$), as in Eggertsson and Krugman (2012) and Benigno et al (2020) among others, implying a potential challenge to the *conventional* dimension of monetary policy, given the effective-lower bound (ELB) on nominal interest rates.

Third. The idiosyncratic uncertainty, despite not affecting the level of potential output, activates additional transmission channels that are reflected in the path of the natural rate of interest, as captured by the second and last terms in equation (84). The natural rate of interest, indeed, accommodates current potential-output shocks more than it does in the benchmark New Keynesian model, in order to absorb fluctuations in consumption inequality that would otherwise put pressures on the output gap, through consumption risk. The degree of over-accommodation is larger: *i*) the larger is the idiosyncratic uncertainty (i.e. lower γ_s), whereby given consumption inequality fluctuations are more relevant for the output gap and *ii*) the larger is the disposable income inequality (i.e. lower ϖ), whereby consumption inequality fluctuations are larger for a given path of potential output. For the same reason – to absorb the fluctuations in consumption inequality that would otherwise put pressures on the output gap – the natural rate also leans against financial shocks more than it would in the case of no idiosyncratic uncertainty (as, for example, in Sims et al. 2021).

Fourth. There is no interest-rate path that supports a socially optimal allocation in which $\pi_t = x_t = \omega_t = 0$ for all t. As clearly implied by equations (83)–(84), in the natural equilibrium (where $\pi_t = x_t = \hat{u}_t = 0$ for all t) as long as there are shocks to potential output or the leverage constraint, there will also be fluctuations in consumption inequality, making this allocation suboptimal.

Therefore, in our economy with heterogeneous households, credit frictions and idiosyncratic uncertainty, there is no path for the *conventional* monetary-policy tool $\hat{\imath}_t^R$ that is consistent with the socially optimal allocation, as the system lacks one degree of freedom to accommodate all three targets at all times. The discussion of the previous Section, however, and in particular equation (71), suggests a role for the *unconventional* dimension of monetary policy to provide the degree of freedom that we need to reconcile stability of consumption inequality with the natural equilibrium. Indeed, recall that we have so far assumed the size of the central bank's balance sheet to be constant – i.e. $\hat{u}_t = 0$ for all t. Relaxing this assumption provides the necessary margin to achieve consistency with the socially optimal allocation.

To see this, let us now focus on such "optimal" (or first-best) allocation where not only inflation and the output gap are on target, but also consumption inequality is constant at the socially optimal steady-state level – i.e. $\pi_t = x_t = \omega_t = 0$ for all t – denoting variables in the corresponding equilibrium with an apex *. Moreover, let the size of the central bank's balance sheet be determined now endogenously in equilibrium. Imposing $x_t = \omega_t = 0$ for all t on the consumption-inequality equation (71) yields

$$\hat{u}_t^* = z(1-\varpi)y_t^* - \hat{\theta}_t \tag{85}$$

which, used in equation (74) finally implies

$$\hat{r}_t^* = \sigma E_t \Delta y_{t+1}^* - E_t \Delta \hat{\xi}_{t+1}. \tag{86}$$

Overall, the analysis above implies the following

Remark 3 *In our economy with heterogeneous households, credit frictions and idiosyncratic uncertainty, an appropriate state-contingent path of the* **unconventional** *tool of monetary policy is a necessary condition to achieve the socially optimal equilibrium.*

Indeed, while there is no stochastic process for the *conventional* monetary-policy tool $\{\hat{\imath}_t^R\}_{t=t_0}^{\infty}$ that can support the first-best equilibrium if the central bank's balance sheet is constant, there exists instead a joint stochastic process for the *conventional* monetary-policy tool $\{\hat{\imath}_t^R\}_{t=t_0}^{\infty}$ and the *unconventional* monetary-policy tool $\{\hat{u}_t\}_{t=t_0}^{\infty}$, that is consistent with the socially optimal allocation, where in particular the conventional tool supports the aggregate efficiency dimension of the socially optimal allocation, and the unconventional tool the distributional dimension.

This joint process implies a short-term interest rate following the same dynamics as it would in the absence of any heterogeneity or credit friction whatsoever, because the pressures on consumption inequality that are implied by shocks to either potential output or the leverage constraint are completely absorbed by an appropriate adjustment in the quantity of reserves.

Therefore, and as in Sims et al. (2021), an appropriate change in central bank's reserves in our economy is in principle able – by itself – to completely absorb the effects of financial shocks, thus insulating the economy without requiring changes in the real interest rate.

More interesting and novel are the implications of fluctuations in potential output, which the optimal allocation requires to be reflected into both the real interest rate and the quantity of reserves. Indeed, as potential output expands, the *optimal* interest rate falls to accommodate the increase in output to a full extent, exactly as in the benchmark New Keynesian model, as implied by equation (86). However, in our economy the effects of the fall in the real interest rate are initially entirely on the savers, which therefore reduce savings and raise current consumption proportionately. Borrowers also benefit from the increase in potential output, though not through the fall in the real interest rate but through a higher disposable labor income. The latter, however, only increases by a fraction ϖ of the rise in output. As a consequence consumption inequality rises, as also implied by equation (71). A rise in central bank's reserves in this case is able to compensate such a lower increase in disposable labor income through lower borrowing costs that in equilibrium end up expanding the borrowers' consumption by exactly as much as the savers', thus stabilising consumption inequality at the optimal steady-state level. On this joint path, therefore, the amount of reserves increases relatively more, for a given shock to potential output, the larger is the disposable labor-income inequality (i.e. lower ϖ).

Note in particular that this distributional role for the unconventional dimension of monetary policy arises as a result of the banks' leverage constraint being binding, but independently of *fluctuations* in the degree to which it binds, i.e. regardless of financial shocks $\hat{\theta}_t$.[20]

4.2 The transmission channels of unconventional monetary policy

To characterize the channels through which unconventional monetary policy transmits to the real economy in our model, use equations (85) and (86) in the IS schedule (74) and in the inequality equation (71), to simplify the aggregate demand block into a single IS-type schedule

$$x_t = \Phi E_t x_{t+1} - \sigma_x^{-1}(\hat{\imath}_t^R - E_t \pi_{t+1} - \hat{r}_t^*) - \delta E_t\{\Delta \hat{u}_{t+1} - \Delta \hat{u}_{t+1}^*\} + z^{-1}\delta(1-\gamma_s)E_t\{\hat{u}_{t+1} - \hat{u}_{t+1}^*\} \quad (87)$$

where we defined $\Phi \equiv 1 - \delta(1-\chi)(1-\gamma_s)$. Equation (87) clarifies the channels through which unconventional monetary policy can affect aggregate demand in our economy.

Remark 4 *In our economy with heterogeneous households, credit frictions and idiosyncratic uncertainty, unconventional monetary policy affects current aggregate demand through three channels:*

i) a "borrowing-cost channel" related to the effects of unconventional monetary policy on borrowers' current demand, through changes in the long-term interest rate – second-to-last term in (87);

ii) an "idiosyncratic-risk channel" related to the effects of unconventional monetary policy on savers' current demand, through changes in consumption risk – last term in the RHS of (87);

iii) a "cyclical-inequality channel" related to the cycle-induced general-equilibrium effects on expected inequality and consumption risk along the planning horizon – first term in the RHS of (87).

Through the "borrowing-cost channel", an expansion in central bank's reserves stimulates aggregate demand because the relaxation of the leverage constraint of financial intermediaries allows borrowers to access cheaper credit and finance an expansion in their consumption, as implied by equations (70) and (72). This is the familiar transmission channel of unconventional monetary policy discussed in Gertler and Karadi (2011, 2013) and Sims et al. (2021), among others.

Through the "idiosyncratic-risk channel", a persistent increase in the size of the central bank's balance sheet stimulates aggregate demand because the positive effect on future borrowers' consumption reduces the consumption risk faced by current savers, who therefore find it optimal to cut precautionary savings and increase current spending. To our knowledge, this is a novel channel, that emphasizes the role of unconventional monetary policy in mitigating the propagation and amplification of negative shocks through their effect on idiosyncratic consumption risk.

Through the "cyclical-inequality channel", a persistent increase in central bank's reserves further stimulates current aggregate demand in general equilibrium, if consumption inequality is countercyclical (i.e. $\chi > 1$). In this case, the fall in expected future inequality is amplified by the increase in the expected future output gap, thereby reinforcing the fall in consumption risk for the savers and their incentive to increase current demand. By the same logic, in the case of procyclical inequality (i.e. $\chi < 1$) this channel would instead reduce the stimulative effects of an expansionary

[20] This is an additional difference with respect to Sims et al (2021), where the only role for unconventional monetary policy is to undo the implications of financial shocks.

unconventional monetary policy, since the increase in expected output gap would dampen the fall in expected inequality.

The latter channel affects the transmission of conventional interest-rate policy as well. As discussed in Bilbiie (2018), idiosyncratic uncertainty (i.e. $\gamma_s < 1$) makes the cyclicality of consumption inequality a key factor for the transmission of expected future changes in interest rates, as implied by the composite parameter Φ: procyclical inequality ($\chi < 1$) translates into expected future changes in interest rates being discounted (i.e. $\Phi < 1$), as opposed to being compounded with countercyclical inequality (i.e. $\Phi > 1$). In our economy with credit frictions, equation (87) generalizes this implication and extends it to the unconventional balance-sheet dimensions of monetary policy as well: not only the transmission of expected future changes in the conventional policy tool $\hat{\imath}^R$ is affected, but also that of expected future changes in the unconventional one, $\hat{u}$.

To see this, and also to facilitate the intuition behind the result on determinacy we derive in the next section, note indeed that we can solve equation (87) forward and write the IS schedule as[21]

$$x_t = E_t\left\{\sum_{k=0}^{\infty}\Phi^k\Big[-\sigma_x^{-1}\left(\hat{\imath}^R_{t+k} - \pi_{t+k+1} - r^*_{t+k}\right)\right.$$
$$\left.- \delta\left(\Delta\hat{u}_{t+k+1} - \Delta\hat{u}^*_{t+k+1}\right) + z^{-1}\delta(1-\gamma_s)\left(\hat{u}_{t+k+1} - \hat{u}^*_{t+k+1}\right)\Big]\right\}. \quad (88)$$

4.3 Equilibrium Determinacy

To understand the monetary-policy options to steer the system towards the optimal equilibrium, this Section evaluates the implications for equilibrium determinacy of feedback policy rules along both the conventional and unconventional dimensions.

Assume that conventional and unconventional monetary policies are set according to the following simple feedback rules:

$$\hat{\imath}^R_t = \phi_\pi \pi_t + \phi_x x_t + v^c_t \quad (89)$$
$$\hat{u}_t = -\psi_\pi \pi_t - \psi_x x_t + v^u_t. \quad (90)$$

Proposition 1 *In our economy with heterogenous households, credit frictions and idiosyncratic uncertainty, a rational-expectations equilibrium is locally determinate if and only if the response coefficients in the feedback policy rules (89)–(90) satisfy the following inequality:*

$$\sigma_x^{-1}\Big[(1-\beta)\phi_x + \kappa(\phi_\pi - 1)\Big] + z^{-1}\delta(1-\gamma_s)\Big[(1-\beta)\psi_x + \kappa\psi_\pi\Big] > (1-\beta)(\Phi - 1). \quad (91)$$

Proof. See Appendix B ■

[21] To be accurate, equation (88) also assumes that the effects of price stickiness vanish asymptotically (and in particular at a rate higher than Φ in the case $\Phi > 1$).

Condition (91) implies the following

Remark 5 *In our economy with heterogenous households, credit frictions and idiosyncratic uncertainty, the conventional and unconventional monetary-policy instruments are "perfect substitutes" in the pursuit of determinacy of the rational-expectations equilibrium.*

The rate of substitution between the conventional and unconventional tools is the ratio between the elasticity of the current output gap with respect to the conventional interest-rate channel (σ_x^{-1}) and that with respect to the unconventional "idiosyncratic-risk channel" ($z^{-1}\delta(1-\gamma_s)$).

Note that the above result is a direct implication of idiosyncratic uncertainty in our economy. Indeed, in the polar case where there is still heterogeneity between savers and borrowers but there is no idiosyncratic uncertainty (i.e. $\gamma_s = 1$, which also implies $\Phi = 1$ regardless of χ), the condition for determinacy reduces to $(1-\beta)\phi_x + \kappa(\phi_\pi - 1) > 0$, i.e. identical to the one derived in the context of the benchmark New Keynesian model by Bullard and Mitra (2002), among others.[22] This implication clarifies that the relevant margin of unconventional policy for equilibrium determinacy is the "idiosyncratic-risk channel", as opposed to the familiar "borrowing-cost channel".

In general, therefore, to assess whether the central bank is responsive enough to grant equilibrium determinacy, in our economy we need to evaluate a convex combination of the conventional and unconventional policy rules, rather than just the conventional interest-rate rule.

The important and novel implication of Remark 5 is that determinacy of the rational-expectations equilibrium can be achieved by means of unconventional tools only, i.e. even in the limiting case of an interest-rate peg (i.e. $\phi_\pi = \phi_x = 0$), or a permanent liquidity trap. All is needed is that the unconventional monetary policy is active enough, meaning it satisfies

$$z^{-1}\delta(1-\gamma_s)\Big[(1-\beta)\psi_x + \kappa\psi_\pi\Big] > (1-\beta)(\Phi - 1) + \sigma_x^{-1}\kappa.$$

To grasp the power of this implication, note that, in the special case where the disposable labor-income inequality ϖ is such that $\Phi = 1 - (1-\beta)^{-1}\sigma_x^{-1}\kappa$, the above condition further simplifies to $(1-\beta)\psi_x + \kappa\psi_\pi > 0$: the central bank in this special case is able to rule out endogenous instability even if the only thing the private sector expects it to do is use its balance sheet to respond to inflation with a positive (however small) coefficient, i.e. if $\psi_x = 0$ and $\psi_\pi > 0$.

