Time Series Analysis

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Class 12

Financial data

- Empirical research highlights some stylized facts that financial data possess (e.g., indexes of major stock markets, exchange rates...)
- Consider the time series of the stock price of Generali, a well known company quoted in the Italian exchange.
- It is clear from the plot of the series that there is no stationarity in mean, rather there is a trend!
- Non stationarity is confirmed when looking at the correlogram that decays slowly.
- Testing the parameters may highlight the presence of a unit root.

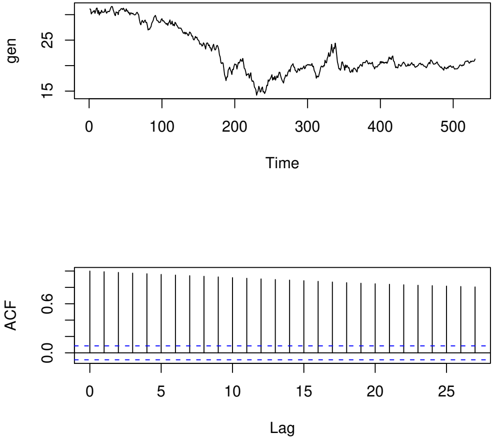

Figure: Daily time series of Generali in 26/10/2001-2/12/2003 and ACF.

- When studying financial time series, the interest is in the variations of the price levels rather than just their levels.
- Variations give information on profits and losses.
- Usually, people look at the log variations which are called in finance returns.
- One of the advantages in modelling returns is that they represent a good approximations of the percentage variations.
- Given the time series of the prices P_t, and defined the log prices $p_t = \log(P_t)$, returns are obtained as follows

$$r_t = \log P(t) - \log P(t-1) = \log \frac{P(t)}{P(t-1)} = (p_t - p_{t-1})(\times 100).$$

- From a statistical point of view, modeling first differences of $P(t)$ is equivalent to modeling those of $\log P(t)$, since, logarithm is a monotone and bijective function.
- With respect to the Box-Jenkins procedure, returns correspond to a first order difference that eliminates the non stationarity in mean.

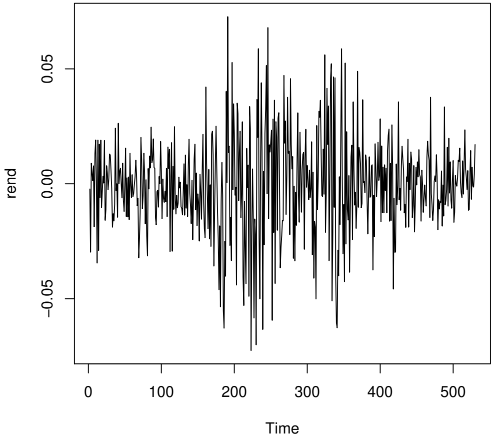

Figure: Generali returns.

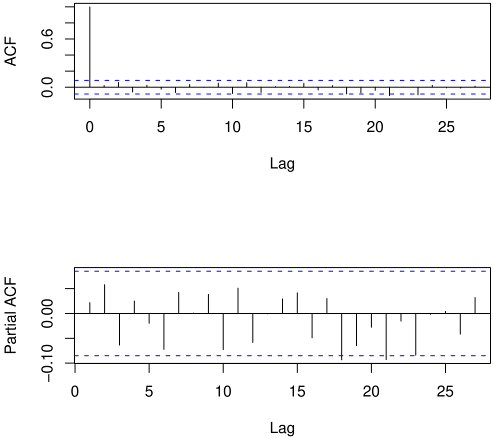

Figure: ACF and PACF of Generali returns.

- The plots show the main feature of the returns.
- They look uncorrelated, as confirmed by the Ljung-Box test.
- By the theory studied so far, we could conclude that series of returns is a *WN*...
- ...this means that returns cannot be modelled!
- What if we look at other forms of dependence different from the linear one ?
- Let's have a look at the correlogram of the squared returns.

