HIGH-PERFORMANCE PEEC ANALYSIS OF ELECTROMAGNETIC SCATTERERS

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The Partial Element Equivalent Circuit (PEEC) formulation can be more convenient than differential ones to compute electromagnetic scattering from arbitrary 3-D bodies. However, the use of the resulting fully-populated matrices may easily lead to unfeasible computational efforts, which can be effectively reduced adopting *H*-matrix compression methods. An example, the study of the electromagnetic scattering from a high-contrast dielectric sphere is presented.

Partial Element Equivalent Circuit

The Dual-PEEC formulation is briefly reviewed for a frequency-domain study [1]. The conducting domain Ω is discretized into a primal cell complex K made up of n nodes, e edges, f faces and v tetrahedra. The dual grid $\widetilde{K}$, of $\tilde{n} = v$ nodes, $\tilde{e} = f$ edges and $\tilde{f} = e$ faces, is constructed by taking a barycentric subdivision of the primal mesh.

Introducing the discrete differential operators $\mathbf{D}$ (Divergence) between pairs (v, f) on the primal complex and $\widetilde{\mathbf{G}} = -\mathbf{D}^{\mathrm{T}}$ (Gradient) between pairs $(\tilde{e}, \tilde{n})$ on $\widetilde{K}$, the circulations of the total and scattered electric field along dual edges, $\tilde{\mathbf{e}}_{\mathrm{tot}}$ and $\tilde{\mathbf{e}}_{\mathrm{sca}}$ respectively, are expressed as

$$\tilde{\mathbf{e}}_{\mathrm{tot}} = \mathbf{R}\,\mathbf{i}\,, \qquad \tilde{\mathbf{e}}_{\mathrm{sca}} = -\mathrm{j}\omega\tilde{\mathbf{a}} - \widetilde{\mathbf{G}}\tilde{\phi}\,, \tag{1}$$

where the array $\boldsymbol{i}$ stores the equivalent currents across primal faces, $\tilde{\boldsymbol{a}}$ is the circulation of the magnetic vector potential along dual edges and $\tilde{\phi}$ expresses the electric scalar potential on dual nodes. $\mathbf{R}$ is a sparse matrix obtained by discretizing local constitutive relationships.

Then, the electromagnetic potentials are expressed as a function of their sources by means of the *Partial Inductance* $\mathbf{L}$ and *Partial Coefficient of Potential* $\mathbf{P}$ dense matrices:

$$\tilde{\mathbf{a}} = \mathbf{L}\,\mathbf{i}\,, \qquad \tilde{\phi} = -\frac{1}{\mathrm{j}\omega}\mathbf{P}\mathbf{D}\mathbf{i}\,. \tag{2}$$

By letting (2) in (1), the discrete *Electric Field Integral Equation* holds:

$$\left[\mathbf{R} + \mathrm{j}\omega\mathbf{L} + \frac{1}{\mathrm{j}\omega}\mathbf{D}^{\mathrm{T}}\mathbf{P}\mathbf{D}\right]\mathbf{i} = \tilde{\mathbf{e}}_{\mathrm{ext}}\,, \tag{3}$$

where $\tilde{\mathbf{e}}_{\mathrm{ext}}$ stores the electromotive forces (on dual edges) due to external sources.

H-Matrix Compression and Results

Although fully populated, the system matrix in (3) contains low-rank blocks that make the matrix suitable for a data-sparse representation. Among different compression methods, the HLIBPro [2] and STRUMPACK-Dense [3] codes use H^2 and HSS matrix representation, respectively. While the former relies on an admissibility criterion to identify which blocks can be compressed, the latter creates a low-rank approximation of all off-diagonal blocks. This operation is independent from the kernel of the integral expression (3), but requires to test the whole matrix, as well as to reorder all the entries, guaranteeing the off-diagonal blocks to be low-rank.