Moreover, note that condition (91) generalizes another result derived for the conventional monetary policy by Bilbiie (2018): the cyclicality of consumption inequality determines the extent to which monetary policy – intended here as the *combination* of conventional and unconventional policy – needs to be active in order to implement equilibrium determinacy, with countercyclical inequality (i.e. $\chi, \Phi > 1$) requiring a higher degree of responsiveness to rule out sunspot fluctuations.[23]

[22] Note that this result is different than the one in Sims et al. (2021), where the standard conditions for determinacy only arise if the central bank *decides* not to respond with the unconventional tool to inflation or the output gap (i.e. $\psi_x = \psi_\pi = 0$), while here they would arise *in spite of that*, were the case $\gamma_s = 1$. The reason behind this difference is the role of credit frictions for aggregate supply that are implied by the exclusion of borrowers from the labor market assumed in Sims et al. (2021), unlike in our model.

[23] An analogous result is derived in Acharya and Dogra (2020), in a prototypical (though analytical) HANK model.

The interesting complementary insight that we provide is that a countercyclical inequality in our economy does not necessarily make the Taylor Principle insufficient for determinacy, as instead in Bilbiie (2018). Indeed, endogenous unconventional monetary policy in this case improves the central bank's ability to anchor the private-sector expectations, by providing the additional degree of responsiveness that is needed to rule out endogenous instability without deviating from the Taylor Principle, as long as

$$(1-\beta)\psi_x + \kappa\psi_\pi > z(1-\beta)(\chi-1). \tag{92}$$

When inequality is countercyclical, (i.e. $\chi > 1$), and there is idiosyncratic uncertainty, (implying also $\Phi > 1$), an upward revision in expected future income increases current aggregate demand more than one for one, because of the amplification induced through the "cyclical-inequality channel" by the expectation that future inequality will be lower – as also implied by Remark 4.*iii*). This challenges equilibrium determinacy unless monetary policy leans against the extra push on aggregate demand coming from a lower consumption risk for the savers. There are two possible remedies to this: increase the responsiveness of interest-rate policy to inflation beyond the Taylor Principle, as shown by Bilbiie (2018), or use balance-sheet policy sufficiently actively, as implied by (92).

A final implication of condition (91) is that we can evaluate three alternative monetary policy regimes, that can be equally effective in terms of ruling out endogenous instability.

Definition 3 *A* **conventional regime** *– or "old-style regime" – combines a conventional monetary policy specified as a feedback function of endogenous and exogenous processes, $i_t^R = \mathcal{I}(\bar{\mathbf{Y}}_t, \mathbf{X}_t)$ (where the function $\mathcal{I}(\cdot)$ is non-negative for any value of its arguments) with an unconventional monetary policy specified as an exogenous process for the central bank's asset holdings $\{u_t\}_{t=t_0}^{\infty}$.*

Rules (89)–(90) are consistent with a **conventional regime** *if at least one between ϕ_π and ϕ_x is strictly positive, and both $\psi_\pi = \psi_x = 0$, and condition (91) is always satisfied.*

This regime is arguably best describing the "old-style" central banking, before the Great Financial Crisis, when the main monetary policy tool was the short-term interest rate.[24]

Definition 4 *An* **unconventional regime** *combines an unconventional monetary policy specified as a feedback function of endogenous and exogenous processes, $u_t = \mathcal{B}(\bar{\mathbf{Y}}_t, \mathbf{X}_t)$ (where the function $\mathcal{B}(\cdot)$ is non-negative for any value of its arguments) with a conventional monetary policy specified as an exogenous process for the nominal interest rate on reserves $\{i_t^R\}_{t=t_0}^{\infty}$.*

Rules (89)–(90) are consistent with an **unconventional regime** *if at least one between ψ_π and ψ_x is strictly positive, and both $\phi_\pi = \phi_x = 0$, and condition (91) is always satisfied.*

This regime is better describing the policy landscape in the immediate aftermath of the Great Financial Crisis, when the short-term policy rate was stuck at zero and the main margin of policy response to shocks was the unconventional one.

[24] Our notion of old-style and new-style central banking is loosely related to Hall and Reis (2013), who introduce this distinction mostly on the basis of the average maturity structure of the central bank's asset portfolio.

Definition 5 *A* **mixed regime** *– or "new-style regime" – combines a conventional monetary policy specified as a feedback function of endogenous and exogenous processes,* $i_t^R = \mathcal{I}(\bar{\mathbf{Y}}_t, \mathbf{X}_t)$ *(where the function* $\mathcal{I}(\cdot)$ *is non-negative for any value of its arguments) with an unconventional monetary policy specified as a feedback function of endogenous and exogenous processes,* $u_t = \mathcal{B}(\bar{\mathbf{Y}}_t, \mathbf{X}_t)$ *(where the function* $\mathcal{B}(\cdot)$ *is non-negative for any value of its arguments).*

Rules (89)–(90) are consistent with a **new-style regime** *if at least one between* ϕ_π *and* ϕ_x *and at least one between* ψ_π *and* ψ_x *are strictly positive, and condition (91) is always satisfied.*

This last regime is interesting to characterize what seems to be the new normal, where conventional and unconventional tools are both available to respond to the state of the economy.

4.4 The Transmission of Shocks and the Role of Inequality

This section analyzes the transmission of both policy and non-policy shocks through our economy, and the role of the idiosyncratic uncertainty and the cyclicality of inequality in shaping the way in which the monetary-policy regime affects such transmission.

Let the conventional and unconventional policy rules be specified by the simple rules (89)–(90), with non-negative response coefficients ϕ's and ψ's, and where the policy shocks v_t^i, with $i = c, u$, follow an AR(1) process:

$$v_t^i = \rho_i v_{t-1}^i + \varepsilon_t^i, \tag{93}$$

with $\rho_i \in [0, 1]$. We can exploit the tractability of our framework to characterize analytically the effects of both policy and non-policy shocks, study their transmission mechanism and the role of inequality and the monetary-policy regime.

4.4.1 An unconventional policy shock

Recall that a rational-expectations equilibrium in our economy can be locally unique under three alternative monetary-policy regimes, described in Definitions 3–5. We start by considering the general *new-style regime* defined in Definition 5, where both dimensions of monetary policy are endogenous – i.e. the response coefficients are non-zero in both feedback rules (89)–(90), and satisfy condition (91).

The solution of our model, conditional on an unconventional monetary-policy shock is:

$$x_t = \sigma_x \delta (1 - \beta\rho_u) \left[1 - \rho_u + \rho_u \frac{1-\gamma_s}{z}\right] \Psi_u v_t^u \qquad \pi_t = \sigma_x \delta \kappa \left[1 - \rho_u + \rho_u \frac{1-\gamma_s}{z}\right] \Psi_u v_t^u \tag{94}$$

where

$$\Psi_u \equiv \left[\sigma_x (1 - \Phi\rho_u)(1 - \beta\rho_u) + \eta_u\right]^{-1} \tag{95}$$

and η_u captures the overall degree of responsiveness of monetary policy

$$\eta_u \equiv (1 - \beta\rho_u)\phi_x + \kappa(\phi_\pi - \rho_u) + \sigma_x \delta \left(1 - \rho_u + \rho_u \frac{1-\gamma_s}{z}\right) \Big(\kappa\psi_\pi + (1 - \beta\rho_u)\psi_x\Big), \tag{96}$$

with $\Psi_u \geq 0$, as guaranteed by the response coefficients satisfying the determinacy condition (91). An expansionary unconventional monetary-policy shock, therefore, unambiguously increases the output gap and inflation. Such expansionary effects are transmitted through the three channels outlined in Section 4.2.

The "borrowing-cost channel" is captured by the first two terms in the square brackets of (94), whereby a temporary ($\rho_u < 1$) increase in central bank's reserves stimulates aggregate demand through the fall in the long-term interest rate that increases borrowing and raises the consumption of borrowers. The "idiosyncratic-risk channel" is captured by the third term in the square brackets of (94), whereby a persistent ($\rho_u > 0$) increase in reserves reduces consumption risk for the savers and provides an additional push on aggregate demand that reinforces the expansionary effect of the shock. The "cyclical-inequality channel", finally affects coefficient Φ in definition (95), whereby a countercyclical inequality ($\Phi > 1$) implies a general-equilibrium amplification through a larger Ψ_u.

Notice that while the "borrowing-cost channel" is stronger when the shock is more short-lived (smaller $\rho_u < 1$) and progressively fades away as the persistence of the shock approaches the unitary value ($\rho_u = 1$),[25] the "idiosyncratic-risk channel" is instead stronger precisely when the shock is more long-lived. Considering the very high persistence of the balance-sheet policies that we have observed in the past fifteen years, and the effort of central bankers in communicating such persistence to steer the private sector's expectations, this result suggests a key role of the "idiosyncratic-risk channel" in the transmission of such policies.

To visualize the relevance of this point and the two additional channels implied in our economy compared to existing literature, Figure 1 displays a numerical illustration of the effects on the output gap, inflation and consumption inequality of an increase in central bank's reserves worth 1% of steady-state output, with a half-life of about 6 quarters, for a benchmark calibration of the model, and under a *conventional* monetary-policy regime.[26]

The black solid line in the figure shows the responses in the baseline version of the model, where all channels are at work, and where the idiosyncratic risk is calibrated to $p_s = 0.95$, which implies a value for $\Phi = 1.10$.[27] The red dashed line shows the case where we shut off the "cyclical-inequality channel", by forcing a value $\Phi = 1$, and the blue dashed-dotted line shows the case where we also shut off the "idiosyncratic-risk channel", by ruling out consumption risk altogether: $p_s = 1$, which also implies $\Phi = 1$.

The simulation then implies that, while the response of inequality per se is not substantially affected in the three cases, the relevance of its implications for consumption risk and aggregate

[25] Equation (72) clarifies that, in a first-order approximation, a change in reserves only affects the long-term borrowing interest rate when it is transitory.

[26] In particular, we calibrate the steady-state nominal short-term rate to $\bar{\imath}^R = 3.5\%$, the inflation target to 2% and the steady-state term premium to $\bar{\imath}^B - \bar{\imath}^R = 3\%$, all in annual rates; the share of borrowers to $z = 1/3$, the relative-risk aversion to $\sigma = 1$, the inverse-Frisch elasticity to $\varphi = 1$, the Calvo parameter to $\alpha = 0.75$, the income-inequality parameter to $\varpi = 0.6$, the steady-state consumption inequality to $\Gamma = 1.05$ and the steady-state size of the central bank's balance sheet to 10% of output $\bar{u}_Y = 0.1$.

[27] The calibrated value for p_s is consistent with the estimated transition probabilities during downturns within the related framework of Bilbiie et al. (2022). We thank the authors for providing us with the smoothed estimates from their analysis.

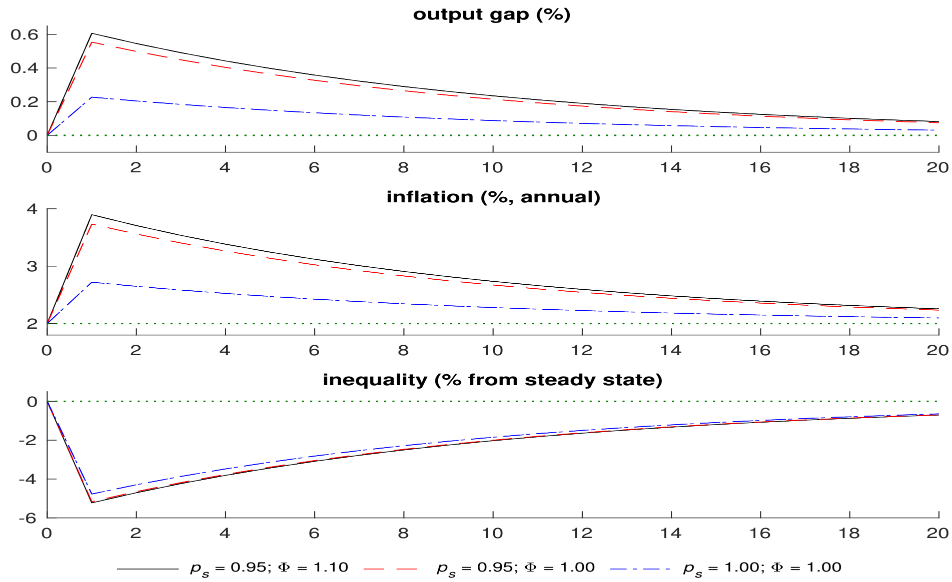

Figure 1: Isolating the channels of transmission of an unconventional monetary policy shock. The blue dashed-dotted line displays the response when the sole "borrowing-cost channel" is at work; the red dashed line displays the response when the "idiosyncratic-risk channel" is switched on; the black solid line displays the response when also the "cyclical-inequality channel" is on.

demand instead is: cyclical inequality and idiosyncratic risk jointly imply a response of inflation and the output gap that is almost three times as large as in the case where the sole "borrowing-cost channel" is at work, with the difference explained essentially by the "idiosyncratic-risk channel".[28]

Moreover, the specific monetary policy regime in place also affects the *ex-post* equilibrium adjustment that supports the expansionary effects on the output gap and inflation. This point can be evaluated by looking at the general-equilibrium solution for consumption inequality and the policy tools. Consider a given degree of overall monetary-policy responsiveness η_u regardless of the specific regime in place, and such that the conditions for determinacy are always satisfied.

