Course Syllabus
Quantitative Financial Modelling
I semester - Fall 2022

Instructor: prof. Sergio Bianchi (sergio.bianchi@uniroma1.it)

Office: 1.09

Office Hours: Tuesday 10 –11:30 a.m. (or by appointment)

Office Phone: 0649766507

Class Hours
Tuesday 12 pm - 2 pm (Aula Matematica Memotef, first floor)
Wednesday 10 am - 12 pm (Aula Master Memotef, fifth floor)
Thursday 10 am - 12 pm (Aula Matematica Memotef, first floor)

Textbooks
[a] Shreve S.E., Stochastic Calculus for Finance I: The Binomial Asset Pricing Model, Springer Finance, 2005
[b] Shreve S.E., Stochastic Calculus for Finance II: Continuous-Time Models, 2nd edition, Springer Finance, 2004
[c] Musiela M., Rutkwoski M., Martingale Methods in Financial Modelling, Stochastic Modelling and Applied Probability, Springer, 2009
[d] Oosterlee C.W., Grzelak L.A., Mathematical Modeling and Computation in Finance (with Exercises and Python and MATLAB computer codes), World Scientific, 2020
[e] Brigo D., Mercurio F., Interest Rate Models – Theory and Practice (With Smile, Inflation and Credit), Springer, 2006

Additional Materials (in parentheses the sub-directory of the classroom page)
- Slides used during the classes (Classes)
- Financial time series (Data)
- Papers focusing on specific topics covered during the course (Further readings)
- Matlab functions and software (Software)
- Websites of interest (Websites)

The additional materials will be available at https://classroom.google.com/c/Mzg3NjU2OTMzOTY5

Prerequisites
Students should know and master:
- the topics of a basic course in Financial Mathematics (with a special concern to the simple and compound interest scheme, both in discrete and continuous time, and to the market structure)
- the basics of probability theory and of stochastic processes theory (a short recall will be given at the beginning of the course)

Final and grade policy
Final Exam:
- Project (with class presentation)
- Individual oral examination

Weights:
- 20% Course participation (attendance and assignments)

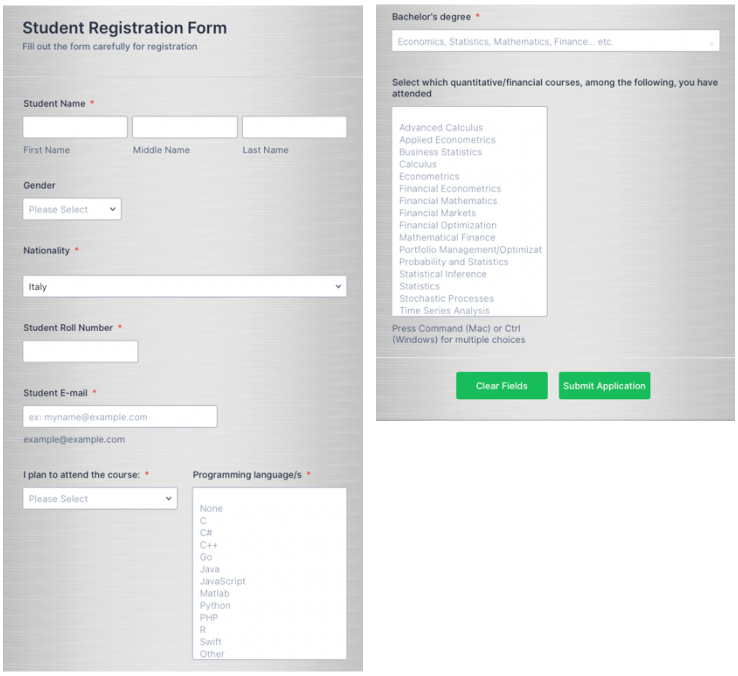
- 30% Project evaluation
- 50% Individual oral examination

Grading scale:

US Scale	%	IT Scale
A	93-100	29-30L
A-	90-92	27-28
B+	87-89	26
B	83-86	25
B-	80-82	24
C+	77-79	23
C	73-76	22
C-	70-72	21
D+	67-69	20
D	63-66	19
D-	60-62	18
F	≤ 60	≤ 18

Registration Area

In Google Classroom a Registration Area is active. Students will be asked to fill a form reporting some details which will be kept in due consideration to constitute the groups for each project and adjust the focus of the lessons, to avoid going into too much obvious explanation or, conversely, taking for granted topics with which the class is less familiar. Students are asked to register within the first week of the course.



