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The House in Four Dimensions is a Theorem

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Abstract
This paper deals with the construction of a possible architectural design in four dimensions. To demonstrate the hypothesis of the real consistency of a new four-dimensional spatiality for architecture, the design process has been analyzed in form of a theorem, starting from the comparison between the elements of the project and the definitions of the referred geometry, acquired from the study of the mathematical treatises on the fourth dimension. The adopted method, following the scientific process, proceeded from the exposition of the problem, to the acquisition of the state of the art, to a description of the steps of the research, to the formulation of the hypothesis and finally, to the proof of the theoretical assumption.

Introduction: The Mathematical Logic of Composition

The present study is an in-depth analysis of the compositional process and how paradigms in architecture change through time. Referring to Kuhnian "paradigm shifts" (Kuhn 1962), we must acknowledge that the developments in the field of creative thinking are strictly related to so-called scientific revolutions. In fact, the design process, founded on the development of a compositional idea and the issues underlying the architectural design as well as a close relationship with the geometry that forms them, arises from theoretical choices that are the product of what Thomas Kuhn might call "normal scientific progress". What he presents as "development by accumulation" of accepted facts and scientific theories is interrupted by periods of revolutionary science, due to the discovery of "anomalies" leading to new paradigms. Both the history of architecture and the personal experience of architects highlight an effective change in reference-form as well as a revision of the ordering principles of compositional thought that goes hand-in-hand with the progress of science and knowledge.

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In order to focus on the connections between mathematics and architecture, rooted in the creative expression of the philosophy they share, we must take into account that in architecture the definitions, terms and logical procedures are weaker, the passages more imprecise and uncertain than they are in mathematics. Sometimes there is a clear and visible identity that derives from the geometric construction of the architectural shape; other times the relationship lies more in a conceptual reference that may be even be traced to other, different fields of knowledge.

The questions arising in the present study concern the relationship between the knowledge of space in architecture and its construction and practical use, between the rational control of the inexhaustible overall problem of how form (in terms of architectural shape) is generated and, ultimately, between mathematics and architecture, not so much in the retrospective "rereading" of works designed and built in a bygone era, but as a generating system of compositional ideas for contemporary interpreters.

The possible application of non-Euclidean and multi-dimensional geometries in architecture is the basis of the study of the genesis of a four-dimensional house design.

The Research: A Design of a House in Four Dimensions

Let's start with a simple question. Should architectural space have more than three dimensions? This theoretical question also has consequences in the field of aesthetics. Addressing it in the phenomenological, perceptive and linguistic fields is only one aspect of the problem, which may entail the need to tackle the ontological aspects of the reality of the physical world and, in general, questions about a mathematical reality that could also correspond to our physical reality.

For these reasons, in order not to be confined within an abstract speculation, the research about a new spatiality in architecture that goes beyond the limits of Euclidean geometry required a thorough study in purely geometric-mathematical terms of the texts of popular understanding of mathematics, in particular those on the fourth dimension, as well as those concerning non-Euclidean geometries.

Theoretical Frame of Reference

In the field of creative thinking, if we consider the different phases of the development of any design we can affirm that *the idea comes always first* because it arises from the author's personal research interests, nourished by cultural references and selected memories. What we design derives in some way from what we know. What we know derives from the schools we attended, from the books we read, the magazines we browsed, where we grew up, of course from our native language and culture, from the places we visited and from the photos and images we came away with. In general the idea comes from our life experiences, from our memories and from the process of knowledge that has linked them. The design idea is therefore not a pure idea, but the result of previous knowledge and its constant implementation over time.