In the *conventional regime* – i.e. $\psi_\pi = \psi_x = 0$ – the general-equilibrium consumption inequality, conditional on an unconventional-policy shock, is

$$\omega_t = -\frac{1}{z(1-z)}\left[\sigma_x \delta \Psi_u (1-\beta\rho_u)\left(1-\rho_u+\rho_u \frac{1-\gamma_s}{z}\right) z(\chi-1)+1\right] v_t^u, \tag{97}$$

[28]This result is markedly different from the main implication of Sims et al (2022), who instead find that the transmission of unconventional monetary policy shocks is not substantially affected by the idiosyncratic uncertainty typical of HANK economies. The reason behind this difference lies in the different mechanism through which unconventional monetary policy is relevant in their economy, where the borrowing agent exposed to the credit friction in the banking sector is a wholesale non-financial firm, while the cross-sectional distribution of consumption across households is essentially unaffected by the credit friction. Unconventional monetary policy in their economy is therefore relevant through its effects on the investment decisions of firms, rather than on the consumption decisions of households, which explains why the idiosyncratic risk faced by the latter does not play a major role.

while the equilibrium real interest rate $\hat{r}_t \equiv \hat{i}_t^R - E_t\pi_{t+1}$ and the quantity of reserves are equal to

$$\hat{r}_t = \sigma_x \delta \Big[1 - \rho_u + \rho_u z^{-1}(1-\gamma_s)\Big] \Big[\kappa(\phi_\pi - \rho_u) + (1-\beta\rho_u)\phi_x\Big] \Psi_u v_t^u \tag{98}$$

$$\hat{u}_t = v_t^u. \tag{99}$$

Therefore, an expansionary balance-sheet policy triggers a contractionary response of conventional monetary policy that unambiguously increases the real interest rate. The expansionary effect of the shock is thereby supported entirely by the equilibrium increase in central bank's reserves and its effects on the consumption of borrowers and savers along the three channels discussed.

In the *unconventional regime* – i.e. $\phi_\pi = \phi_x = 0$ – the equilibrium consumption inequality is

$$\omega_t = -\frac{\Psi_u}{z(1-z)} \left[\sigma_x(1-\beta\rho_u)\left(z(\chi-1)\left(1-\rho_u+\rho_u\frac{1-\gamma_s}{z}\right) + (1-\Phi\rho_u)\right) - \kappa\rho_u\right] v_t^u, \tag{100}$$

where we used (95), while the real interest rate and the quantity of reserves are equal to

$$\hat{r}_t = -\kappa\rho_u\sigma_x\delta\Big[1-\rho_u+\rho_u z^{-1}(1-\gamma_s)\Big]\Psi_u v_t^u \tag{101}$$

$$\hat{u}_t = -\kappa\rho_u\Psi_u v_t^u. \tag{102}$$

Therefore, in this regime an expansionary balance-sheet shock triggers general-equilibrium effects that shape the transmission differently, depending on how persistent the shock is. A transitory, one-time shock (i.e. $\rho_u = 0$) on the one hand leaves the equilibrium real interest rate unchanged and on the other hand cannot trigger any response of consumption risk, since this policy does not affect expected consumption inequality. The expansionary effect in this case is therefore transmitted entirely through the "borrowing-cost channel", implying a temporary fall in the long-term interest rate that expands the current consumption of borrowers. The endogenous response implied by the unconventional regime leaves the *ex-post* equilibrium quantity of reserves unchanged, while the current consumption inequality falls.

A persistent shock affects instead expectations and, thereby, also the real interest rate and consumption risk. The expectation of a persistent fall in inequality, indeed, reduces consumption risk for the savers and further fuels persistent inflationary expectations. The latter, in the absence of a conventional policy response (since we are in an *unconventional regime*), are validated *ex-post* by the fall in the real interest rate that indeed supports the higher levels of output gap and inflation. The general-equilibrium response of inequality, however, is not necessarily a fall. The endogenous response implied by the unconventional regime, indeed, yields an equilibrium lower level of central bank's reserves – the lower the more persistent the shock, as implied by equation (102). The lower the level of equilibrium reserves, the more consumption inequality is pushed upwards, thereby potentially inducing an increase in equilibrium.

4.4.2 A conventional policy shock

Turning to the transmission of conventional monetary policy, the solution of the model conditional on a standard interest-rate shock implies the familiar result that both the output gap and inflation rise in response to an expansionary shock (i.e. a negative ε_t^c):

$$x_t = -(1-\beta\rho_c)\Psi_c v_t^c \qquad\qquad \pi_t = -\kappa\Psi_c v_t^c, \tag{103}$$

where

$$\Psi_c \equiv \Bigg[\sigma_x(1-\Phi\rho_c)(1-\beta\rho_c)+\eta_c\Bigg]^{-1} \tag{104}$$

with

$$\eta_c \equiv (1-\beta\rho_c)\phi_x + \kappa(\phi_\pi - \rho_c) + \sigma_x\delta\left(1-\rho_c+\rho_c\frac{1-\gamma_s}{z}\right)\Big(\kappa\psi_\pi + (1-\beta\rho_c)\psi_x\Big) \tag{105}$$

and with $\Psi_c \geq 0$. Definitions (104)–(105) imply the familiar result that when monetary policy is more responsive, the effects of exogenous shocks are smaller.

Consider again a given degree of overall monetary-policy responsiveness η_c regardless of the specific regime in place, and such that the conditions for determinacy are always satisfied. The equilibrium consumption inequality conditional on a conventional monetary policy shock can be written as

$$\omega_t = \frac{\Psi_c}{z(1-z)}\Big[\Big(z(\chi-1)-\psi_x\Big)(1-\beta\rho_c)-\kappa\psi_\pi\Big]v_t^c. \tag{106}$$

Therefore, whether an expansionary conventional monetary-policy shock reduces inequality depends on a structural feature of the economy but also on the specific monetary-policy regime. It decreases consumption inequality, unless the latter is structurally procyclical ($\chi < 1$) and/or the endogenous component of unconventional monetary policy is strong enough (i.e. ψ_π and ψ_x large enough).

In the *conventional regime* ($\psi_\pi = \psi_x = 0$), a countercyclical inequality is sufficient to imply that an expansionary interest-rate shock reduces it, through the expansionary effect on aggregate income and its disproportionate effect on the consumption of high-MPC borrowers with respect to low-MPC savers. In the *unconventional regime*, on the other hand, an expansionary conventional policy triggers a contractionary unconventional response that hurts borrowers relatively more than savers, thereby potentially pushing inequality in the opposite direction.

As for the case of unconventional monetary policy, the general-equilibrium adjustment that supports *ex-post* the real expansion depends on the underlying monetary-policy regime. To see this, consider that in the *conventional regime*, the equilibrium real interest rate is

$$\hat{r}_t = \sigma_x(1-\Phi\rho_c)(1-\beta\rho_c)\Psi_c v_t^c \tag{107}$$

while the equilibrium quantity of reserves is constant at the steady state level: $\hat{u}_t = 0$. Compared to the benchmark New Keynesian model, idiosyncratic uncertainty here plays an important role through parameter Φ. Indeed, while in the benchmark New Keynesian environment the real interest rate unambiguously declines after a persistent expansionary conventional-policy shock, in a THANK model it needs to increase if inequality is sufficiently countercyclical ($\Phi > \rho_c^{-1}$), in order to counteract the extra push on demand coming from lower consumption risk for the savers. In this case, therefore, the positive effects of an expansionary conventional-policy shock are transmitted entirely through the fall in expected inequality, which is so strong that it challenges the stability of the equilibrium, requiring a compensating increase in the real interest rate.

In the *unconventional regime*, on the other hand, the endogenous balance-sheet response of the central bank reduces the need for an increase in the real interest rate. An expansionary conventional-policy shock, in this case, triggers a contractionary balance-sheet response:

$$\hat{r}_t = \left[\sigma_x(1-\Phi\rho_c)(1-\beta\rho_c) + \sigma_x\delta\left(1-\rho_c+\rho_c\frac{1-\gamma_s}{z}\right)\left(\kappa\psi_\pi + (1-\beta\rho_c)\psi_x\right)\right]\Psi_c v_t^c \tag{108}$$

$$\hat{u}_t = \left[\kappa\psi_\pi + (1-\beta\rho_c)\psi_x\right]\Psi_c v_t^c. \tag{109}$$

As a consequence, inequality falls less even when it is countercyclical, posing less of a challenge for equilibrium determinacy. In this regime, therefore, it is mostly the fall in the real interest rate that channels the expansionary effect of the cut in the policy rate to the output gap and inflation.

4.4.3 A productivity shock

Let the log-productivity index a_t follow the AR(1) process $a_t = \rho_a a_{t-1} + \varepsilon_t^a$, with $\rho_a \in [0,1]$. Accordingly, the definition of the output gap $y_t^* \equiv \frac{1+\varphi}{\sigma+\varphi}a_t$ implies that the latter follows the process

$$y_t^* = \rho_a y_{t-1}^* + \varepsilon_t^*, \tag{110}$$

with $\varepsilon_t^* \equiv \frac{1+\varphi}{\sigma+\varphi}\varepsilon_t^a$.

The solution of our model, conditional on a technology shock, implies the familiar result that both the output gap and inflation rate fall in response to an increase in productivity:

$$x_t = -\sigma_x\delta(1-\beta\rho_a)\left[(1-\rho_a)(1-z\varpi) + \rho_a(1-\gamma_s)(1-\varpi)\right]\Psi_a y_t^* \tag{111}$$

$$\pi_t = -\sigma_x\delta\kappa\left[(1-\rho_a)(1-z\varpi) + \rho_a(1-\gamma_s)(1-\varpi)\right]\Psi_a y_t^* \tag{112}$$

where

$$\Psi_a \equiv \left[\sigma_x(1-\Phi\rho_a)(1-\beta\rho_a) + \eta_a\right]^{-1} \geq 0 \tag{113}$$

with

$$\eta_a \equiv (1-\beta\rho_a)\phi_x + \kappa(\phi_\pi - \rho_a) + \sigma_x\delta\left(1-\rho_a+\rho_a\frac{1-\gamma_s}{z}\right)\left(\kappa\psi_\pi + (1-\beta\rho_a)\psi_x\right). \tag{114}$$

As in the benchmark New Keynesian model, the level of actual output increases less than potential because of the imperfect accommodation implied by the simple policy rules that prevents the necessary intertemporal adjustment of demand to track the dynamics of productivity, and therefore the effects are larger when monetary policy is less responsive to the state of the economy (i.e. when η_a is smaller). Compared to the benchmark New Keynesian model, however, in our economy the disinflationary pressures and the decline in the output gap are further reinforced by the contraction in demand implied – through an increase in consumption risk – by the idiosyncratic uncertainty, as captured by the second term in the square brackets in both (111) and (112).

To isolate this effect, consider a permanent shock to potential output (i.e. $\rho_a = 1$). While in the benchmark New Keynesian model a permanent shock does not affect either the output gap or inflation because the natural rate of interest remains unaffected – since there is no need for an intertemporal reallocation of demand – and therefore no policy response is needed, in our economy consumption risk for the savers rises permanently, thus inducing a permanent equilibrium decline in the output gap and inflation:

$$x_t = -\sigma_x \delta (1-\beta)(1-\gamma_s)(1-\varpi)\Psi_a y_t^* \tag{115}$$

$$\pi_t = -\sigma_x \delta \kappa (1-\gamma_s)(1-\varpi)\Psi_a y_t^*. \tag{116}$$

Key to this transmission in our economy is the underlying disposable labor-income inequality implied by the need for borrowers to pledge some of their labor income to service their outstanding debt (i.e. $\varpi < 1$). To build some intuition on this, consider (71), and notice that a positive productivity shock implies un upward pressure on consumption inequality, because the savers benefit from the increase in labor income implied by higher real wages more than the borrowers. When the shock is permanent, the upward pressures on expected inequality remains just as strong as it is on current inequality, regardless of how far in the future expectations are taken. This implies a permanent increase in consumption risk for the savers, which then permanently increase their precautionary savings and cut their consumption. In the first-best equilibrium, such negative pressures on demand are prevented altogether by an appropriate adjustment in central bank's reserves, which increase to compensate the under-response of borrowers' disposable labor income through more borrowing, whereby the optimal interest rate remains unaffected as in the benchmark New Keynesian model, as shown by (86). Under the simple policy rules (89)–(90), these negative pressures on demand instead do materialize and induce a monetary-policy response that may involve conventional or unconventional tools, depending on the underlying policy regime.

To show this, we consider a constant value of η_a regardless of the specific policy regime, as in the previous Sections, and evaluate the equilibrium monetary-policy stance, defined as the deviations of the real interest rate and central bank's reserves from their optimal levels, conditional on a permanent productivity shock. In the *conventional regime*, the latter are, respectively

$$\hat{r}_t - \hat{r}_t^* = -\sigma_x \delta(1-\gamma_s)(1-\varpi)\Big[\kappa(\phi_\pi - 1) + (1-\beta)\phi_x\Big]\Psi_a y_t^* \tag{117}$$

$$\hat{u}_t - \hat{u}_t^* = -z(1-\varpi)y_t^*, \tag{118}$$

while the general-equilibrium level of consumption inequality is

$$\omega_t = \frac{1-\varpi}{1-z}\left[\beta\sigma_x(\Phi - 1) + \kappa(\phi_\pi - 1) + (1-\beta)\phi_x\right]\Psi_a y_t^*. \tag{119}$$

Therefore, in this policy regime, the unconventional monetary-policy stance (118) is contractionary, as it fails to accommodate the increase in the optimal level of reserves needed to prevent the increase in consumption risk for the savers, which ultimately translates into lower output gap and inflation. On the other hand, the conventional policy stance (117) is expansionary and partially leans against these downward pressures on demand.

