# Analysis of the electromagnetic scattering from arbitrarily-shaped objects by using an analytical Dirichlet-to-Neumann map

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The numerical evaluation of the electromagnetic scattering from objects of arbitrary shape is a classical problem that is still of interest for practical applications, such as the efficient EMC simulation of complex systems [1]. When such a problem is solved by means of a differential formulation, an efficient technique to close it is the use of the so-called Dirichlet-to-Neumann (DtN) operator, i.e. a relation between a Dirichlet condition to a corresponding Neumann one onto the boundary of the solution domain. Such a map allows to truncate the computational domain from an infinite one to a finite one. Specifically, it introduces an exact boundary condition on the truncated computational domain rather than an approximate one as it happens when using classical absorbing boundary conditions. Moreover, the boundary condition is exact regardless the size of the truncation domain that, in principle, may be chosen coincident with the boundary of the scatterer. Compared to classical approaches based on absorbing boundary conditions, such as the Perfect Matching Layer (PML) [2], for a targeted accuracy such a technique reduces the computational burden, both in terms of number of unknowns and of memory allocation.

The problem under investigation is the evaluation of the scattered field, $w^s$, from an object of arbitrary shape, hit by a known incident field, $w^i$. In particular, the problem must be solved at any distance from the scatterer, e.g., both in the near and the far zone. Indicating with $B$ the solution domain and with $\partial B$ its boundary, the scattering from an object of arbitrary shape is formulated through the Helmholtz equation [3]. The numerical model is implemented with a differential formulation coupled with a DtN operator $\Lambda$, defined as $\Lambda$: $u|_{\partial B} \rightarrow \partial_n u|_{\partial B}$. The DtN operator may be calculated in analytically on spherical boundaries. In the simple case of a 2D problem (where the boundary $\partial B$ reduces to a circumference of radius $R$), the DtN may be written as follows [3]:

$$\Lambda(u|_{\partial B}) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \frac{H_n^{(2)\prime}(kR)}{H_n^{(2)}(kR)} \int_0^{2\pi} u(R,\theta')\, e^{-jn\theta'} d\theta' \tag{1}$$

where $k$ is the wavenumber, and $H_n^{(2)}$ is the Hankel function of the second kind.

Figure 1a refers to a penetrable U-shaped object, infinitely long in the *z*-direction, with a dielectric of relative permittivity of $\varepsilon_r = 4$ and illuminated by a TM plane wave. The object dimensions w.r.t. to the wavelength are assumed to be $h = \lambda$, $H = 2\lambda$, $l = \lambda$, $L = 1.5\lambda$. Given the aspect-ratio, a circular boundary is a suitable choice for the DtN: here we assume as truncating boundary $\partial B$ the circle of radius $R = 1.2\lambda$.

The reference solution is derived by a simple MoM implementation of the solution in [4], and is reported in Fig.1b. It is evident the presence of hot spots located at the corners of the scatterer.

The DtN numerical solution of the above problem is obtained with a FE numerical model using a standard triangular mesh, and test functions of the first order.

Finally, the result is compared to a PML-based numerical solution obtained by means of a commercial simulator (Comsol Multiphysics [5]), which imposes an artificial layer of thickness δ to be added at the exterior of the boundary $\partial B$.

The relative error of the DtN and PML solutions compared to the reference one (versus the total number of unknowns, $N_{DOF}$) is reported in Fig.2. For a fixed value of $N_{DOF}$, the error associated to the DtN shows to be smaller than that related to the PML approach.

(a) (b)

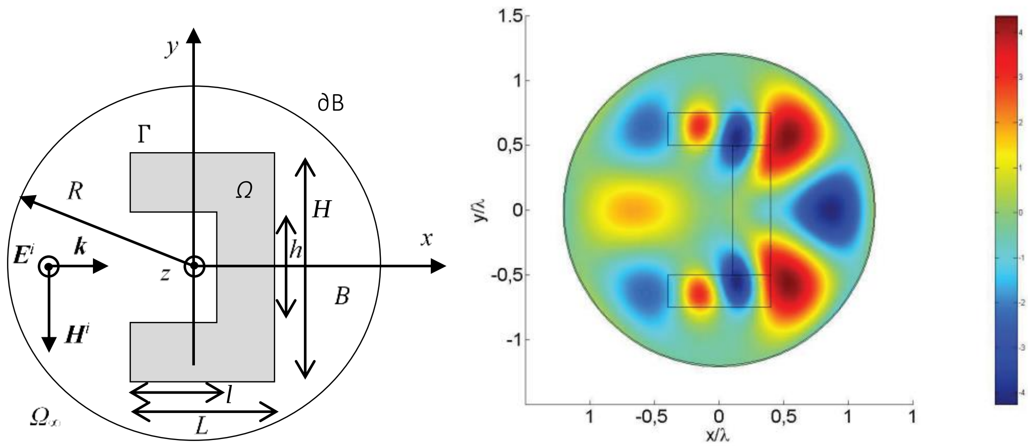


Fig. 1. Scattering from an infinitely long U-shaped dielectric object: (a) geometry; (b) map of scattered field (dimensions are normalized to the wavelength)

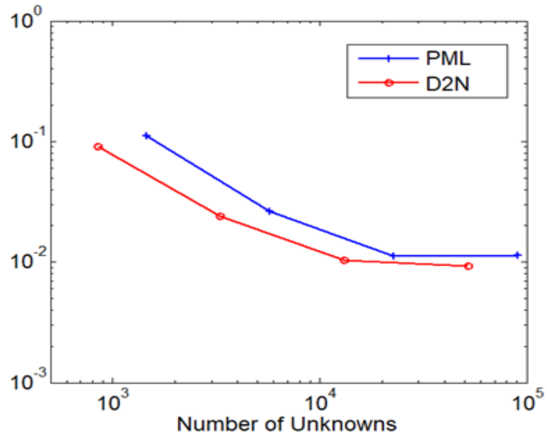


Fig. 2. Relative error as a function of the total number of unknowns, for DtN and PML approaches.

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