My research began with the reading of the treaties on the fourth dimension published between the late nineteenth century and the early years of the twentieth. More recent treatises on the fourth dimension have not been published. An explanation for this was given some fellow teachers in my university who believe that higher-dimension geometries are only important in the context of non-Euclidean geometry and that in the courses of study at the faculty of mathematics the fourth dimension is merely mentioned (and not always, as something implicit) as part of some basic courses of analytic space geometry.[1]

The books collected and studied as the cultural mathematical references, together with other academic dissertations and literary works that explored this topic for artistic purposes, are as follows:

- Charles H. Hinton, *Scientific Romances* (1886);
- Charles H. Hinton, *A New Era of Thought* (1888);
- Esprit Jouffret, *Traité Élémentaire de Géométrie a Quatre Dimensions* (1903);
- Charles H. Hinton, *The Fourth Dimension* (1904);
- Alfred T. Schofield, *Another World; Or, The Fourth Dimension* (1905);
- Henry P. Manning, *The Fourth Dimension Simply Explained* (1910);
- Henry P. Manning, *Geometry of Four Dimensions* (1914);
- Cora L. Williams, *Creative Involution* (1916);
- Eric H. Neville, *The Fourth Dimension* (1921);
- Robert Reeves, *Space and the Fourth Dimension* (1922).

The authors were esteemed academics, lecturers and chairs of mathematics in important universities and colleges. They published treatises of analytic geometry which progressed from plane geometry to solid geometry and finally arrived at the exposition and systematization of a geometry in four dimensions and higher. Sometimes the books were philosophical dissertations on the nature of reality, introducing the concept of "multiverses", or simply the concepts of parallel universes and alternative realities. This is the case of Charles Hinton who, in parallel with the publishing of a rigorous, strictly geometrical exposition of the problem, developed an interest in the potential of science fiction to spread scientific knowledge also to non-mathematicians. Thus in 1886 he published his *Scientific Romances. Speculations on the Fourth Dimension.* The "romances" begin with the same questions we are examining: first of all, "how are our actions largely influenced by our theories?", and then "what is knowledge?". The same questions were addressed by Kuhn in his theoretical synthesis of scientific revolutions. In Hinton's *Romances* we read, "Why should there be three and only three directions? Space, as we know

[1] The interest of the present research is indeed the question linked to the description of the space, on which I am currently working on a book provisionally entitled *Define Space*, a collection of annotated interviews with mathematicians, physicists, architects, musicians, people of science and of literature, on the definition of space within their respective disciplines.

it, is subject to a limitation" (1886: 5–6). The question arises after the description of the possible movement in a room makes this limitation evident.

Among the mathematicians listed above, Henry Manning's *The Fourth Dimension Simply Explained* (1910) was particularly valuable for my development of an architectural design in four dimensions. In this book he introduced and commented on the results of a competition for the best definition of the fourth dimension sponsored by the editor of the prestigious magazine *Scientific American*. The competition was entered by 245 candidates; among the 22 selected writings published, one that is worthy of mention is that presented by architect Claude Fayette Bragdon[2] (Manning 1910: 91–99; Ellis 2015). Bragdon's paper is centered on the fact that in order to speak about the fourth dimension, the idea itself of space must be reformed and that we have to think of *spaces* (plural), "each one generated from one next below it" (Manning 1910: 91–92), in a way that makes it possible to formulate a definition of any dimension. The winner of the first prize was Lieut. Col. Graham Denby Finch of the US Corps of Engineering. After what we might call a "common" description of the fourth dimension, proceeding in analogy with lower dimensions, with a detailed exposition of each spatial property, he refers to the geometrical construction through rotations according to reference points, lines, surfaces, and volumes. Interestingly, Finch describes, in the novelistic manner depicted, for example, by Lewis Carroll in *Alice through the Looking-Glass*, the fact that freedom of movement is greater in hyperspace than in our space because a body in hyperspace can pass in and out an enclosed space without going through the surface surrounding it. This provides an interesting hint for the construction of a new, complex spatiality in architecture. The point, as stated towards the end of the winning essay, is whether hyperspace has a real physical existence. If so, concluded Finch, it would simplify certain scientific theories, and could help to explain certain chemical phenomena as well as the change of body in the polarization of light, thus introducing a fundamental necessary interchange between the research in physics and that in mathematics. The topic therefore revolves around the question of mathematical reality and in this particular case of the physical reality of four-dimensional geometry. The physical space and that of architecture are analogous and are certainly real. To answer the question of whether hyperspace, as a geometric mathematical reality, can be considered as having a real physical existence Manning concluded the long introduction to the book *The Fourth Dimension simply explained,* as follows:

> The question of the existence of space of four dimensions is one which we cannot escape. It may be well to remind the reader that this is not a mathematical question, though the most interesting of all. The possibility that we are part of a four-dimensional space with physical limitations which confine us to a three-dimensional space, and with limitations of our senses

[2] Claude Fayette Bragdon (1866–1946) was an American "organic" architect who created the "projective ornament," a system of geometric patterns designed to serve as a universal form-language integrating architecture, art, and design.