The relative strengths of the latter, moreover, depends on the cyclicality of consumption inequality, and implies a stronger interest-rate fall if inequality is countercyclical ($\chi, \Phi > 1$). In this case, indeed, the increase in equilibrium inequality is amplified by the cyclical response – as shown by equation (119) – and not only induces a stronger downward pressure on demand but also challenges determinacy by implying compounding of expected future falls in demand. Therefore, the interest rate declines relatively more in this case to offset both these pressures.[29]

Under the *unconventional regime*, instead, the equilibrium conventional and unconventional monetary-policy stances are, respectively

$$\hat{r}_t - \hat{r}_t^* = \sigma_x \delta\kappa(1-\gamma_s)(1-\varpi)\Psi_a y_t^* \tag{120}$$

$$\hat{u}_t - \hat{u}_t^* = z(1-\varpi)\Big[\kappa + \sigma_x(1-\beta)(\Phi - 1)\Big]\Psi_a y_t^*, \tag{121}$$

while the general-equilibrium level of consumption inequality is

$$\omega_t = \frac{1-\varpi}{1-z}\left[\beta\sigma_x(\Phi - 1) - \kappa\right]\Psi_a y_t^*. \tag{122}$$

Therefore, in this policy regime, the general-equilibrium conventional monetary-policy stance is always contractionary, as the nominal interest rate does not respond, and the disinflationary expectations drive the real interest rate up. The unconventional stance, on the other hand, reflects the endogenous response that accommodates the increase in the optimal level of central bank's reserves. Whether the policy stance ($\hat{u}_t - \hat{u}_t^*$) turns out to be overall expansionary in equilibrium, depends however on how strong the downward pressures on aggregate demand coming from the increase in consumption risk are. If these pressures are strong enough, like in the case of countercyclical

[29] This is captured by a larger Ψ_a in equation (117), that is implied by a larger Φ.

inequality (i.e. $\chi, \Phi > 1$), the equilibrium unconventional stance is overall expansionary.[30] The contractionary effects of the shock are therefore mostly transmitted to the output gap and inflation through the real interest rate.

4.4.4 A deleveraging shock

A deleveraging shock in our economy occurs when the leverage constraint of the financial intermediaries becomes more severe. In our model, this is captured by an exogenous fall in $\hat{\theta}_t$, which follows the AR(1) process $\hat{\theta}_t = \rho_\theta \hat{\theta}_{t-1} + \varepsilon_t^\theta$, with $\rho_\theta \in [0, 1]$.

The solution of our model, conditional on a deleveraging shock, implies

$$x_t = \sigma_x \delta (1 - \beta \rho_\theta) \left[1 - \rho_\theta + \rho_\theta \frac{1 - \gamma_s}{z} \right] \Psi_\theta \hat{\theta}_t \qquad \pi_t = \sigma_x \delta \kappa \left[1 - \rho_\theta + \rho_\theta \frac{1 - \gamma_s}{z} \right] \Psi_\theta \hat{\theta}_t \tag{123}$$

where

$$\Psi_\theta \equiv \left[\sigma_x (1 - \Phi \rho_\theta)(1 - \beta \rho_\theta) + \eta_\theta \right]^{-1} \geq 0 \tag{124}$$

with

$$\eta_\theta \equiv (1 - \beta \rho_\theta) \phi_x + \kappa (\phi_\pi - \rho_\theta) + \sigma_x \delta \left(1 - \rho_\theta + \rho_\theta \frac{1 - \gamma_s}{z} \right) \left(\kappa \psi_\pi + (1 - \beta \rho_\theta) \psi_x \right). \tag{125}$$

When the financial intermediaries are forced to deleverage, therefore, the economy experiences recessionary and deflationary pressures, whose intensity depends on several factors.

Notice first that, compared to an economy with no idiosyncratic risk (i.e. $\gamma_s = 1$) where the heterogeneity between borrowers and savers is "static", in our economy the deflationary and contractionary effects of a persistent deleveraging shock are deeper. This is due to the additional contraction in demand induced by the partial-equilibrium increase in consumption inequality, that translates into higher consumption risk for the savers, who then cut their demand, as captured by the last term in the square brackets in (123). Whether the general-equilibrium inequality

$$\omega_t = \frac{\Psi_\theta}{z(1 - z)} \Bigg[z(1 - \chi) \sigma_x (1 - \beta \rho_\theta) \left(1 - \rho_\theta + \rho_\theta \frac{1 - \gamma_s}{z} \right) \\ - (1 - \beta \rho_\theta) \left(\sigma_x (1 - \Phi \rho_\theta) + \phi_x \right) - \kappa (\phi_\pi - \rho_\theta) \Bigg] \hat{\theta}_t \tag{126}$$

actually increases, however, depends on the underlying monetary-policy regime in place, and on the structure of the economy, in particular with respect to the cyclicality of inequality. In this respect, the analysis of the transmission mechanism of the shock follows the same lines as for the unconventional monetary policy shock, to which we refer the reader.

[30]Note that a countercyclical inequality is not necessary for the unconventional stance to be expansionary, as we only need inequality to not be too procyclical, i.e. if $\Phi > 1 - \sigma_x^{-1}(1 - \beta)^{-1}\kappa$, as implied by equation (121).

Indeed, the solution of the model conditional on a deleveraging shock is identical to the solution conditional on a negative unconventional-policy shock, for equal persistence (i.e. $\rho_\theta = \rho_u$). As in Sims et al. (2021), this suggests the result that an unconventional monetary-policy expansion may be the most appropriate response to a deleveraging shock.

We complement this result by noticing that, while the natural interest rate falls in response to a deleveraging shock, as shown by equation (84), the optimal interest rate does not respond at all, as shown by equation (86). This is an important implication, considering that most of the literature studying the monetary-policy response to a financial crisis in the small-scale New Keynesian model typically uses a fall in the natural rate as the primitive shock to respond to. Indeed, our analysis suggests that observing a fall in the natural interest rate is generally not sufficient to infer the appropriate monetary-policy response, and that the monetary-policy regime plays an important role in the stabilization of the shock.

To see this, consider the general-equilibrium response of the policy rate in the conventional and the new-style regimes:

$$\hat{i}_t^R = \sigma_x \delta \left(1 - \rho_\theta + \rho_\theta \frac{1-\gamma_s}{z}\right) \left[\kappa \phi_\pi + (1 - \beta \rho_\theta)\phi_x\right] \Psi_\theta \hat{\theta}_t \tag{127}$$

which implies that the conventional policy tool in this regime is an increasing function of the shock. A deleveraging shock strong enough is therefore able to bring the policy rate to the ELB, exacerbating the negative effects on inflation and the output gap. In the conventional regime this is the end of the story, as the balance sheet of the central bank does not respond at all (i.e. $\hat{u}_t = 0$) and the standard analysis from the benchmark New Keynesian model applies.

In the new-style regime, instead, the general-equilibrium level of central bank's reserves responds to the shock according to

$$\hat{u}_t = -\sigma_x \delta \left(1 - \rho_\theta + \rho_\theta \frac{1-\gamma_s}{z}\right) \left[\kappa \psi_\pi + (1 - \beta \rho_\theta)\psi_x\right] \Psi_\theta \hat{\theta}_t. \tag{128}$$

To see how state-contingent unconventional monetary-policy can help insulate the economy from the ELB and sterilize the shock, consider the case of an inflation-targeting central bank. Figure 2 provides a numerical illustration of this case, and displays the response of our economy to a deleveraging shock bringing the natural interest rate to -6%, under three alternative regimes: a conventional inflation targeting, an unconventional inflation targeting, and a regime combining the conventional inflation targeting with an unconventional "inequality targeting".

Under a "conventional inflation-targeting" regime, i.e. the limiting case of $\phi_\pi \to \infty$, it is straightforward to show that, for a given level of unconventional response coefficients ψ_π and ψ_x the following holds:

$$\lim_{\phi_\pi \to \infty} \Psi_\theta = 0 \qquad\qquad \lim_{\phi_\pi \to \infty} \left[\kappa \phi_\pi + (1 - \beta \rho_\theta)\phi_x\right] \Psi_\theta = 1, \tag{129}$$

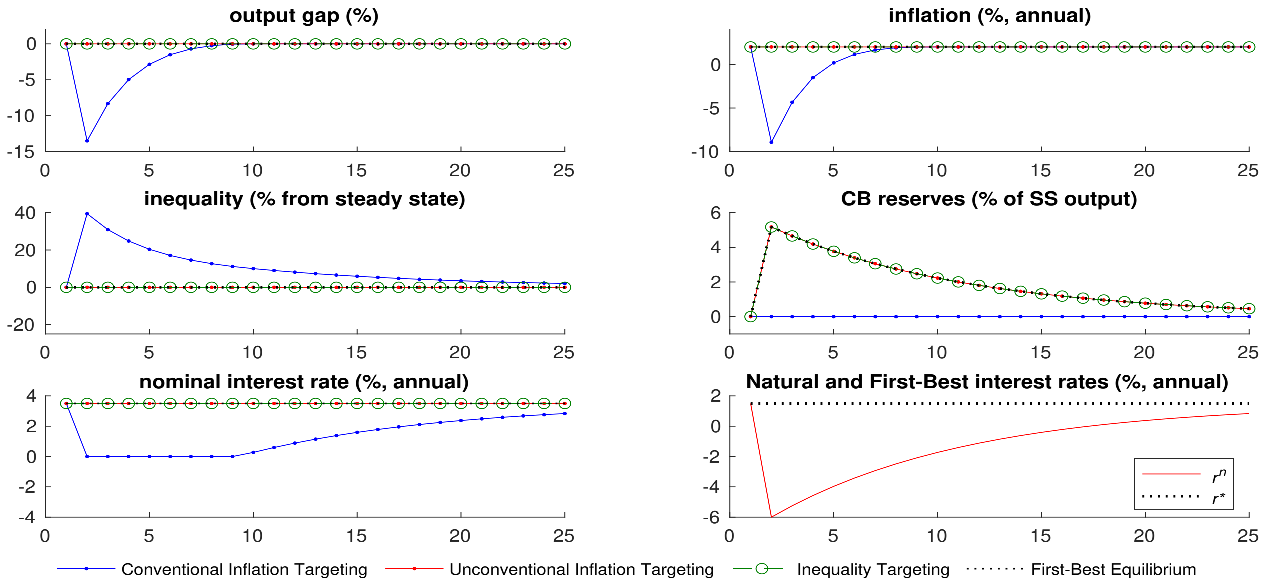

Figure 2: The response of the economy to a deleveraging shock. Blue lines with dots: conventional inflation targeting; red lines with dots: unconventional inflation targeting; circled green lines: conventional inflation targeting and unconventional inequality targeting; dotted black lines: first-best equilibrium. The bottom right panel displays the natural interest rate r_t^n (red line) against the first-best real interest rate r_t^* (dotted black line).

which implies

$$\hat{\imath}_t^R = \sigma_x \delta \left(1 - \rho_\theta + \rho_\theta \frac{1-\gamma_s}{z}\right) \hat{\theta}_t \qquad\qquad \hat{u}_t = 0. \tag{130}$$

As in the benchmark New Keynesian model, an inflation-targeting central bank resorting to its conventional tool only is unable to hit the inflation target, as the ELB in this case prevents the policy rate to track the natural rate in case the latter goes negative. As shown by the blue lines in Figure 2, the policy rate stays at the zero-lower bound for two years, during which the economy experiences a very deep recession and a strong deflation, accompanied by a stark increase in consumption inequality. Note that the increase in inequality and idiosyncratic risk amplify the recessionary and deflationary effects of the ELB, compared to the benchmark New Keynesian model, for a given fall in the natural interest rate. Indeed, in our economy a deleveraging shock hits the borrowers first and foremost, implying a persistent fall in their consumption. The ensuing persistent increase in inequality then also hits the savers, through the increase in consumption risk that reduces their current spending as well.

Moreover, the bottom-right panel shows that, although the natural rate falls to negative values, the optimal interest rate does not move (dotted black line). On the other hand, the dotted line in the middle-right panel shows that the first-best level of reserves rises. This suggests an alternative, more appropriate way to hit the inflation target: unconventional policy.

Consider then the case of an “unconventional inflation-targeting” regime, that is when $\psi_\pi \to \infty$.

In this case, for a given level of conventional response coefficients ϕ_π and ϕ_x, it is easy to show that:

$$\lim_{\psi_\pi \to \infty} \Psi_\theta = 0 \tag{131}$$

$$\lim_{\psi_\pi \to \infty} \Big[\kappa\psi_\pi + (1-\beta\rho_\theta)\psi_x \Big] \Psi_\theta = \left[\sigma_x \delta \left(1 - \rho_\theta + \rho_\theta \frac{1-\gamma_s}{z} \right) \right]^{-1}. \tag{132}$$

As a direct implication of the above, the general-equilibrium levels of the nominal interest rate and central bank's reserves, conditional on a deleveraging shock, in this limiting case are

$$\hat{\imath}_t^R = 0 \tag{133}$$

$$\hat{u}_t = -\hat{\theta}_t, \tag{134}$$

which are consistent with the first-best equilibrium, as implied by equations (85)–(86). And indeed – as also shown by the red lines in the figure – under unconventional inflation targeting not only both the output gap and inflation are at their respective targets (since $\Psi_\theta = 0$ in (123) implies $x_t = \pi_t = 0$) but also equilibrium consumption inequality is stabilised at its steady-state level, as implied by (126). The commitment to appropriately adjust central bank's reserves to hit the inflation target – where "appropriately" means so as to track the first-best level $\hat{u}_t^*$, as shown in the middle-right panel – is able to completely absorb the effects of the deleveraging shock and prevent its pass through to consumption risk and aggregate demand.

