

Stochastic Processes

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Surname and name _____

Identification number _____

Solve 3 exercises. Time: 2 hours

1. Let X_1 and X_2 be independent discrete uniform random variables assuming values on $\{1, 2, 3, 4, 5\}$. Let $Y = \max(X_1, X_2)$ be the maximum of (X_1, X_2)

- A. Find the distribution of Y
 - B. Find the expected value of Y
 - C. Find $E(Y|X_2 = 2)$
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2. Let (X, Y) be a joint random variable such that $P(X = 1) = \omega$ and $Y|X = 1$ is a Bernoulli random variable with probability $1 - e^{-\lambda}$ while $Y|X = 0$ is Poisson(λ)

- A. Find the expected value of Y
 - B. Write the distribution of Y
 - C. Find $P(X = 1|Y = 1)$
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3. Let (X, Y) be a random variable with density

$$f(x, y) = \begin{cases} k(x + y) & 0 \leq x \leq 3, \quad 0 \leq y \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

- A. Find the value of k .
 - B. Calculate $P(X > 2Y)$
 - C. Say if X and Y are independent
 - D. Find the distribution of $X|Y = 2$
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4. Consider the random vector (X, Y) with density

$$f(x, y) = \begin{cases} 2 \exp(x + y) & x \leq y \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

- A. Find the marginal density of X and Y

- B. Find the expected value $E(X|Y = y)$
 - C. Find the expected value of $Z = XY$
 - D. Find the distribution of $W = Y - X$
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5. You have 2 urns, A and B . In the urn A there are 3 white balls, in the urn B there are 2 black balls. Let X_0 be the number of white balls in urn B , that is $X_0 = 0$. Suppose that you modify the urn composition in the following way. In each trial, you first take a ball from urn B and put the ball in the urn A , then you take a ball from urn A and you put the ball in urn B . Let X_1 be the number of white balls in urn B after the first complete switching and X_n be the number of white balls in urn B after n trials.
- A. Write the distribution of X_1
 - B. Write the transition matrix P of the Markov Chain X_n
 - C. Find the invariant distribution of the chain
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