> which prevent us from perceiving anything outside of this space – this possibility excites the interest of all who are inclined to abstract speculation. Attempts may be made to discover physical proofs of such a space, to build up theories on its basis that will explain discoveries of modern Physics as yet but little understood, or by it to account for various mysterious phenomena. Most of us are satisfied that no real proofs of the existence of space of four dimensions will be found along these lines. … But we do say that the existence of space of four dimensions can never be disproved by showing that it is absurd or inconsistent; for such is not the case. Nor, on the other hand, will the most elaborate development of the analogies of different kinds ever prove that it does exist (Manning 1910: 40).

Architects are always inclined to abstract speculation. This is one of the reasons that motivates the development of a design *in search for the spatial quality of higher dimension*. The charm of the new geometries won artists in the first half of the twentieth century. They gave voice and form to new geometries. While figurative art works are well-known, lesser-known—except, of course, the novels by Edwin Abbott and Lewis Carroll—are those literary works describing environments built in the fourth dimension, which lead us to the awareness, just mentioned, that in the fourth dimension objects can pass through surfaces without breaking the surface. This is the case of the magic of the looking glass through which Alice crosses the three-dimensional reality (Carroll 1872). It is also the case of the Square of Abbott's novel *Flatland* (1884) who came first to an understanding of the third dimension and then of other many possible dimensions starting from the observation of the sphere crossing the horizon in Flatland. This is why the present design experimentation has built an architectural model starting from the reading of the fundamentals of geometry in multiple dimensions drawn by mathematicians who formulated their characters and properties.

The description of the design process in the form of a theorem aims to verify the consistency of the axiomatic system in order to establish the conditions of existence of a four-dimensional architectural space. If discovering and proving theorems represents one of the fundamental purposes of mathematical activity—because theorems are the most significant part of a theory—this study of the fourth dimension in architecture intends to analyze the compositional process as a result of a logical process that—between inspiration and method—follows analogous mathematical rules. It proceeds from a hypothesis/idea to its proof/construction, by adopting a hypothetical-deductive system.

The Theorem: the Construction of a House in Four Dimensions

Our term "theorem" is derived from the Greek $\theta\epsilon\acute{\omega}\rho\eta\mu\alpha$, meaning "what one looks at, that on which one speculates". That is to say, a theorem is a precept. If we refer to a dictionary of mathematical terms we read that a theorem is a statement that has been proven on the basis of previously established statements. In the same way, the hypothesis of a design that arises from an abstract idea such as the geometric fourth le pagg da 6 a 12 non fanno parte di questa anteprima per diritti di copyrigh

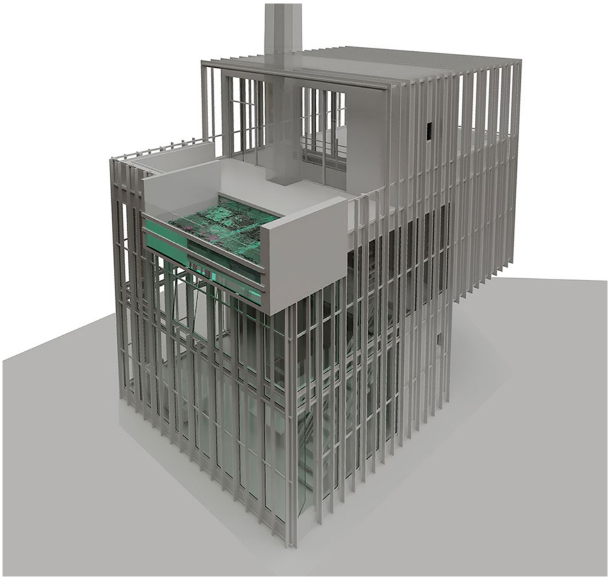

Fig. 5 Digital model of a four-dimensional house developing the concept born from the reading of Heinlein's novel, with reference to the description of the fourth dimensional geometry in the treatises of Hinton, Manning and Veronese