Notice that this power of unconventional monetary policy is derived from its distributional nature, which makes it particularly effective in response to shocks that have mostly distributional effects, such as a deleveraging shock. To show this, Figure 2 shows a third monetary-policy regime, displayed by the green circled lines, which combines the conventional inflation targeting with an unconventional "inequality targeting", i.e. a commitment to fully stabilise consumption inequality my means of central bank's reserves. This regime can be characterised as a case where $\phi_\pi \to \infty$ in rule (89) and the following unconventional feedback rule replaces (90):

$$\hat{u}_t = -\psi_\omega \omega_t + v_t^u \tag{135}$$

with $\psi_\omega \to \infty$. As the figure shows, under this regime the equilibrium outcome is the same as under the unconventional inflation targeting. Note in particular that the commitment to stabilise consumption inequality is enough to achieve the first-best equilibrium, without the need to move the nominal interest rate at all. Finally, note that equations (67)–(68) imply that fluctuations in consumption inequality are essentially due to fluctuations in the credit spread

$$\omega_t = (\gamma_s + \gamma_b - 1) E_t \omega_{t+1} + \sigma^{-1} \left(E_t \hat{\imath}_{t+1}^B - \hat{\imath}_t^R \right). \tag{136}$$

Accordingly, the feedback rule (135) is equivalent to one where central bank's reserves are contingent on the credit spread, instead of consumption inequality: a commitment to use the unconventional monetary policy to "close the spread" is therefore able, in our economy, to implement the first-best allocation in response to deleveraging shocks.[31]

4.4.5 A discount-factor shock

To further scrutinize the claim of the previous Section, that observing a fall in the natural interest rate is generally not sufficient to infer the appropriate monetary-policy response, here we consider the response of our economy to a preference shock hitting the discount factors, $\hat{\xi}_t$, which follows the AR(1) process $\hat{\xi}_t = \rho_\xi \hat{\xi}_{t-1} + \varepsilon_t^\xi$, with $\rho_\xi \in [0, 1]$.

This shock is typically used in the New Keynesian literature to capture disruptions in financial markets that take the natural rate in the negative territory, exposing the economy to the ELB issue. In our economy, as shown in Section 4.1, a preference shock affects the reference interest rate in both the natural and the first-best equilibria. Under simple policy rules (89)–(90), the general-equilibrium solution for the output gap and inflation, conditional on preference shock, are

$$x_t = (1 - \beta\rho_\xi)(1 - \rho_\xi)\,\Psi_\xi \hat{\xi}_t \qquad\qquad \pi_t = (1 - \rho_\xi)\,\Psi_\xi \hat{\xi}_t \tag{137}$$

where

$$\Psi_\xi \equiv \left[\sigma_x(1 - \Phi\rho_\xi)(1 - \beta\rho_\xi) + \eta_\xi\right]^{-1} \geq 0 \tag{138}$$

and

$$\eta_\xi \equiv (1 - \beta\rho_\xi)\phi_x + \kappa(\phi_\pi - \rho_\xi) + \sigma_x\delta\left(1 - \rho_\xi + \rho_\xi\frac{1 - \gamma_s}{z}\right)\left(\kappa\psi_\pi + (1 - \beta\rho_\xi)\psi_x\right), \tag{139}$$

while the general-equilibrium solutions for consumption inequality and the real interest rates are, respectively

$$\omega_t = \frac{1 - \rho_\xi}{z(1 - z)}\left[(1 - \beta\rho_\xi)\Big(z(1 - \chi) + \psi_x\Big) + \kappa\psi_\pi\right]\Psi_\xi \hat{\xi}_t \tag{140}$$

$$\hat{r}_t = (1 - \rho_\xi)\left[(1 - \beta\rho_\xi)\phi_x + \kappa(\phi_\pi - \rho_\xi)\right]\Psi_\xi \hat{\xi}_t. \tag{141}$$

As for any other shock, therefore, also the equilibrium adjustment that supports *ex post* the transmission of a negative discount-factor shock depends on the underlying monetary-policy regime. Under the conventional regime the real interest rate falls to lean against the contraction in demand, the more so when consumption inequality is less procyclical and/or idiosyncratic uncertainty is higher (as Ψ_ξ is increasing in Φ). Under the unconventional regime, on the other hand, the real interest rate actually rises and accommodates the fall in demand. It is in this case the unconventional response that leans against the shock, by inducing a persistent fall in inequality that partially offsets the contractionary effects of the shock, through a reduction in consumption risk for the savers that stimulates their current spending.

[31] For an analysis of the role of credit spreads in a related environment, see Cúrdia and Woodford (2011, 2016).

Moreover, looking at the equilibrium path of the two policy tools in the mixed regime, conditional on a preference shock provides some additional insight:

$$\hat{i}_t^R = (1-\rho_\xi)\Big[(1-\beta\rho_\xi)\phi_x + \kappa\phi_\pi\Big]\Psi_\xi\hat{\xi}_t \tag{142}$$

$$\hat{u}_t = -(1-\rho_\xi)\Big[(1-\beta\rho_\xi)\psi_x + \kappa\psi_\pi\Big]\Psi_\xi\hat{\xi}_t. \tag{143}$$

As in the benchmark New Keynesian model (and as in the case of a deleveraging shock), a strong enough negative preference shock is able to bring the nominal interest rate down to the ELB if the interest rate responds to the shock. This in general amplifies the deflationary and contractionary effects of the shock. As in the case of a deleveraging shock, the availability of the unconventional tool offers in principle an alternative to shield the economy from the effects of the ELB.

To see this, consider again an inflation-targeting regime implemented using either the conventional or the unconventional tools. In the case of a "conventional inflation-targeting regime" (i.e. the limiting case $\phi_\pi \to \infty$) one can easily show that

$$\lim_{\phi_\pi\to\infty}\Psi_\xi = 0 \qquad\qquad \lim_{\phi_\pi\to\infty}\Big[(1-\beta\rho_\xi)\phi_x + \kappa\phi_\pi\Big]\Psi_\xi = 1, \tag{144}$$

which implies

$$\hat{i}_t^R = (1-\rho_\xi)\hat{\xi}_t \qquad\qquad \hat{u}_t = 0. \tag{145}$$

Under an "unconventional inflation-targeting regime" (i.e. when $\psi_\pi \to \infty$), instead we get

$$\lim_{\psi_\pi\to\infty}\Psi_\xi = 0 \tag{146}$$

$$\lim_{\psi_\pi\to\infty}\Big[\kappa\psi_\pi + (1-\beta\rho_\theta)\psi_x\Big]\Psi_\xi = \left[\sigma_x\delta\left(1-\rho_\xi+\rho_\xi\frac{1-\gamma_s}{z}\right)\right]^{-1}, \tag{147}$$

which finally implies

$$\hat{i}_t^R = 0 \tag{148}$$

$$\hat{u}_t = -(1-\rho_\xi)\left[\sigma_x\delta\left(1-\rho_\xi+\rho_\xi\frac{1-\gamma_s}{z}\right)\right]^{-1}\hat{\xi}_t. \tag{149}$$

On the one hand, as in the benchmark New Keynesian model, a strict inflation-targeting regime implemented through the *conventional* tool only is unable to hit the target in the face of a strong enough negative preference shock, due to the ELB on nominal interest rates. On the other hand, in our economy with heterogeneous households, credit frictions and idiosyncratic uncertainty, a strict inflation targeting regime implemented through the *unconventional* tool is able to hit the inflation target regardless of the size of the shock, thus insulating the economy from the ELB.

Figure 3 provides a numerical illustration and displays the response of the economy under the three regimes of Figure 2, to a discount-factor shock that brings the natural interest rate to −6%,

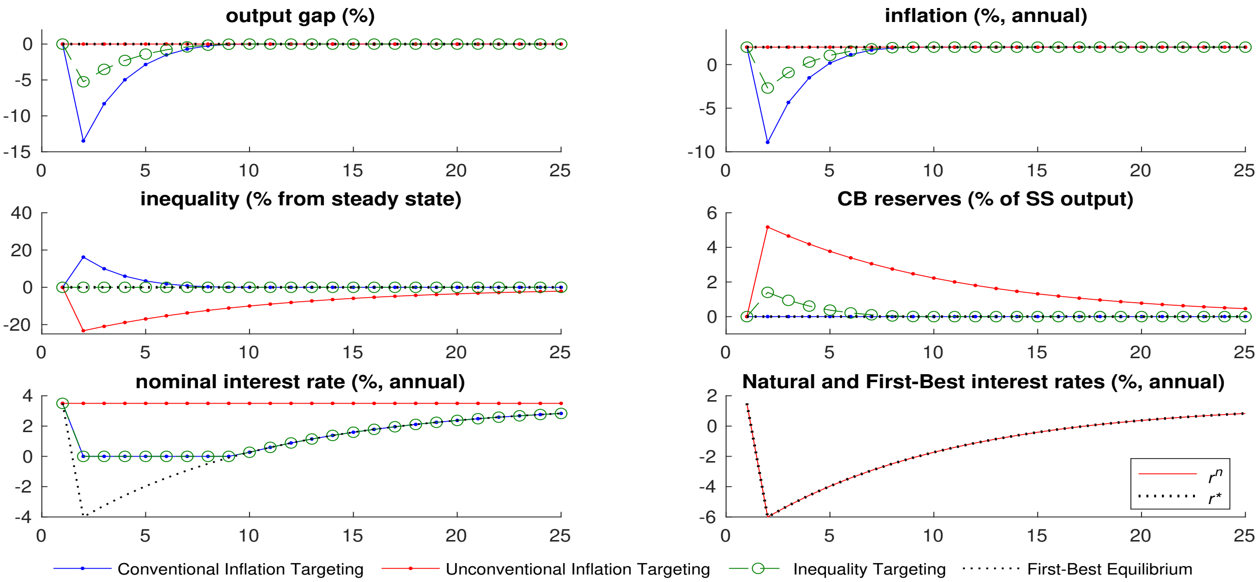

Figure 3: The response of the economy to a negative discount-factor shock. Blue lines with dots: conventional inflation targeting; red lines with dots: unconventional inflation targeting; circled green lines: conventional inflation targeting and unconventional inequality targeting; dotted black lines: first-best equilibrium. The bottom right panel displays the natural interest rate r_t^n (red line) against the first-best real interest rate r_t^* (dotted black line).

as in the previous Section.

A few insights can be drawn from the figure. First, the path of the natural interest rate in the bottom-right panel – which is the same as in the previous Section, by assumption – in this case coincides with that of the first-best rate as well, as also implied by equations (84) and (86). The path of the first-best level of central bank's reserve is instead flat at zero (dotted line in the middle-right panel). This suggests in principle that the conventional interest rate be the appropriate policy tool to respond to this shock. In the case of a small shock, this policy indeed achieves full stabilization of all the relevant variables.

Second, since the path of the natural rate is identical as in the case of a deleveraging shock, the aggregate response of output, inflation and the nominal interest rate is also identical as in that case. The transmission is however different. Indeed, the cross-sectional distribution of the response is much milder, as captured by an increase in consumption inequality that is about 50% as large as in the case of a deleveraging shock (see the middle-left panel in the figure). This results from the fact that a discount-factor shock has a symmetric effect, on impact, on both savers and borrowers, which implies a smaller impact on inequality and therefore a weaker amplification through consumption risk. The weaker impact on savers' spending decisions through consumption risk is however compensated by a stronger transmission through inter-temporal substitution, triggered by a higher-than-optimal nominal interest rate (as displayed in the bottom-left panel by the solid blue and dotted black lines).

Third, committing to stabilize inflation through *unconventional* policy allows the central bank to hit the inflation target in spite of the ELB, as in the previous Section (see red lines in the figure). However, while the equilibrium paths of the output gap, inflation and real reserves are the same as in the case of a deleveraging shock, the overall equilibrium outcome is different. Indeed, in the case of a deleveraging shock, the path of real reserves was consistent with the first-best equilibrium, implying that not only the output gap and inflation were on target, but also consumption inequality was fully stabilised, making that equilibrium outcome socially optimal. In the case of a negative discount-factor shock, instead, the equilibrium outcome is not socially optimal in spite of the fact that output gap and inflation are on target. The reason is that in order to hit the inflation target, the central bank in this case must exploit the "idiosyncratic-risk channel" of unconventional monetary policy to compensate for the inability to accommodate the intertemporal substitution called for by the preference shock, because of the ELB. Thereby, the central bank needs to increase real reserves more than in the first-best equilibrium, in order to trigger a strong and persistent decline in equilibrium consumption inequality, relative to the optimal level, according to

$$\omega_t = \frac{1-\rho_\xi}{z(1-z)}\left[\sigma_x\delta\left(1-\rho_\xi+\rho_\xi\frac{1-\gamma_s}{z}\right)\right]^{-1}\hat{\xi}_t. \tag{150}$$

Note that, while a central bank endowed with a dual mandate including only inflation and the output gap would find it optimal to use the unconventional policy to stabilize inflation, a central bank seeking to maximize social welfare would face a tradeoff that makes the unconventional inflation-targeting regime suboptimal, unlike in the case of a deleveraging shock. A central bank concerned with all welfare-relevant variables might want to use its conventional and unconventional tools selectively on aggregate and distributional targets. This is captured in the figure by the circled green line displaying the response under a policy regime where the conventional dimension of policy targets inflation and the unconventional one targets consumption inequality (or the credit spread).

As the figure shows, under such a regime the central bank is able to perfectly stabilise consumption inequality through a mild increase in central bank's reserves and, thereby, it also substantially improves inflation and output-gap stabilisation, compared to the conventional inflation-targeting regime. The reason is that, by stabilising consumption inequality, the central bank completely offsets the consumption-risk dimension of the shock's transmission, which accounts for two-thirds of the effects on inflation and the output gap in the conventional inflation-targeting regime.[32]

Complementing the conventional inflation-targeting regime with an unconventional inequality targeting is therefore unambiguously welfare improving. However, the figure suggests that the central bank could use unconventional monetary policy to raise welfare even more, by tolerating a reduction in consumption inequality below its optimal level in order to stabilise inflation and the output gap some more, through a reduction in consumption risk. In order to study this kind of policy tradeoffs more formally, in the next Section we turn to the analysis of optimal monetary policy in a linear-quadratic framework.

[32]The response of inflation and the output gap in this case coincides with the one we would observe in the benchmark New-Keynesian model.

