

Let $X_t = \varphi X_{t-2} + \varepsilon_t$ where $\varepsilon_t \sim WN(0, \sigma^2)$

1. Find conditions on φ to have a stationary process.

2. Calculate $f(h)$ for $h=0, h=1, h=2, h=3, h=4$.

Is there any rule to write down $f(h)$ vs h ?

Let $X_t = a X_{t-1} + \varepsilon_t$ where $|a| < 1$ and $\varepsilon_t \sim WN(0, \sigma^2)$

Assume that $Z_t = X_t - X_{t-1}$, calculate:

1. $E(Z_t)$

2. $\text{Var}(Z_t)$

3. $\text{Cov}(Z_t, Z_{t-k})$

Let $X_t = \varepsilon_t + b(\varepsilon_{t-1} + \varepsilon_{t-2} + \dots)$ an $\pi A(\infty)$ process where b is constant.

1. Calculate $E(X_t)$, $\text{Var}(X_t)$ and $f(h)$ of X_t and evaluate if the process is stationary.

2. Calculate the first differences and show that the resulting process is $\pi A(1)$. Find its mean, variance and ACF.