

Stochastic Processes

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Surname and name:

Identification number

Time 2 hours. Solve 2 exercises

1. Consider a random walk with $S_0 = 0$ and

$$S_n = S_0 + X_1 + X_2 + \cdots + X_n \quad n \geq 1$$

where the random variables X_i are independent and identically distributed each taking either the value -1 with probability q or the value 1 with probability p .

Find

- A. $P(S_1 \geq 0, S_2 \geq 0, S_3 \geq 0, S_4 \geq 0)$
 - B. $P(S_1 \neq 0, S_2 \neq 0, S_3 \neq 0, S_4 \neq 0)$
 - C. $P(S_1 \leq 2, S_2 \leq 2, S_3 \leq 2, S_4 \leq 2)$
 - D. $P(|S_n| \leq 2, \text{ per } n = 1, 2, 3, 4);$
 - E. Let $Y_n = S_{2n} \forall n$. Verify that $\{Y_n, n \geq 1\}$ is a Markov chain and find the transition probability
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2. Fifteen students are to be randomly distributed among three groups of five. Twelve students are female and 3 are males.

- A. How many different groups of students can be formed
 - B. What is the probability that the three men are in the same group
 - C. What is the probability that in each group there is a man?
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3. Let X be an exponential random variable with mean θ . Let $Y = \log X$.

- A. Find the support of the r.v. Y
 - B. Find the density of the r.v. Y
 - C. Find $P(Y > 0)$
 - D. Find the median of the r.v. Y
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4. Let X and Y be independent $\text{Uniform}(0, 1)$ random variables. Moreover consider

$$Z = \min(X, Y) \quad W = \max(X, Y)$$

(a) Find the density of $S = X - Y$

(b) Find $P(W > 3/4 | Z < 1/4)$.

(c) Find the distribution of W conditional on the event $Z \leq z$.

Hint: Write separately the joint distribution $P(W \leq w, Z \leq z)$ for the cases $w \leq z$ and $w > z$.