4.5 Optimal Monetary Policy

The optimal policy problem of a central bank concerned with social welfare can be characterized as the minimization of loss (80) subject to the aggregate-demand block described by

$$x_t = E_t x_{t+1} - \sigma_x^{-1}(\hat{\imath}_t^R - E_t\pi_{t+1} - \hat{r}_t^*) - \Phi_\omega E_t\omega_{t+1} - \delta E_t\{\Delta\hat{u}_{t+1} - \Delta\hat{u}_{t+1}^*\} \tag{151}$$

$$\omega_t = (1-z)^{-1}\left[(1-\chi)\,x_t - z^{-1}(\hat{u}_t - \hat{u}_t^*)\right], \tag{152}$$

where we defined $\Phi_\omega \equiv \delta(1-z)(1-\gamma_s)$, the aggregate supply

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t, \tag{153}$$

and the non-negativity constraint on the nominal interest rate:

$$\hat{\imath}_t^R \geq -\frac{\bar{\imath}^R}{1+\bar{\imath}^R}. \tag{154}$$

Consider first the case of discretion. In this case, the central bank chooses a path for its policy instruments and the three target variables in order to minimize the period loss-function

$$\frac{\sigma+\varphi}{2}(x_t^2 + \lambda_\pi\pi_t^2 + \lambda_\omega\omega_t^2)$$

such that

$$x_t = K_{x,t} - \sigma_x^{-1}\hat{\imath}_t^R + \delta\hat{u}_t \tag{155}$$

$$\pi_t = K_{\pi,t} + \kappa x_t \tag{156}$$

$$\omega_t = K_{\omega,t} + (1-z)^{-1}\left[(1-\chi)\,x_t - z^{-1}\hat{u}_t\right] \tag{157}$$

$$\hat{\imath}_t^R \geq -(1+\bar{\imath}^R)^{-1}\bar{\imath}^R, \tag{158}$$

where $K_{x,t}$, $K_{\pi,t}$, $K_{\omega,t}$ collect terms that are either exogenous or expectational.

Consider now the case of an "old-style" central bank that only uses its conventional, interest-rate policy and keeps the size of its balance sheet constant. In this case, $\hat{u}_t = 0$ for all t in the system of constraints above, and the first-order conditions for the problem above are:

$$x_t = \kappa\mu_{2,t} + (1-z)^{-1}(1-\chi)\mu_{3,t} - \mu_{1,t} \tag{159}$$

$$\mu_{2,t} = -\lambda_\pi\pi_t \tag{160}$$

$$\mu_{3,t} = -\lambda_\omega\omega_t \tag{161}$$

$$\mu_{1,t} = -\sigma_x\mu_{4,t}, \tag{162}$$

where $\mu_{j,t}$, for $j = 1, ..., 4$ are the Lagrange multipliers on the contraints (155)–(158). Thus, the optimal targeting rule in this case requires

$$x_t + \kappa\lambda_\pi \pi_t + (1-z)^{-1}(1-\chi)\lambda_\omega \omega_t = \sigma_x \mu_{4,t}. \tag{163}$$

The above implies that, even when the ELB is not binding (i.e. $\mu_{4,t} = 0$), there is an endogenous tradeoff between inflation and output stability on the one hand, and consumption inequality on the other hand. This endogenous tradeoff, however, is relevant in response to leverage and productivity shocks only. Indeed, in the absence of productivity and leverage shocks (i.e. $\hat{u}_t^* = 0$) consumption inequality becomes proportional to the output gap, as in Bilbiie (2021), so that by stabilizing inflation and the output gap also inequality is stabilized. On the other hand, fluctuations in productivity or the bank's leverage constraint makes it impossible to stabilize at the same time all three targets using only the conventional instrument – as already implied by the analysis of Section 4.1 – and the central banks finds it optimal to induce some volatility in inflation and output in order to reduce that in inequality.[33]

The picture changes for a "new-style" central bank: if the central bank chooses also the path of reserves $\hat{u}_t$, the solution of the optimal problem requires the following additional condition:

$$\mu_{3,t} = \delta z(1-z)\mu_{1,t}. \tag{164}$$

To understand the implications of (164), consider first the case in which the ELB is not binding. In this case $\mu_{4,t} = 0$ and, through equation (162), also $\mu_{1,t} = 0$, as in the case of an old-style central bank. However, now, condition (164) also requires $\mu_{3,t} = 0$, which finally requires $\omega_t = 0$, by (171).

Therefore, with two policy tools, the central bank can hit two targets at once. As a consequence, the optimal targeting regime now includes two rules:

$$x_t + \kappa\lambda_\pi \pi_t = 0 \tag{165}$$

and

$$\omega_t = 0. \tag{166}$$

The targeting rule (165) is identical to the one implied by the benchmark New Keynesian model, and implies the optimal dynamics of the nominal interest rate, which – as in the benchmark New Keynesian model – takes care of the inflation-output tradeoff, if there is any. The targeting rule (166), on the other hand, implies the optimal path for central bank's reserves that is needed to hit the consumption-inequality target, and it follows the feedback rule

$$\hat{u}_t = \hat{u}_t^* - z(\chi - 1)x_t, \tag{167}$$

[33] Analogous results concerning the implications of productivity shocks for this kind of tradeoffs can be found in Nisticò (2016). Notice however that here the role of productivity shocks is entirely due to the presence of disposable labor-income inequality. Were $\varpi = 1$, indeed, productivity shocks would be irrelevant for $\hat{u}_t^*$ and the only shock implying an endogenous tradeoff would be $\hat{\theta}_t$.

where the optimal response coefficient to the output gap is negative if consumption inequality is countercyclical (i.e. $\chi > 1$).

Moreover, notice that using equations (166) and (167) into (151) implies that the IS schedule collapses to the one arising in the benchmark New Keynesian model

$$x_t = E_t x_{t+1} - \sigma^{-1}(\hat{\imath}_t^R - E_t \pi_{t+1} - \hat{r}_t^*). \tag{168}$$

Therefore, considering the above and the targeting rule (165), it follows that the optimal path for the interest rate on reserves is also equivalent to the one arising in the benchmark New Keynesian model. As implied by the analysis of Section 4.1, thus, the availability of two policy tools allows the central bank to use them selectively on different welfare-relevant targets: the conventional tool to pursue aggregate targets such as inflation and the output gap, and the unconventional one to pursue distributional targets related to consumption inequality.

Remark 6 *Therefore, in our economy with heterogeneous households, credit frictions and idiosyncratic uncertainty, as long as the ELB is not binding, unconventional monetary policy is optimally used to completely offset the additional distortions, compared to the benchmark New Keynesian model, implied by the credit friction, and induces the same equilibrium outcome that would arise in its absence.*

Notice that the above result is not limited to the case of discretion. To see this, consider the first-order conditions for an optimum under full commitment, that minimizes loss (80) subject to equations (151), (153), (152) and (154):

$$x_t = \kappa \mu_{2,t} + (1-z)^{-1}(1-\chi)\mu_{3,t} - \mu_{1,t} + \beta^{-1}\mu_{1,t-1} \tag{169}$$

$$\mu_{2,t} = -\lambda_\pi \pi_t + \sigma_x^{-1}\beta^{-1}\mu_{1,t-1} + \mu_{2,t-1} \tag{170}$$

$$\mu_{3,t} = -\lambda_\omega \omega_t - \beta^{-1}\Phi_\omega \mu_{1,t-1} \tag{171}$$

$$\mu_{1,t} = -\sigma_x \mu_{4,t} \tag{172}$$

$$\mu_{3,t} = \delta z(1-z)(\mu_{1,t} - \beta^{-1}\mu_{1,t-1}) \tag{173}$$

Therefore, also under full commitment, when the ELB is not binding (i.e. $\mu_{4,t} = 0$ for all t) the IS equation (151) and consumption inequality (152) are not relevant constraints for the optimal policy, implying $\mu_{1,t} = \mu_{3,t} = 0$ for all t. As a consequence, the targeting regime under commitment includes the two rules

$$x_t - x_{t-1} + \kappa \lambda_\pi \pi_t = 0 \tag{174}$$

and

$$\omega_t = 0, \tag{175}$$

where, again, the targeting rule related to aggregate targets is identical to the one arising in the benchmark New Keynesian model, where credit frictions are absent.

Consider now the case in which the ELB is binding, i.e. $\mu_{4,t} > 0$. In this case, the central bank effectively looses one of its policy tools, and, with it, the possibility to completely offset the financial distortions. Indeed, when $\mu_{4,t} > 0$ the optimal targeting regime includes the rules

$$x_t + \kappa\lambda_\pi\pi_t + (1-z)^{-1}(1-\chi)\lambda_\omega\omega_t = \sigma_x\mu_{4,t} \tag{176}$$

and

$$\lambda_\omega\omega_t = \delta\sigma_x z(1-z)\mu_{4,t} \tag{177}$$

which can be reduced into

$$x_t + \kappa\lambda_\pi\pi_t = z^{-1}\lambda_\omega\omega_t. \tag{178}$$

When the ELB is binding, therefore, the endogenous tradeoff arises again: as in the case of an old-style central bank, when the ELB is binding even a new-style central bank does not have enough policy tools to completely offset the distortions coming from the credit friction, and it has to trade off some aggregate stability for a more stable consumption inequality.

As a consequence, we can use (178) and (152) to solve for the optimal path of central bank's reserves:

$$\hat{u}_t = \hat{u}_t^* + z\left[(1-\chi) - \frac{z(1-z)}{\lambda_\omega}\right]x_t - \frac{z^2(1-z)}{\lambda_\omega}\kappa\lambda_\pi\pi_t. \tag{179}$$

Therefore, when the central bank looses its conventional tool because it is stuck at the ELB, it should expand its reserves in order to counteract the deflationary and recessionary effects of the shock that brought the economy into the liquidity trap.

Notice, finally, that equations (178) and (179) also clarify the open question at the end of Section 4.4.5, when a negative preference shock is large enough that the conventional tool cannot sterilize it fully because of the ELB.

On the one hand, a central bank endowed with a traditional dual mandate (i.e. $\lambda_\omega = 0$) can indeed resort to unconventional monetary policy and completely stabilize inflation and the output gap, as implied by equation (178) once we impose $\lambda_\omega = 0$. As shown in Section 4.4.5, this can be done with a strong enough commitment to hit the inflation target by means of the central bank's reserves only, without using the conventional tool at all.

On the other hand, a central bank concerned with social welfare would find that option sub-optimal as it would induce eccessive fluctuations in consumption inequality. Such a central bank would find it optimal instead to let inflation and the output gap take some of the effects of the shock in order to reduce fluctuations in consumption inequality. The way to do this is implied by equations (178) and (179): use the conventional tool until it reaches the ELB, and complement that interest-rate policy with an unconventional one that expands the central bank's reserves as implied by a feedback rule with a larger (though finite) response to inflation and the output gap.

We provide a numerical illustration of these mechanisms in the case of full commitment. As in Eggertsson and Woodford (2003) – and most of the ELB literature that followed – we consider the

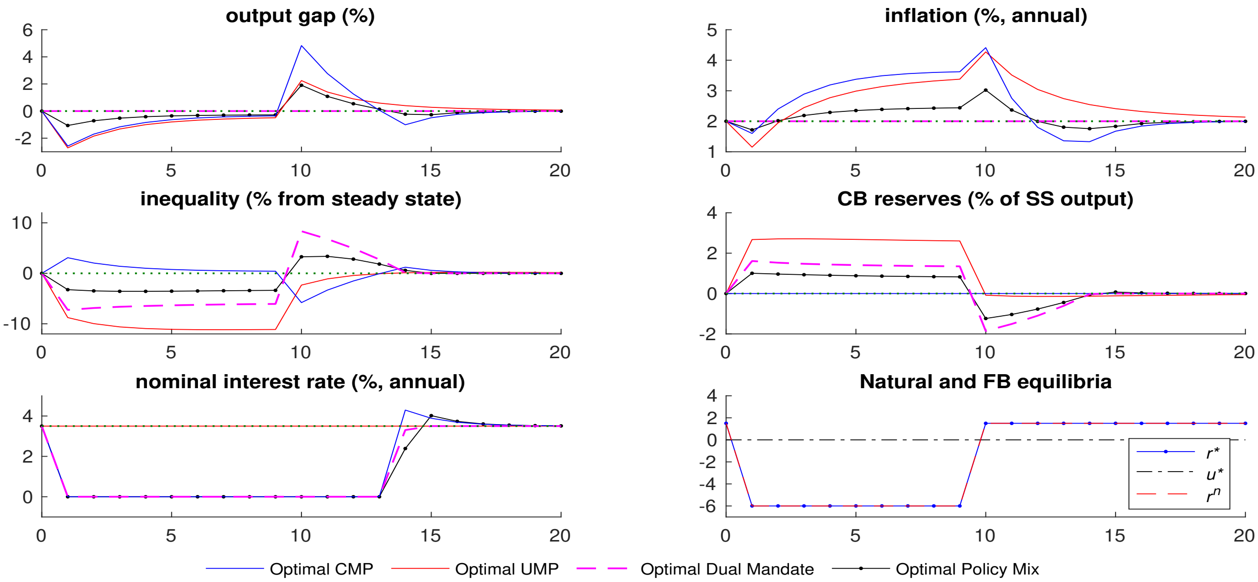

Figure 4: The response of the economy to a fall in the natural interest rate under optimal commitment. The fall in the natural rate is induced by a negative discount-factor shock. Blue solid line: optimal conventional policy only; red solid line: optimal unconventional policy only; magenta dashed line: optimal policy mix conditional on $\lambda_\omega = 0$; black line with dots: unconditional optimal policy mix.

case of a stochastic shock that follows a two-state Markov Process with an absorbing state.[34]

Figure 4 displays the response of selected variables under full commitment to a discount-factor shock that hits in period 1 and takes the natural (and first-best) real interest rate to -6%, in annual terms. When the shock hits, it is expected to last 6 quarters (i.e. the probability of the natural rate staying negative is about 0.83 per quarter). Ex post, it reverts to steady state in quarter 10 and from that moment onward nothing else happens. The figure shows the response of the output gap (in percentage points), the inflation rate (in annualized percentage points), consumption inequality (in percentage deviations from the steady-state level), central bank's reserves (in percentage deviations from the steady-state level, as a share of steady-state output), and the nominal interest-rate on reserves (also in annualized percentage points). The bottom-right panel shows the effect of the shock on the first-best interest rate $\hat{r}_t^*$ (blue line with dots), the first-best level of reserves $\hat{u}_t^*$ (dash-dotted black line) and the natural interest rate $\hat{r}_t^n$ (dashed red line).