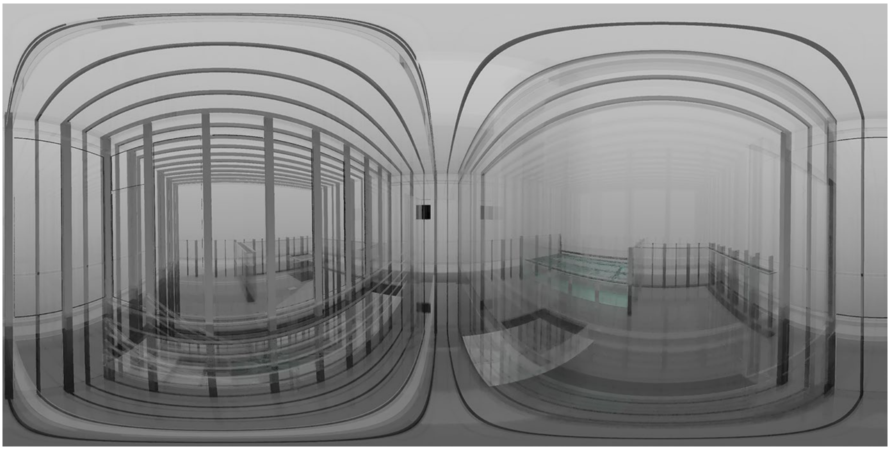

Fig. 6 Digital perspective of the interiors showing the identity and simultaneity of views of inside and outside and the identical dual view in both directions by simply using a semitransparent mirror wall situated along the vertical surface that intercepts the exit towards the garden terrace

Conclusion

The aim of this research is to discuss the innovative components of design thinking when it combines art and science, and in this case architecture and mathematics.

It is also centered on the question of whether the problem of building a four-dimensional (or higher) architectural space concerns only its proper visualization and the perceptual effects that results (Arnheim 1954), or if a projection in three-dimensional space of four-dimensional geometry can prove the hypothesis of the real consistency of a new four-dimensional spatiality for architecture.

In the first case, the theorem will find its proof in the field of projective geometry as a tool to design and previously make visible the quality of the space in terms of coherence with the concept. It is more than a *trompe l'oeil*, more than a fiction, but is not a magic formula.

In the second, the theorem will find its proof in the space–time fourth dimension, which is something already experienced in terms of kinematic perception without the need to declare it as a form of hyperspace. It is in fact impossible to understand architecture all at once, at a single glance. Only by walking through and observing an environment in time can we accumulate all the information necessary to comprehend it. Time is in all respects a scientifically recognized fourth dimension which has changed the vision of the universe and given a more precise exegesis of reality. In this regard it is worth reiterating that the concept of time as a fourth dimension derives from studies of non-Euclidean geometries and corresponds to the theory of relativity by contrasting a multidimensional, curved, open, dynamic, relative space to a three-dimensional, linear, closed space which is static and hierarchical. Thus a link is established between the physical reality and the reality of mathematics.

But the theorem, in the second case, can also be proved in the correspondence of the images and in the correspondence of the concepts, that is, the correspondence between the characteristics of the architectural and the geometric space. In the preface to his *Fondamenti di Geometria a più dimensioni* (1891),[8] Giuseppe Veronese argued that the foundations and laws of space in *n* dimensions are the same foundations and laws of space in general, and that the truths deduced from axioms are theorems.

Therefore, we can say that the four-dimensional house, having been deduced from the principle of the existence of the geometry of fourth dimension, and established as the foundation of the logical-deductive theory linked to the very structure of the design presented here (axiom), is a theorem.

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[8] I am not aware of any English versions of this work by Veronese, whose title translated from Italian is: "Foundations of Geometry in multiple dimensions and in multiples species of rectilinear units, set forth in elementary form".

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