The two libraries are compared evaluating the Radar Cross Section (RCS) of a high-contrast homogeneous dielectric sphere (radius 1 m, $\varepsilon_r = 80$), excited by a linearly polarized wave $\mathbf{E}_{\mathrm{ext}} = \mathrm{e}^{-\mathrm{j}k_0 x}\mathbf{u}_z$, where $k_0 = 2\pi f\sqrt{\varepsilon_0\mu_0}$, $f = 10$ MHz. The RCS is then compared with the *MIE* [4] analytical solution. A fine mesh of 83927 DoFs is used.

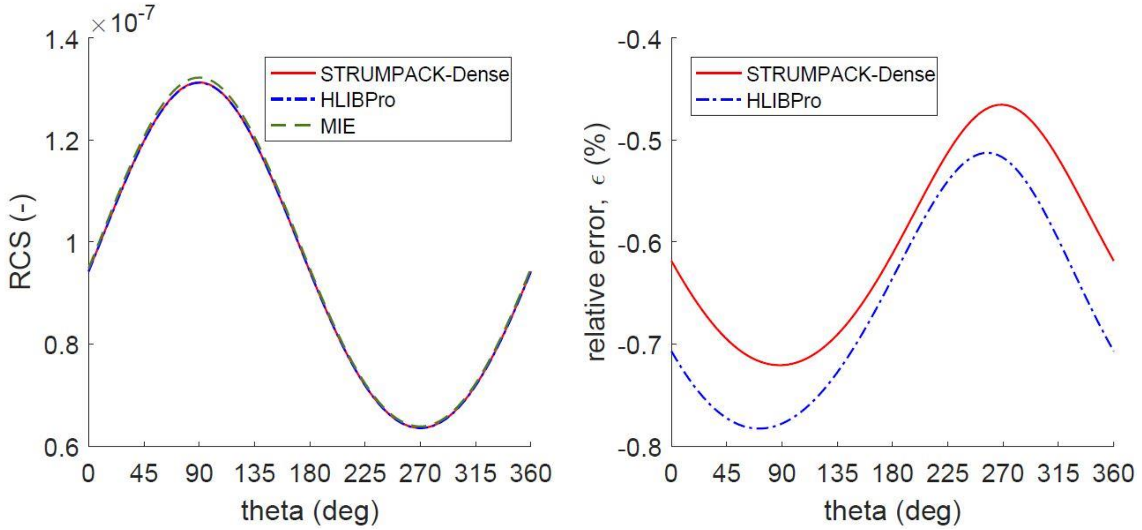

Figure 1: RCS along a circular target on the *yz* plane, radius 500 m. In green: MIE, in red: STRUMPACK-Dense, in blue: HLIBPro

Table 1: Compression Data. Full Matrix Size: 112.7 Gb, Compression Tolerance: $\epsilon = 1E - 07$

Library	Compression time (s)	*H*-matrix factors (MB)
HLIBPro	41473	35080 (31.1 %)
STRUMPACK-Dense	45579	10198 (9.1 %)

The accuracy of the results, as well as the compression ratio, show how the PEEC formulation can be effectively coupled with low-rank approximations: although the computational effort has been reduced, the necessary data in (3) to catch the right oscillations of the EM wave are preserved.

References

[1] R. Torchio, et al., “A 3-D PEEC Formulation Based on the Cell Method for Full Wave Analyses with Conductive, Dielectric, and Magnetic Media,” *IEEE Trans. Magn.*, vol. 54, no. 3, Mar. 2018, Art. no. 7201204.

[2] R. Kriemann, *HLIBPro*, Available on: http://www.hlibpro.com

[3] F. H. Rouet, X. S. Li, P. Ghysels and A. Napov, *STRUMPACK*, Available on: http://portal.nersc.gov/project/sparse/strumpack/

[4] G. Mie, “Beiträge zur Optik trüber Medien, speziell kolloidaler Metallösungen,” *Annalen der Physik*, vol. 330, no. 3, pp. 377-445, 1908.