Course Objectives

The course provides some of the most relevant theoretical tools for quantitative analysis of financial markets at the advanced master's level. Ideally, it will be structured into three parts:

Part 1. A prerequisite to dealing with the mathematical modeling of financial markets is their knowledge, hence the first part of the Course will be dedicated to an overview of the structure of financial markets, the types of contracts traded therein, and the general principles of modeling the price dynamics of financial assets. Special emphasis will be given to topics such as the Efficient Market Hypothesis and its analytical relationship to the martingale model, the financial markets microstructure, and the notion of arbitrage in market models of increasing generality. In this part of the course, students will also develop the ability to analyze, recognize and test the main stylized facts, whose parry and thrust drive much modeling of financial time series.
Part 2. In its second part, the course will cover pricing models in which price evolves both in discrete time, as in the so-called binomial model, and in continuous time, such as the one leading to the famous Black-Scholes formula. The analysis of such models is unified by the fundamental principle of no arbitrage opportunities, which allows for formulas for valuation and hedging (pricing and hedging) of various derivative securities. The role and the calculation of several measures of sensitiveness (the so-called Greeks) of the option price will also be focused and some financial puzzles such as the behavior of implied volatility and so-called rough volatility will be explored.
Part 3. The third part of the course will be devoted to yield curve modeling. Some of the basic one-factor spot-interest rate models will be reviewed (time-homogeneous models: Vasicek, Cox Ingersoll Ross (CIR), Exponential Vasicek (EV); models with time-varying coefficients: Hull and White's extended Vasicek model, extensions of the CIR model, Black and Karasinski's (BK) extended EV model). A hint will also be given to the Heath-Jarrow-Morton (HJM) framework as a theoretical approach for developing a no-arbitrage interest-rate theory.

Expected learning objectives and skills

- Access, organize, and analyze with advanced quantitative methods and tools the relevant patterns exhibited by financial data.
- Critically analyze, question, and evaluate implications of alternative and new financial models to address trading and risk management issues.
- Develop an awareness of the implications that potential abuse of financial contracts can have in systemic risk and substantial negative spillovers on society.
- Convey mathematical and financial models clearly, and in high-quality written form.
- In-depth understanding of the no-arbitrage principle and of its role in asset pricing theory.
- Acquire a robust conceptual knowledge of the fundamental issues determining the valuation and behavior of the main derivatives contracts.
- Develop knowledge and skill sufficient for correct application and analysis of continuous-time stochastic models involving stochastic integrals and stochastic differential equations.

Assignments and Assessment

Students will be asked to turn in three assignments at the end of each month of course. The assignments will concern the topics covered in each part of the course. For each assignment it will be settled a non-extendible deadline at the midnight of the due date. The evaluation will concern correctness, clearness, effectiveness of the individual project. The assignments will concur to determine the final mark for a share equal to 20% (see grade policy).

Group project

Provided that the number of students attending allows for this, as a part of the final exam, students will be subdivided into small groups of four-five participants. Each group will be asked to develop a project on a topic randomly chosen among those covered by the course. Each group designates a representative for all communication with the instructor. The project is articulated into three parts:

- a short paper (no longer than six, one-sided A4 pages, Times New Roman 12 or equivalent);
- a computer program implementing the task outlined in the project theme;
- a PPT/PDF/LATEX presentation (no longer than 15 slides) that will be used by the group to illustrate the project to the class. The duration of each presentation is 25 minutes + 5 minutes of discussion.