Moreover, the figure considers four alternative policy regimes. The blue solid line displays the optimal policy when the central bank only uses the conventional interest rate; the red solid line the optimal policy when it only uses the unconventional tool (i.e. reserves); the magenta dashed line the optimal policy when the central bank uses both conventional and unconventional tools and it seeks to maximize a standard, dual-objective loss function (i.e. when $\lambda_\omega = 0$); the black line with dots finally displays the unconditional optimal policy, when the central bank seeks to maximize social welfare and uses both conventional and unconventional tools.

The first implication of this exercise is that, despite the response of reserves in the first-best equilibrium signals in principle no role for unconventional policy (as shown in the bottom-right

[34]In particular, we compute the solution of the model using the toolkit provided in Eggertsson et al. (2021).

panel), the effectiveness of a lower-bound on the conventional policy tool opens some room for a stabilization role of central bank's balance-sheet policy. Indeed, when the central bank only responds with the policy rate (blue solid lines), the zero-lower bound implies a persistent recession that the central bank addresses with *forward guidance*, i.e. the commitment to keep the nominal rate at zero for an additional 4 quarters after the natural rate has reverted back to steady state.

Compared to this outcome (familiar from the benchmark New Keynesian model), the figure shows that under the optimal policy (the black lines with dots) the central bank is able to reduce substantially the fluctuations in both the output gap and inflation. Key to this achievement is the "idiosyncratic-risk channel" of transmission of unconventional monetary policy: the central bank complements the zero-interest-rate policy with a persistent expansion in real reserves that lasts as long as the natural rate remains negative. The expansion in reserves is key to dampen the response of the output gap and inflation to the shock because the persistent fall in expected inequality that it implies reduces consumption risk for the savers and thus stimulates their demand.[35] When the natural rate returns positive (period 10 in the figure), the expansionary conventional stance, implied by the commitment to keep the interest rate at zero, is mirrored by an unconventional tightening that mitigates the upward jumps in output and inflation that is typically associated with forward-guidance policies in this class of models.

Notice that both instruments are used under the optimal policy. The use of the unconventional tool only (red lines in the figure), indeed, would not be able to substantially improve output-gap and inflation stabilization, compared to the use of the sole interest rate, and it would come at the cost of a much larger drop in consumption inequality relative to the optimal steady-state.

The policy mix that maximizes social welfare then combines conventional and unconventional policies to optimally trade off some output-gap and inflation stability for lower fluctuations in consumption inequality. A central bank endowed with a traditional dual mandate (which is nested in our model when the welfare weight on consumption inequality in equation (80) is forced to be $\lambda_\omega = 0$) can instead use the full power of unconventional monetary policy in this economy. Indeed, as shown by the magenta dashed line in Figure 4, a central bank only concerned with output gap and inflation stability can achieve full stabilization of both these objectives if it complements the zero-interest-rate policy with a stronger expansion in real reserves, that can exploit the "idiosyncratic-risk channel" as much as needed to close the output gap completely.

In Section 4.4 we discussed the different implications that a given fall in the natural interest rate might have for the output gap and inflation depending on the underlying fundamental shock. Figure 5 further scrutinizes that argument from an optimal policy perspective and displays the response to a fall in the natural rate of interest that is caused by a correlated shock to both the discount factor and the leverage constraint of financial intermediaries. In particular, the natural rate falls to -6% as before, driven down to -3% by a negative discount-factor shock and for the remaining 3 percentage points by a correlated deleveraging shock.

[35] In the absence of idiosyncratic risk, the role of central bank's reserves would be smaller – consistently with their smaller effectiveness through the "borrowing-cost channel" only – and the optimal response would imply a longer zero-interest-rate policy. Simulations available upon request.

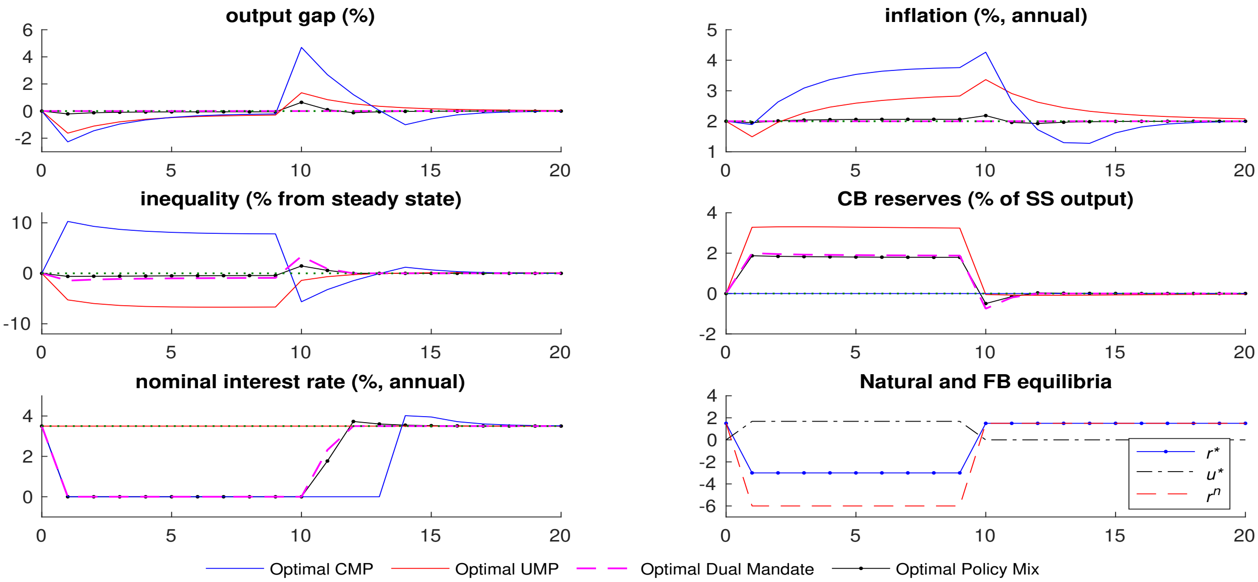

Figure 5: The response of the economy to a fall in the natural interest rate under optimal commitment. The fall in the natural rate is induced by a combination of correlated negative discount-factor and deleveraging shocks. Blue solid line: optimal conventional policy only; red solid line: optimal unconventional policy only; magenta dashed line: optimal policy mix conditional on $\lambda_\omega = 0$; black line with dots: unconditional optimal policy mix.

We can appreciate the first difference with respect to the case of Figure 4 in the bottom-right panel: while the path of the natural rate is the same as before, the first-best equilibrium is now different, with the optimal real interest rate $\hat{r}_t^*$ falling to -3% and the optimal level of real reserves $\hat{u}_t^*$ rising to 1.7%, both until the fundamental shocks revert to the absorbing state.

Under the "optimal conventional policy" – that is when the central bank aims at minimizing welfare losses using only the nominal interest rate (blue line in the figure) – the response of the economy is very similar to Figure 4 with respect to inflation and the output gap. A notable difference is however the response of consumption inequality: while a discount-factor shock hits symmetrically savers and borrowers, a deleveraging shock hurts proportionately more the borrowers, thereby implying a substantial increase in consumption inequality. This raises consumption risk for the savers and induces stronger downward pressures on the output gap, which the optimal conventional policy is able to address through a more inflationary commitment: the path of the inflation rate, indeed, lies above the one in Figure 4, thereby implying stronger downward pressures on the real interest rate that counteract the contractionary consequences of the increase in consumption risk.

The fact that about a half of the fall in the natural rate is transmitted to the real economy through the increase in consumption risk to the savers explains the meaningful difference arising under the "optimal unconventional policy" – that is when the central bank seeks to minimize welfare losses using only its balance sheet (red line in the figure). Indeed, using only the unconventional tool allows the central bank to more effectively lean against the deleveraging component of the fall in the natural rate that pushes consumption risk up, and to improve the stabilization of the output gap in particular, compared to the conventional optimal policy. In this case, the nominal interest rate does not move at all and the equilibrium consumption inequality has to fall substantially in order to lean against the fall in the natural rate.

Particularly worth noticing are the implications of the "optimal policy mix" – that is when the central bank seeks to maximize social welfare optimally combining both types of policy tools (black line with dots). Appropriately complementing the zero-interest-rate policy with an expansion in real reserves allows the central bank to substantially shorten the duration of the liquidity trap and lift the nominal interest rate only one quarter after the shock has reverted to steady state. The expansion in reserves indeed addresses the deleveraging component of the fall in the natural rate, and is able to substitute the power of *forward guidance* on nominal rates with a reduction in consumption risk for the savers that almost completely offsets the short-run contractionary effects of the shock. The early liftoff of the policy rate, moreover, also reduces the expansionary stance of the conventional policy in period 10, when the shock is back in the absorbing state, and thus mitigates the upward jump of the output gap and inflation without the need of a sizable unconventional tightening.[36]

This result echoes Benigno and Benigno (2022) where, too, optimally setting the path of central bank's reserves in the face of a fall in the natural interest rate is able to shorten the duration of the liquidity trap. In their environment, however, central bank reserves affect real activity through their utility value for a representative agent, and therefore can shorten the liquidity trap regardless of the source of the fall in the natural rate. Here instead we point out that this result critically depends on whether the natural rate falls in response to a fundamental shock – such as a deleveraging shock – that is mainly transmitted through inequality and consumption risk, that is the same theoretical channel that makes unconventional monetary policy most effective.

Finally notice that in this case the outcome implied by a central bank endowed with a traditional dual-objective mandate is very similar to the unconditional optimal policy mix along every dimension: output gap and inflation in this case can be completely stabilized by an appropriate combination of conventional and unconventional monetary policy, in spite of the liquidity trap, with reserves increasing only slightly more to exploit the "idiosyncratic-risk channel" of unconventional monetary policy to a full extent.

5 Conclusion

This paper studies the interplay between unconventional monetary policy and income and consumption inequality. We derive the monetary-policy implications of an economy where households are heterogeneous and face idiosyncratic risk, financial intermediaries channel funds from savers to borrowers but are limited by some leverage constraints, and the central bank controls both the interest rate on its reserves and the size of its balance sheet. Accounting for idiosyncratic risk and cyclical inequality opens room for two additional channels of transmission of central banks' balance-sheet

[36] Note that the difference in the duration of the liquidity trap compared to Figure 4 is not simply due to the milder discount-rate shock. Indeed, simulating the response to a preference shock that takes the natural rate to -4%, without the correlated deleveraging shock, would require a duration of the zero-interest-rate policy one quarter longer than in Figure 5, suggesting a more meaningful interaction of the two shocks in opening room for the stabilization power of unconventional monetary policy. Simulations available upon request.

policies, that critically amplify the familiar channel related to the relaxation of the banks' leverage constraint and the reduction of the borrowing interest rate.

The idiosyncratic-risk channel in particular is key for the transmission of persistent balance-sheet policies: improving the consumption opportunities for borrowers reduces consumption risk for the savers which, in turn, find it optimal to cut their precautionary savings and expand their current spending as well. This critically amplifies the expansionary effect of an unconventional monetary policy shock that initially only affects the borrowers.

Through this channel, unconventional monetary policy also improves the ability of the central bank to anchor the private sector's expectations and rule out endogenous instability. Endogenous unconventional balance-sheet policies in this economy are a perfect substitute for conventional interest-rate feedback rules in the implementation of a (locally) unique rational-expectations equilibrium. As a consequence, appropriately specified balance-sheet policy rules allow the equilibrium to be determinate even in the case of an interest-rate peg, or a permanent liquidity trap.

Unconventional monetary policy allows the central bank to fully stabilise inflation and the output gap even in the face of shocks that the conventional dimension of policy would find impossible to sterilise due to the existence of an effective lower bound on nominal interest rates. We show, however, that (unconventional) strict inflation targeting is not necessarily an optimal policy regime from a welfare perspective, as it may require strong and persistent effects on consumption inequality that are detrimental for social welfare. Nevertheless, optimal unconventional monetary policy within a new-style regime improves the ability of the central bank to reduce fluctuations in inflation and the output gap during a liquidity trap, and may promote a swifter exit from zero-interest rate policies.

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Appendix

A The Welfare-Based Monetary-Policy Loss Function

In this section we provide details on the derivation of equation (80). We consider an efficient steady state maximizing the social welfare (76) subject to the resource and technological constraints, as in equation (77). Such a steady state satisfies:

$$(1-\tilde{z})U_c(\bar{C}_s) = (1-z)\bar{\lambda} \tag{180}$$

$$\tilde{z}U_c(\bar{C}_b) = z\bar{\lambda} \tag{181}$$

$$(1-\tilde{z})V_l(\bar{L}_s) = (1-z)\bar{\lambda}\frac{\bar{Y}}{\bar{L}_s} \tag{182}$$

$$\tilde{z}V_l(\bar{L}_b) = z\bar{\lambda}\frac{\bar{Y}}{\bar{L}_b} \tag{183}$$

where $\bar{\lambda}$ is the Lagrange multiplier on the contraint (77), evaluated at the steady state.

