Statistics fo Health Economics

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February 24, 2020

Main topics of the course

- Probability
- R
- Statistical inference
- Cost-effectiveness analysis in health economics
- Decision modeling in health economics

Main References

- Introduction to Statistical Thought. Micheael Lavine
  - The book is free from the the author's web site Link
- Bayesian Methods In Health Economics. Gianluca Baio
  - Author's web site Link
- Decision Modeling for Health Economic Evaluation. Andrew Briggs, Mark Sculpher, Karl Claxton
  - Book web site Link
- Slides and other material can be downloaded from my institutional web page Link

  https://web.uniroma1.it/memotef/statistics-health-economics-andrea-tancredi

Exam evaluation

- Presentation of a research paper published on international journals:
  - Health economics Link
  - Journal of health economics Link
  - Value in health Link
  - ....
- Homeworks
  - Statistical exercises
  - R exercise
  - ...
- Oral exam

Probability

- Let $\mathcal{X}$ be a set $\mathcal{F}$ a collection of subset of $\mathcal{X}$. A *probability measure*, or simply a *a probability* on $(\mathcal{X}, \mathcal{F})$ is a function

$$\mu : \mathcal{F} \to [0, 1]$$

  - To every set in $\mathcal{F}$ μ assigns a probability between 0 an 1
  - μ is a set function

- To be a probability μ must satisfy

  1. $\mu(\emptyset) = 0$ ($\emptyset$ is the empty set)
  2. $\mu(\mathcal{X}) = 1$
  3. if A_1 e A_2 are disjoint then $\mu(A_1 \cup A_2) = \mu(A_1) + \mu(A_2)$

- We can show that property 3 holds for any finite collection of disjoint sets

$$\mu(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} \mu(A_i)$$

- It is common practice to assume that 3 hold for countable collections of disjoint sets

$$\mu(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{n} \mu(A_i)$$

- When $\mathcal{X}$ is a finite or countably finite set, μ is a discrete probability
- When $\mathcal{X}$ is an interval, either finite or infinite, μ is a continuous probability
- In the discrete case, $\mathcal{F}$ usually contains all possible subsets of $\mathcal{X}$. But in the continuous case, technical complications prohibit $\mathcal{F}$ from containing all possible subsets of $\mathcal{X}$.

- In practical examples $\mathcal{X}$ is the set of outcomes of an experiment and μ is determined by experience, logic or judgement
- Rolling a six-sided die, the set of outcomes is $\mathcal{X} = \{1, 2, 3, 4, 5, 6\}$.
  - If we believe the die to be fair we may assign

$$\mu\{1\} = \mu\{2\} = \mu\{3\} = \mu\{4\} = \mu\{5\} = \mu\{6\} = \frac{1}{6}$$

  - The laws of probability then imply, e.g.

$$\mu\{1, 2\} = 1/6 + 1/6 = 1/3 \;\; \mu\{1, 2, 3\} = 1/6 + 1/6 + 1/6 = 1/2$$

- Setting $\mu\{i\} = 1/6$ is not automatic simply because a die a six faces, we have to believe the die to be fair. For example we could also have

$$\mu\{1\} = \frac{1}{2} \quad \mu\{2\} = \mu\{3\} = \mu\{4\} = \mu\{5\} = \mu\{6\} = \frac{1}{10}$$

- We usually use the word probability or the symbol P in place of μ.
- Equivalent statements
  - The probability that the die lands 1
  - $P(1)$
  - $P([\text{the die lands 1}])$
  - $\mu\{1\}$
- We also use the word distribution in place of probability measures

The game of craps

- Craps is a gambling game played with two dice
  - For the dice thrower (shooter) the object of the game is to throw a 7 or an 11 on the first roll (a win) and avoid throwing a 2, 3 or 12 (a loss).
  - If none of these numbers (2, 3, 7, 11 or 12) is thrown on the first throw (the Come-out roll) then a Point is established (the point is the number rolled) against which the shooter plays.
  - The shooter continues to throw until one of two numbers is thrown, the Point number or a Seven.
  - If the shooter rollsthe Point before rolling a Seven he/she wins, however if the shooter throwsa Seven before rolling the Point he/she loses.

- Ultimately we would like to calculate

$$P(\text{shooter wins})$$

but for now let's calculate

$$P(\text{shooter wins on Come-out roll}) = P(7 \textit{ or } 11)$$

- The set of possible outcomes is the set of ordered pairs (d_1, d_2) where $d_1 \in \{1, 2, 3, 4, 5, 6\}$ is the outcome of the first die and $d_2 \in \{1, 2, 3, 4, 5, 6\}$ is the outcome of the second die,

$$\begin{array}{cccccc}
(6,6) & (6,5) & (6,4) & (6,3) & (6,2) & (6,1) \\
(5,6) & (5,5) & (5,4) & (5,3) & (5,2) & (5,1) \\
(4,6) & (4,5) & (4,4) & (4,3) & (4,2) & (4,1) \\
(3,6) & (3,5) & (3,4) & (3,3) & (3,2) & (3,1) \\
(2,6) & (2,5) & (2,4) & (2,3) & (2,2) & (2,1) \\
(1,6) & (1,5) & (1,4) & (1,3) & (1,2) & (1,1)
\end{array}$$

- If the dice are fair, then the pairs are all equally likely and $P(d_1, d_2) = 1/36$ since thera are 36 pairs.
- By the probability law

$$P(7 \text{ } or \text{ } 11) = P(7 \bigcup 11) = P(7) + P(11)$$

and

$$\begin{aligned} P(7) &= P((6,1) \cup (5,2) \cup (4,3) \cup (3,4) \cup (2,5) \cup (1,6)) \\ &= P(6,1) + P(5,2) + P(4,3) + P(3,4) + P(2,5) + P(1,6) \\ &= 6/36 \end{aligned}$$

$$P(11) = P((6,5) \cup (5,6)) = P(6,5) + P(5,6) = 2/36$$

so that $P(7 \,\mathrm{or}\, 11) = 6/36 + 2/36 = 8/36 = 2/9$