

# Stochastic Processes

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May 2023

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**Solve 2 exercises: time 2 hours.**

1. Let  $X$  be a random variable with density

$$f(x) = \begin{cases} c|x| & -2 < x < 2 \\ 0 & otherwise \end{cases}$$

- Find  $c$ ,  $E(|X|)$  and  $E(X)$
- Find the density of  $Y = X^2$
- Find  $P(X \leq 0 | X^2 \leq 1/3)$

2. Let  $X$  and  $Y$  independent and identically distributed Bernoulli random variables with parameter  $p$ . Consider  $U = X + Y$  and  $V = (-2)^X$

- Find the p.m.f of  $(U, V)$
- Find the covariance between  $U$  and  $V$
- Find  $P(U - V > 0)$  and  $P(U = 0 | V = 1)$

3. Let  $X$  and  $Y$  be independent exponential random variables with parameters  $\lambda$  and  $\mu$  Find the density of  $Z = X/Y$  and  $P(X - Y > 1)$

4. A Markov chain with 3 states  $\{0, 1, 2\}$  has transition matrix

$$P = \begin{pmatrix} 1/2 & 1/3 & 1/6 \\ 0 & 1/3 & 2/3 \\ 1/2 & 0 & 1/2 \end{pmatrix}$$

Suppose that  $P(X = 0) = P(X = 1) = 1/4$

- Find  $E(X_2)$
  - Find  $Cov(X_1, X_2)$
  - Is the chain irreducible? In case find the invariant distribution. Moreover, If the chain is also aperiodic find  $\lim_{n \rightarrow \infty} P(X_n > 0)$
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