Along with the above materials, upon completion of the work, each group participant will individually send to the instructor a self-assessment sheet and an evaluation form of the other group members. The forms will be made available with the Project requirements. **The information on the self-assessment sheet and the evaluation form will be strictly confidential and in no way will be disclosed to any student enrolled in the course.** It is therefore recommended that evaluations be made with the utmost intellectual honesty.
The project will concur to determine the final mark for a share equal to 30% (see grade policy).

Preliminary Course Calendar

Note:
"Theory" and "Applications" refer to Chapters/paragraphs of the suggested books; students can find "Data and software" and "Websites" in the relevant section of Google Classroom; samely, "Further readings" refer to papers, presentations, or other material that students can find in the section Further readings of Google Classroom.

Week	Topics
1.	Stochastic processes. Joint distribution function. Probability mass function of order k. Ensemble and time mean. Gaussian processes. Discrete-time Martingales. Total variation. Quadratic variation. Continuous-time limit of the random walk model. Standard and Fractional Brownian motion (distribution, increments, continuity, non-differentiability, martingale property, quadratic variation, law of iterated logarithm, scaling, and self-similarity). Geometric Brownian motion. GBM as a stochastic model for the value of a stock. Simulations.

Suggested additional readings:

Theory	**[a]** Ch. 2, 2.4, 2.5, Ch. 5, 5.1, 5.2; **[b]** Ch. 3
Applications	**[d]** Ch. 1, 1.2; Ch. 2, 2.1, 2.3, 9.1, 9.1.1
Data and software	Fraclab (Tool running in Matlab licensed by INRIA) Scaling.m (matlab routine)
Further readings	• Fractional Brownian motions, fractional noises and applications (B.B. Mandelbrot, J.W. Van Ness) • Arbitrage with fractional Brownian motion (L.C.G. Rogers) • Simulation of fractional Brownian motion (master's thesis Ton Dieker)
Websites	https://kluge.in-chemnitz.de/tools/sharesim/black_scholes.php https://www.maa.org/press/maa-reviews/kiyosi-it-selected-papers https://project.inria.fr/fraclab/ (to download Fraclab)

Week	Topics
2.	Itô process. Financial interpretation of the Itô integral. Itô's lemma. Higher-Dimensional Itô's Lemma. Examples. Euler-Maruyama approximation method.

Theory	**[b]** Ch. 4, 4.1, 4.2, 4.3, 4.4; **[c]** Appendices B; **[e]** Appendices C, C.1, C.2, C.3, C.4
Applications	**[d]** Ch. 1, 1.3, 1.4; Ch. 2, 2.1, 9.1.2, 9.2
Further readings	• Robert Jarrow and Philip Protter, A short history of stochastic integration and mathematical finance: The early years, 1880–1970, Institute of Mathematical Statistics Lecture Notes – Monograph Series, 45, 75–91, (2004) • Hans Föllmer, On Kiyosi Itô's Work and its Impact

	• Vlad Gheorghiu, Itô calculus in a nutshell
Websites	https://www.maa.org/press/maa-reviews/kiyosi-it-selected-papers

3. Financial markets. Taxonomy, size, and instruments. Modeling financial markets. Assumptions. Return and risk. Efficient Market Hypothesis. A first look at arbitrage. Law of one price. Stylized facts.

Theory	**[c]** Ch. 1, **[e]** Ch. 2, from 2.1 to 2.6
Applications	
Software	Stylized_facts.m
Further readings	• Fama E., Efficient capital markets: A review of theory and empirical work, The Journal of Finance 25 (2) (1970) 383-417. • Malkiel B.G., The Efficient Market Hypothesis and Its Critics, Journal of Economic Perspectives, 17, 1, 59–82 (2003) • Guillaume D.M., Dacorogna M.M., Davè R.R:, Müller U.A., Olsen R.B., Pictet O.V., From the bird's eye to the microscope: A survey of new stylized facts of the intra-daily foreign exchange markets, Finance and Stochastics, 1, 95–129 (1997) • Lux T., Chapter 3 - Stochastic Behavioral Asset-Pricing Models and the Stylized Facts, in Handbook of Financial Markets: Dynamics and Evolution, 161-215 (2009) • Cont R., Empirical properties of asset returns: stylized facts and statistical issues, Quantitative Finance, 1(2), 223-236 (2001) • Rickles D., Econophysics and the Complexity of Financial Markets, Philosophy of Complex Systems, 531-565 (2011)
Websites	http://finance.martinsewell.com/stylized-facts/ https://www.youtube.com/watch?v=wnCxllQjT-s (A Random Walk Down Wall Street, B. Malkiel, Talks at Google)

