Eddy Current Tomography Based on Monotonicity of Time Constants

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This manuscript discusses a real-time tomography method in time-domain eddy current testing. This tomography is based on the monotonicity between time constants and the defect shape. The mathematical model, monotonicity and imaging algorithm, and a realistic 3D reconstruction example will be presented in this paper.

Eddy current testing is a well-established nondestructive evaluation method. It is very commonly used in many industry aspects because of its non-contact nature and hence, capable of high-speed inspection and application in hazardous environments. While the detection of defects are relatively simple and straightforward in eddy current testing, the tomography, i.e. finding a representation/reconstruction of the defect profile can be challenging. State-of-the-art technique of eddy current tomography try to find the defect profile in an iterative manner, but this kind of method requires intensive computation effort and local minima is a risk because of the non-linear nature of eddy current problems.

On the other hand, non-iterative methods are capable of real-time imaging. Monotonicity based tomography methods is a specific non-iterative method and was first introduced in 2002 [1]. It is a topic of active research and has been shown to be promising by many investigators, to name a few, Garde and Staboulis [2] on electrical impedance imaging and Flores-Tapia *et al*. [3] on breast cancer imaging. Monotonicity based methods have been broadly applied to electrical resistance tomography [1], eddy current tomography in large skin-depth regime [4,6] and small skin-depth regime [5], and microwave tomography [7]. This study describes the monontonicity of eddy current problem in time domain and completes the application of motononiticity based tomography in parabolic partial differential equation (PDE) problems.

Assuming that the conductive material is linear, spatially and temporally non-dispersive, isotropic, non-magnetic, and neglecting the displacement current, the eddy current **J** satisfies the following conditions:

$$\int_D \mathbf{J}'(\mathbf{r})\cdot\eta\mathbf{J}(\mathbf{r},t)\mathrm{d}V = -\partial_t \int_D \mathbf{J}'(\mathbf{r})\cdot\mathcal{A}_i[\mathbf{J}]\,\mathrm{d}V - \partial_t \int_D \mathbf{J}'(\mathbf{r})\cdot A_c(\mathbf{r},t)\mathrm{d}V, \forall \mathbf{J}' \in \hat{H} \tag{1}$$

where $\hat{H} = \{\mathbf{v} \in H(\mathrm{div}; D)\,|\nabla\cdot\mathbf{v} = 0, \mathbf{v}\cdot\hat{\mathbf{n}} = 0 \text{ on } \partial D\}$ and $\mathbf{J}(\cdot,t) \in \hat{H}$ for any given t, D is the conducting domain, $\eta(\cdot)$ is the electrical resistivity of the conductor, φ is the electrical scalar potential, $\mathbf{A}_c$ is the magnetic vector potential due to source currents and $\mathcal{A}_i$ is a properly defined integral operator [4].

The natural modes of the eddy current problem are the solutions of the homogeneous equation, i.e. when the source term is vanishing. It can be proved that the solution can be expressed as

$$\mathbf{J}(\mathbf{r},t) = \sum_{i=1}^{+\infty} c_i \mathbf{J}_i(\mathbf{r}) e^{-t/\tau_i} \tag{2}$$

where c_i's are related coefficients, $\mathbf{J}_i(\mathbf{r})$'s the natural modes and the time constants τ_i's are real, positive, bounded and approach to zero if arranged in decreasing order.

With reference to eddy current tomography, given a conducting domain V and two possible defects D_α and D_β (Figure 1), hereafter we assume the resistivity of the defects is larger than that of the background, the monotonicity property of time constants is [8]

$$D_\beta \subseteq D_\alpha \Rightarrow \tau_i^{D_\beta} \geq \tau_i^{D_\alpha}, \text{ for any } i \tag{3}$$

From the physical standpoint Eq. (3) means if we enlarge the size of one anomaly, all of the time constants related to this anomaly decrease. This gives rise to a straightforward reconstruction scheme: An unknown defect V can be estimated as sum of voxels, or test elements, T_j for such T_j's that their time constants are all greater than or equal to the time constant of V, i.e.

$$V_U = \bigcup \{T_j | \tau_j^k \geqslant \tau_V^k, \forall k\} \tag{4}$$

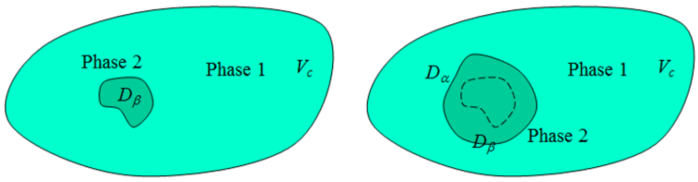
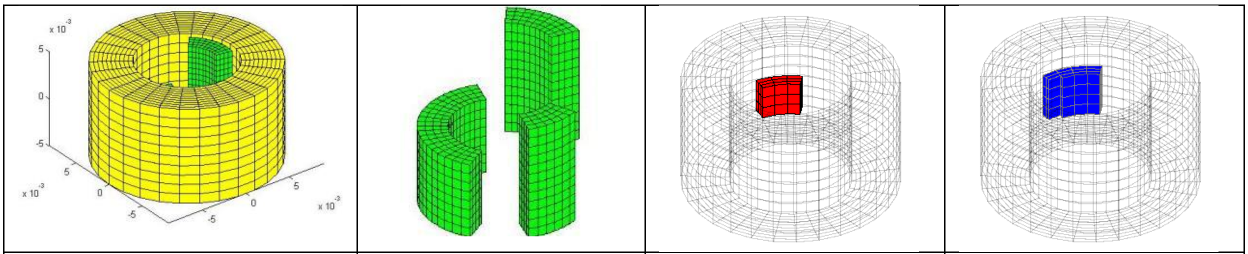


Figure 1. Two defects in a condutive domain, one includes the other.

Finally, figure 2 demonstrate an realistic 3D example of finding the defect in a pipeline section. Notice that time constants is an inherent and global property of the specimen under test, which does not depend on the excitation system. This renders our tomography method insensitive to probe tilting, lift-off or even excitation waveform. The reconstruction results, figure 2(c) and 2(d), demonstrate the feasibility of this approach.



(a)	(b)	(c)	(d)

Figure 2. 3D pipe section tomography example. (a) pipeline section (b) conductor patches used to break the symmetry (c) red voxels highlight the unknown defect (d) blue voxels highlight the reconstructed defect profile.

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