Stochastic Processes

Written exam July 2021

Surname and name:

Solve at least 2 of the following 5 exercises

1.1 A census shows that in the USA, 80 % of the daughters of working women also have a steady job, while only 30 % of the daughters of unemployed women have a steady job.

  A. Write the transition matrix for the Markov process describing the working status of an american woman

  B. In the lung run, what is the proportion of american women with a steady job

1.2 Bob eats his lunch every day near his office. He goes to three places: a Chinese restaurant (C), a Mexican Tacos (T) and a salad place (I). How he chooses where to go depends on how much he ate the day before according to the transition matrix

$$\begin{pmatrix} & C & T & I \\ C & .15 & .6 & .25 \\ T & .4 & .1 & .5 \\ I & .1 & .3 & .6 \end{pmatrix}$$

Last Monday he ate Tacos.

  A. What is the probability that Bob will eat in each one of the three places next Wednesday (that is after four days)

  B. In the long run, what is the distribution of the three restaurants

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2 The joint density of the double random variable (X, Y) is

$$f_{X,Y}(x, y) = Cy \exp\{-xy\} \quad 0 < x < 1; 0 < y < 1$$

and 0 otherwise.

  A. Find the constant Cì such that $f_{X,Y}$ is a density

  B. Calculate $P(1/2 < X < 1, 0 < y < 1/2)$

  C. ì Find the marginal density of X and Y

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3 There is a bank with only two clerks. Three customers arrive, Anna, Bruna, and Cinzia, one after the other and enter in this order. Anna and Bruna go to the two clerks while Cinzia waits for the first clerk available.

Suppose that the two clerks work independently and that for the first clerk the duration times of each operation are independent Exponential random variable with mean equal to 3 minutes while for the other clerk the duration times are independent Exponential with mean equal to 6 minutes.

   A. Find the expected value of the random variable { Time spent by Cinzia inside the bank}?[1]

   B. What is the average amount of time it takes for all three customers to be out of the bank?

   C. What is the probability that Anna, Bruna and Cinzia respectively will be the last to leave the bank?

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4 N dice are rolled where N is a discrete Uniform random variable that takes values on $\{1, 2, \ldots, 10\}$ Let S_i be the result for the ith die and let $S = \sum_{i=1}^{N} S_i$ be the total sum.

   A. Calculate $P(N = 2|S = 4)$

   B. Calculate $P(S = 4|N$ è pari$)$

   C. Calculate $P(S = 4, S_1 = 1|N = 2)$

   D. Calculate $E(S)$

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5 The probability that any child in a certain family will have blue eyes is 1/4, and this feature is inherited independently by different children in the family. Suppose that the family have five children

   A. What is the most probable number of children with blue eyes?

   B. If it is known that at least one of these children has blue eyes, what is the probability that at least three of the children have blue eyes?

   C. If it is known that the youngest child in the family has blue eyes, what is the probability that at least three of the children have blue eyes?

   D. Explain why the answer in part (C) is different from the answer in part (B).

FINE COMPITO

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[1]Consider also the duration of her bank operation ...