4. Financial derivatives. Taxonomy. Forward and Futures. Standardization and mark to market. Forward (future) price (via no arbitrage argument). Hedging. Swaps. Interest Rate Swaps. Pricing. Use of swaps in hedging, speculation, and comparative advantage. Options. Taxonomy. A first example of pricing via no arbitrage argument.

Theory	**[b]** Ch. 5, 5.6 **[c]** Ch. 1, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, Ch. 9, 9.4
Applications and software	**[d]** Ch. 3, 3.1.1
Papers	René M. Stulz, Demystifying Financial Derivatives, The Milken Institute Review, 2005
Websites	https://ec.europa.eu/info/business-economy-euro/banking-and-finance/financial-markets/post-trade-services/derivatives-emir_en

5. Risk-Neutral Probability Measure (or Equivalent Martingale Measure). Radon-Nikodým derivative process. Put-Call parity. Elementary Market Model. General single-period market models. Trading strategies and arbitrage-free models. Wealth process of a trading strategy. Gain process. Discounted Stock price and value process. Discounted gain process. Arbitrage.

Theory	**[a]**, Ch. 3, 3.1, 3.2 **[b]** Ch. 4, 4.5.6, Ch. 5, 5.1, 5.2, 5.3
Applications and software	**[d]**, Ch. 2.3, Ch. 7, 7.2
Papers	• Buhlmann H., Delbaen F., Embrechts P., Shiryaev A., No arbitrage Change of Measure and Conditional Esscher Transforms, 1996
Websites	

6. First fundamental Theorem of Asset Pricing. Market model with arbitrage. Stochastic volatility model. Completeness of market models. Algebraic criterion for market completeness. Second fundamental Theorem of Asset Pricing (probabilistic criterion for market completeness).

Theory	**[b]** Ch. 5, 5.4 **[c]** Ch. 2, 2.6, 2.7
Applications and software	
Papers	• Yan Zeng, Fundamental Theorem of Asset Pricing in a Nutshell: With a View toward Numéraire Change (2009)
Websites	http://pi.math.cornell.edu/~mec/Summer2008/spulido/fftap.html

7. Cox-Ross-Rubinstein (CRR) pricing model. Bernoulli process. Bernoulli Counting process. The CRR Call Option pricing formula (with Cox-Ross-Rubinstein parametrization and with Jarrow-Rudd parametrization).

Theory	**[a]** Ch. 1, 1.1, 1.2, 1.3, 1.4 **[c]** Ch. 2, 2.1, 2.2
Applications and software	
Papers	• Cox J.C., Ross S.A., Rubinstein M., Option pricing: A simplified approach, Journal of Financial Economics, 7(3), 1979, 229-263 • Rutkwoski (slides), Binomial market model
Websites	https://financial-calculators.com/options-calculator https://www.macroption.com/jarrow-rudd-formulas/

8. Black-Scholes-Merton model. Assumptions for the risk-free bond, for the stock price and for the self-financing trading strategy. Martingale measure for the stock. Admissible trading strategies. Attainable contingent claims. Black-Scholes option pricing formula (based on both replicating strategy and risk-neutral measure). Terminal boundary conditions.