Now take a second-order approximation of the social-welfare function (76) around this efficient steady state, to get:

$$\begin{aligned} U_t = \bar{U} + (1-\tilde{z}) &\left[U_c(\bar{C}_s)(C_{s,t}-\bar{C}_s) + \frac{1}{2}U_{cc}(\bar{C}_s)(C_{s,t}-\bar{C}_s)^2\right] \\ &+ \tilde{z}\left[U_c(\bar{C}_b)(C_{b,t}-\bar{C}_b) + \frac{1}{2}U_{cc}(\bar{C}_b)(C_{b,t}-\bar{C}_b)^2\right] \\ &- (1-\tilde{z})\left[V_l(\bar{L}_s)(L_{s,t}-\bar{L}_s) + \frac{1}{2}V_{ll}(\bar{L}_s)(L_{s,t}-\bar{L}_s)^2\right] \\ &- \tilde{z}\left[V_l(\bar{L}_b)(L_{b,t}-\bar{L}_b) + \frac{1}{2}V_{ll}(\bar{L}_b)(L_{b,t}-\bar{L}_b)^2\right] + \mathcal{O}(||\vartheta||)^3, \end{aligned} \tag{184}$$

where an upper-bar denotes a variable in the efficient steady state and the term $\mathcal{O}(||\vartheta||)^3$ collects terms in the expansions that are of an order higher than the second.

Using conditions (180)–(183) in equation (184), the latter reads

$$\begin{aligned} U_t = \bar{U} + (1-z)\bar{\lambda} &\left[(C_{s,t}-\bar{C}_s) + \frac{1}{2}\frac{U_{cc}(\bar{C}_s)}{U_c(\bar{C}_s)}(C_{s,t}-\bar{C}_s)^2\right] \\ &+ z\bar{\lambda}\left[(C_t^b-\bar{C}^b) + \frac{1}{2}\frac{U_{cc}(\bar{C}_b)}{U_c(\bar{C}_b)}(C_{b,t}-\bar{C}_b)^2\right] \\ &- (1-z)\bar{\lambda}\frac{\bar{Y}}{\bar{L}_s}\left[(L_{s,t}-\bar{L}_s) + \frac{1}{2}\frac{V_{ll}(\bar{L}_s)}{V_l(\bar{L}_s)}(L_{s,t}-\bar{L}_s)^2\right] \\ &- z\bar{\lambda}\frac{\bar{Y}}{\bar{L}_b}\left[(L_{b,t}-\bar{L}_b) + \frac{1}{2}\frac{V_{ll}(\bar{L}_b)}{V_l(\bar{L}_b)}(L_{b,t}-\bar{L}_b)^2\right] + \mathcal{O}(||\vartheta||)^3 \end{aligned} \tag{185}$$

Now define $x_t \equiv \ln X_t/\bar{X}$, for $X = Y,\ L,\ A$, which implies

$$\frac{X_t - \bar{X}}{\bar{X}} = x_t + \frac{1}{2}x_t^2 + \mathcal{O}(||\vartheta||)^3. \tag{186}$$

Moreover, define $c_{s,t} \equiv (C_{s,t} - \bar{C}_s)/\bar{Y}$ and $c_{b,t} \equiv (C_{b,t} - \bar{C}_b)/\bar{Y}$, which imply, together with the resource constraint (47):

$$(1-z)c_t^s + zc_t^b = y_t + \frac{1}{2}y_t^2 + \mathcal{O}(||\vartheta||)^3. \tag{187}$$

Using the above in (185) we get:

$$\begin{aligned} U_t = \bar{U} + \bar{\lambda}\bar{Y}\left[y_t + \frac{1}{2}y_t^2\right] - \frac{1}{2}\bar{\lambda}\bar{Y}\sigma\left[(1-z)c_{s,t}^2 + zc_{b,t}^2\right] \\ - (1-z)\bar{\lambda}\bar{Y}\left[l_{s,t} + \frac{1}{2}(1+\varphi)l_{s,t}^2\right] - z\bar{\lambda}\bar{Y}\left[l_{b,t} + \frac{1}{2}(1+\varphi)l_{b,t}^2\right] + \mathcal{O}(||\vartheta||)^3 \end{aligned} \tag{188}$$

where we used $U_{cc}(\bar{C}^s)/U_c(\bar{C}^s) = U_{cc}(\bar{C}^b)/U_c(\bar{C}^b) = -v$, $\bar{L}_s V_{ll}(\bar{L}_s)/V_l(\bar{L}_s) = \bar{L}_b V_{ll}(\bar{L}_b)/V_l(\bar{L}_b) = \varphi$ and $\sigma \equiv v\bar{Y}$.

Now notice that the aggregate production function $Y_t\Delta_t^p = A_t L_t = A_t L_{s,t}^{1-z}L_{b,t}^z$ implies that the following holds exactly:

$$y_t = (1-z)l_{s,t} + zl_{b,t} + a_t - \ln\Delta_t^p,$$

which allows us to simplify the linear terms in equation (188), and write:

$$\frac{1}{2}\frac{U_t - \bar{U}}{\bar{\lambda}\bar{Y}} = y_t^2 - \sigma[(1-z)c_{s,t}^2 + zc_{b,t}^2] - (1+\varphi)[(1-z)l_{s,t}^2 + zl_{b,t}^2] - \ln\Delta_t^p + \text{t.i.p.} + \mathcal{O}(||\vartheta||)^3 \tag{189}$$

where "t.i.p." collects terms independent of policy. To evaluate the second-order terms in the equation above, we can use a first-order approximation of the resource constraint and of the aggregate production function:

$$c_{s,t} = y_t + z\omega_t \tag{190}$$

$$c_{b,t} = y_t - (1-z)\omega_t \tag{191}$$

$$l_{s,t} = y_t - a_t - z\frac{\sigma}{1+\varphi}\omega_t \tag{192}$$

$$l_{b,t} = y_t - a_t + (1-z)\frac{\sigma}{1+\varphi}\omega_t, \tag{193}$$

where $\omega_t \equiv c_{s,t} - c_{b,t}$ and in the last two equations we used

$$l_{s,t} - l_{b,t} = -\frac{\sigma}{1+\varphi}\omega_t, \tag{194}$$

as implied by a first-order approximation of the equilibrium condition in the labor market, i.e.

$$\frac{L_{s,t}^{1+\varphi}}{v\exp(-vC_{s,t})} = \frac{L_{b,t}^{1+\varphi}}{v\exp(-vC_{b,t})}. \tag{195}$$

Using the above in equation (189), after some algebra we can write:

$$-\frac{1}{2}\frac{U_t-\bar{U}}{\bar{\lambda}\bar{Y}} = (\varphi+\sigma)x_t^2 + z(1-z)\sigma\frac{1+\varphi+\sigma}{1+\varphi}\omega_t^2 + \ln\Delta_t^p + \text{t.i.p.} + \mathcal{O}(||\vartheta||)^3, \tag{196}$$

where $x_t \equiv y_t - y_t^*$ and $y_t^* \equiv \frac{1+\varphi}{\sigma+\varphi}a_t$.

Making use of the familiar result about the relative-price dispersion Δ_t^p,

$$\sum_{t=t_0}^{\infty}\beta^{t-t_0}\ln\Delta_t^p = \frac{\epsilon(\sigma+\varphi)}{2\kappa}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\pi_t^2 + \text{t.i.p} + \mathcal{O}(||\vartheta||)^3, \tag{197}$$

where κ is the slope of the New Keynesian Phillips Curve, we finally obtain equation (80):

$$\mathcal{L}_{t_0} \equiv -\frac{1}{2}E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\frac{U_t-\bar{U}}{\bar{\lambda}\bar{Y}}\right)\right\} = \frac{\sigma+\varphi}{2}E_{t_0}\left\{\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Big(x_t^2+\lambda_\pi\pi_t^2+\lambda_\omega\omega_t^2\Big)\right\}, \tag{198}$$

which ignores terms independent of policy and of higher order, and where

$$\lambda_\pi \equiv \frac{\epsilon}{\kappa} \tag{199}$$

$$\lambda_\omega \equiv \sigma\frac{z(1-z)(1+\varphi+\sigma)}{(\sigma+\varphi)(1+\varphi)}. \tag{200}$$

B Proof of Proposition 1: the conditions for determinacy.

Consider the system of equations describing the private-sector equilibrium conditions

$$x_t = \Phi E_t x_{t+1} - \sigma_x^{-1}(\hat{\imath}_t^R - E_t\pi_{t+1} - \hat{r}_t^*) - \delta E_t\{\Delta\hat{u}_{t+1} - \Delta\hat{u}_{t+1}^*\} + z^{-1}\delta(1-\gamma_s)E_t\{\hat{u}_{t+1} - \hat{u}_{t+1}^*\} \tag{201}$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t \tag{202}$$

and the following monetary-policy rules

$$\hat{\imath}_t^R = \hat{r}_t^* + \phi_\pi\pi_t + \phi_x x_t \tag{203}$$

$$\hat{u}_t = \hat{u}_t^* - \psi_\pi\pi_t - \psi_x x_t. \tag{204}$$

Substituting the policy rules in the IS equation, we can write the system more compactly in matrix form as

$$\mathbf{A}\begin{bmatrix}x_t\\ \pi_t\end{bmatrix} = \mathbf{B}\begin{bmatrix}E_t x_{t+1}\\ E_t\pi_{t+1}\end{bmatrix}, \tag{205}$$

where let

$$\mathbf{A} \equiv \begin{bmatrix}1+\sigma_x^{-1}\phi_x+\delta\psi_x & \sigma_x^{-1}\phi_\pi+\delta\psi_\pi\\ -\kappa & 1\end{bmatrix}$$

and

$$\mathbf{B} \equiv \begin{bmatrix} \Phi + z^{-1}\delta(\gamma_s + z - 1)\psi_x & \sigma_x^{-1} + z^{-1}\delta(\gamma_s + z - 1)\psi_\pi \\ 0 & \beta \end{bmatrix}.$$

System (205) admits $x_t = \pi_t = 0$ for all t as a (locally) unique solution if and only if the two eigenvalues of matrix $\mathbf{D} \equiv \mathbf{B}^{-1}\mathbf{A}$ are both outside the unit circle, where

$$\mathbf{D} = (\det \mathbf{B})^{-1} \begin{bmatrix} \mathbf{d}_{11} & \mathbf{d}_{12} \\ \mathbf{d}_{21} & \mathbf{d}_{22} \end{bmatrix}, \tag{206}$$

with

$$\det \mathbf{B} = \beta(\Phi + z^{-1}\delta(\gamma_s + z - 1)\psi_x) \tag{207}$$

and

$$\mathbf{d}_{11} \equiv \beta(1 + \sigma_x^{-1}\phi_x + \delta\psi_x) + \kappa(\sigma_x^{-1} + z^{-1}\delta(\gamma_s + z - 1)\psi_\pi) \tag{208}$$

$$\mathbf{d}_{12} \equiv \beta(\sigma_x^{-1}\phi_\pi + \delta\psi_\pi) - (\sigma_x^{-1} + z^{-1}\delta(\gamma_s + z - 1)\psi_\pi) \tag{209}$$

$$\mathbf{d}_{21} \equiv -\kappa(\Phi + z^{-1}\delta(\gamma_s + z - 1)\psi_x) \tag{210}$$

$$\mathbf{d}_{22} \equiv \Phi + z^{-1}\delta(\gamma_s + z - 1)\psi_x. \tag{211}$$

As proved in Woodford (2003), among others, this condition is satisfied if all of the following holds

$$i)\ \det \mathbf{D} > 1 \qquad ii)\ \det \mathbf{D} - \operatorname{tr} \mathbf{D} > -1 \qquad iii)\ \det \mathbf{D} + \operatorname{tr} \mathbf{D} > -1. \tag{212}$$

Consider now that $\det \mathbf{D} = (\det \mathbf{B})^{-1} \det \mathbf{A}$, with

$$\det \mathbf{A} = 1 + \sigma_x^{-1}\phi_x + \delta\psi_x + \kappa(\sigma_x^{-1}\phi_\pi + \delta\psi_\pi).$$

Accordingly, condition 212.ii) can be written as

$$\det \mathbf{A} - (\mathbf{d}_{11} + \mathbf{d}_{22}) > -\det \mathbf{B}$$

and it requires

$$\begin{aligned} &1 + \sigma_x^{-1}\phi_x + \delta\psi_x + \kappa(\sigma_x^{-1}\phi_\pi + \delta\psi_\pi) - \beta(1 + \sigma_x^{-1}\phi_x + \delta\psi_x) \\ &\quad - \kappa(\sigma_x^{-1} + z^{-1}\delta(\gamma_s + z - 1)\psi_\pi) - \Phi - z^{-1}\delta(\gamma_s + z - 1)\psi_x > -\beta(\Phi + z^{-1}\delta(\gamma_s + z - 1)\psi_x), \end{aligned}$$

which, after some algebra, yields condition (91):

$$\sigma_x^{-1}\Big[(1-\beta)\phi_x + \kappa(\phi_\pi - 1)\Big] + z^{-1}\delta(1-\gamma_s)\Big[(1-\beta)\psi_x + \kappa\psi_\pi\Big] > (1-\beta)(\Phi - 1). \tag{213}$$

Moreover, condition 212.i) requires

$$\sigma_x^{-1}\phi_x + \delta\Big[1-\beta+z^{-1}\beta(1-\gamma_s)\Big]\psi_x + \kappa(\sigma_x^{-1}\phi_\pi + \delta\psi_\pi) > \beta\Phi - 1, \tag{214}$$

which is always satisfied for $\Phi \leq \beta^{-1}$ and it is generally implied by (213) also for $\Phi > \beta^{-1}$, as long as Φ is not too large, in which case condition (214) becomes necessary and (213) is implied.

Finally, condition 212.iii) requires

$$\begin{aligned} &1+\sigma_x^{-1}\phi_x + \delta\psi_x + \kappa(\sigma_x^{-1}\phi_\pi + \delta\psi_\pi) + \beta(1+\sigma_x^{-1}\phi_x + \delta\psi_x) \\ &\quad + \kappa(\sigma_x^{-1} + z^{-1}\delta(\gamma_s + z - 1)\psi_\pi) + \Phi + z^{-1}\delta(\gamma_s+z-1)\psi_x > -\beta(\Phi + z^{-1}\delta(\gamma_s+z-1)\psi_x), \end{aligned}$$

which is always satisfied for non-negative response coefficients ϕ's and ψ's. As a consequence, for Φ not too large, the equilibrium is determinate if and only if condition (91) is satisfied.