STOCHASTIC PROCESSES

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Surname and name:

Identification number:

Time: 2 hours. Solve at least 2 exercises.

1. Let (X, Y) be a random variable with joint density

$$f_{X,Y}(x,y) = \begin{cases} 4x^2y + 2y^5 & 0 \le x \le 1; 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}.$$

Consder $Z = X + Y^2$ and $W = X - Y^2$

A. Find the joint density of (Z, W) and specify the set where the density is positive

B. Find the marginal density of Z.

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2. You throw three regular dice. Let S be the random variable indicating the total score

A. Indicate the set of values taken by S. Calculate the expected value and the variance of S.

B. Find $P(S = 10)$

C. Find $P(S = 10|\text{Dice A is equal to 4})$

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3. The duration time in days of an electronic component is a Gamma random variable X with parameter $\alpha = 5$ and $\theta = 1/10$, i.e.

$$f_X(x) = \frac{\theta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\theta x}, \qquad x > 0.$$

When the component breaks, it is immediately replaced by a new one.

A. Use the central limit theorem to obtain the probability that 40 components are enough to last at least 6 years (assume that each year has 365.25 days)

B. By the central limit theorem, find an expression for the expected value of the number of components which will be used to last at least 6 years. (Use the fact that for a positive r.v Y, $E(Y) = \sum_{y=0}^{\infty} P(Y > y)$)

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4. Let (N, Y) be such that $Y|N = n \sim Binomial(n, p)$ and $N \sim \text{Poisson}(\lambda)$.

   A Write the probability mass function of the double random variable. Specify the points where the probability mass function is positive

   B Find the marginal distribution of Y

   C Find the conditional distribution of $N|Y = y$

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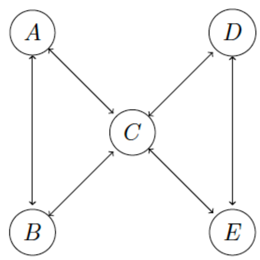
5. A particle follows a random walk on the set described in the following figure



At time n, the particle moves to one of the adjacent nodes with equal probability

   A. Write the transition matrix of the chain

   B. Find the invariant distribution

   C. Suppose that $X_0 = C$. Find the expected value of the first passage time in A

FINE COMPITO