Theory	**[b]** Ch. 4, 4.5, 4.6 **[c]** Ch.2, 2.3, 3
Applications and software	**[d]** Ch. 3, 3.1.2, 3.1.3, 3.2, 3.3
Papers	• Black, Fischer; Myron Scholes (1973). "The Pricing of Options and Corporate Liabilities". Journal of Political Economy. 81 (3): 637–654 • Ajay Shah, Black, Merton and Scholes: Their work and its consequences (1997)
Websites	

9. The Greeks (delta, gamma, theta, rho, vega). Delta-neutral and gamma-neutral portfolios. Volatility. Implied volatility. Volatility “measures”

Theory	**[b]** Ch. 4, 4.5.5 **[c]** Ch. 7, 7.1, 7.6
Applications and software	**[d]** Ch. 3, 3.2.1, Ch. 4, 4.1, 4.2 4.3, Ch. 8, Ch. 9.5
Papers	• Elisa Alòs, Maria Elvira Mancino, Tai-Ho Wang, Volatility and volatility-linked derivatives: estimation, modeling, and pricing, Decisions in Economics and Finance (2019) 42:321–349 • Various Authors, Taxonomy of Global Risk, Uncertainty, and Volatility Measures, International Finance Discussion Papers, Board of Governors of the Federal Reserve System, 1216, 2017 • Gatheral, J. (2006). The volatility surface: a practitioner's guide (Vol. 357). John Wiley & Sons.
Websites	https://www.optionseducation.org/toolsoptionquotes/optionscalculator https://www.macroption.com/option-greeks/ https://www.macroption.com/volatility/

<table>
<tr><td>10.</td><td>Interest rates models. Simple spot rate. Continuously compounded spot rate. Simple forward rate. Continuously compounded forward rate. Instantaneous forward rate. Forward curve. Short rate. Forward vs. future rates. Forward Rate Agreement (FRA). Term structure (or yield curve). Bootstrapping. Stochastic models for bond prices. arbitrage-free family of bond prices.
<table>
<tr><td>Theory</td><td>[b] Ch. 9, 9.4 [c] Ch. 9, 9.1, 9.5. 9.6</td></tr>
<tr><td>Applications and software</td><td>[d] Ch.11</td></tr>
<tr><td>Papers</td><td>Lesniewski A., Interest Rate and Credit Models (slides)</td></tr>
<tr><td>Websites</td><td>https://financetrainingcourse.com/education/2016/10/bootstrapping-zero-curve-forward-rates/</td></tr>
</table></td></tr>
<tr><td>11.</td><td>Short-rate models. Affine models. Stylized facts on yield curve. Vasiček model. Cox-Ingersoll-Ross (CIR) model. Dothan model. Ho-Lee model. Hull-White extension of Vasiček model. Hull-White extension of CIR model. Instantaneous forward rate models (Heat-Jarrow-Morton, HJM, framework). HJM forward rate specification. Theorem HJM.
<table>
<tr><td>Theory</td><td>[b] Ch.10, 10.1, 10.2, 10.3 [c] Ch. 7, 7.4, Ch.9, 9.6, Ch.10, 10.1, 10.2, Ch. 11, 11.1 [e] Ch. 3, 3.1, 3.2, 3.3, 3.4, Ch. 5</td></tr>
<tr><td>Applications and software</td><td>[d] Ch.8, 8.1, 8.2, 8.3, Ch. 9, 9.3, 9.4, Ch.11</td></tr>
<tr><td>Papers</td><td>Chan K.C., Karolyi G.A., Longstaff F.A., Sanders. A.B., An Empirical Comparison of Alternative Models of the Short-Term Interest Rate, The Journal of Finance, 47(3), 1992, 1209-1227</td></tr>
<tr><td>Websites</td><td>https://data.oecd.org/interest/short-term-interest-rates.htm
https://www.ecb.europa.eu/stats/financial_markets_and_interest_rates/euro_short-term_rate/html/index.en.html
https://www.federalreserve.gov/releases/h15/</td></tr>
</table></td></tr>
<tr><td>12.</td><td>Project presentation and discussion</td></tr>
</table>