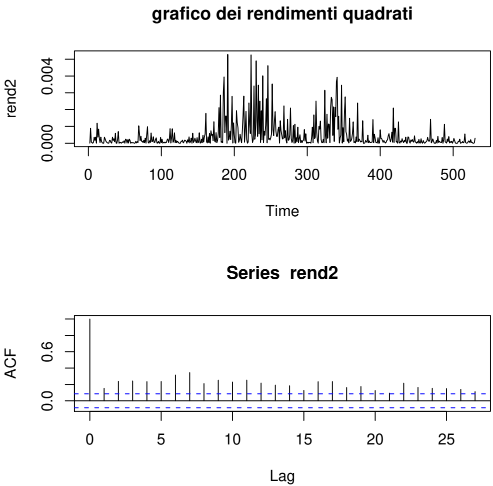

Figure: Squared returns and ACF.

- The plots show the presence of linear dependence in the squared returns.
- This suggests that the *ARMA* class of models can be applied to study the series of r_t^2.
- In practice, it is common to assume that returns have zero mean.
- The plot also suggests that it could be useful to model the variance of r_t, or their volatility.

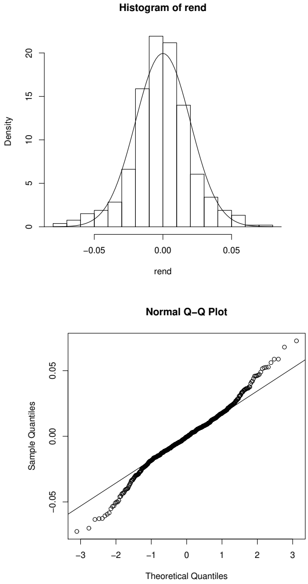

Figure: Histogram and qqplot of the returns series.

- The returns' distribution is not Gaussian, but it is leptokurtic, that is more pointed than the Gaussian case and with heavier tails.
- Returns' distribution often exhibits asymmetry as a result from the fact that negative shocks have higher impact on volatility than positive shocks, a phenomenon called leverage effect.
- The most used index to evaluate kurtosis is

$$K_r = \mathbb{E}\left[\frac{(r_t - \mu)^4}{\sigma^4}\right].$$

- For a Gaussian distribution the kurtosis is equal to 3. By contrast, financial data have

$$\hat{K}_r = \frac{1}{n}\sum_{i=1}^{T}\frac{(r_t - \hat{\mu})^4}{\hat{\sigma}^4} > 3.$$

- The most used index to evaluate asymmetry is

$$SK = \mathbb{E}\left[\frac{(r_t - \mu)^3}{\sigma^3}\right].$$

- For a Gaussian distribution the asymmetry is zero. By contrast, financial data have

$$\hat{SK} = \frac{1}{T}\sum_{i=1}^{T}\frac{(r_t - \hat{\mu})^3}{\hat{\sigma}^3} \neq 0.$$

- In the case of Generali,

$$\hat{SK} = -0.11$$

$$\hat{K} = 4.12$$

- This means that high and low returns in absolute value are more frequent than those expected for a Gaussian distribution.

- Such statistical aspect has strong financial implications. This means that high returns in absolute value represent huge gains or losses.
- The Jarque and Bera Normality test based on the sample skewness and kurtosis allows to test simultaneously if the skewness and kurtosis of the data are coherent with those under the Gaussian hypothesis

$$JB = \frac{T-1}{6}\left[\hat{SK}^2 + \frac{1}{4}(\hat{K}-3)^2\right] \sim_{H_0} \chi^2_2.$$

- Financial times series show a feature denoted as volatility clustering, that is periods of high volatility tend to persist and be followed by persistent periods of relative stability.

- Since returns are not constant, an intuitive risk factor is given by their variability. The higher their variability the higher the risks of losses and the chances of gains.
- A simple possible risk indicator is then given by the conditional variance

$$\sigma_t^2 = Var(r_t|\mathcal{F}_{t-1}) = \mathbb{E}(r_t^2|\mathcal{F}_{t-1}) - \mathbb{E}(r_t|\mathcal{F}_{t-1})^2$$

- Assuming zero mean returns (or $(r_t - \mu)$) then the conditional variance becomes

$$\sigma_t^2 = \mathbb{E}(r_t^2|\mathcal{F}_{t-1})$$

- Autocorrelation in the squared returns means that the value of the volatility "today" is informative about its value "tomorrow".
- therefore, it is important to capture and being able to forecast volatility.