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STABILIZATION AND EXPANDED COMMITMENT: A THEORY OF FORWARD GUIDANCE FOR ECONOMIES WITH RATIONAL EXPECTATIONS

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ABSTRACT

In this paper we construct a general theory of forward guidance in economic policy making, in order to provide a framework to explain the role and strategic advantages of including forward guidance as an explicit part of policy design. We do this by setting up a general policy problem in which forward guidance plays a role and then examine the consequences for performance when that guidance is withdrawn. Following results in Acocella et al (2013), who extend the theory of economic policy to a world with rational expectations, we show that forward guidance provides enhanced controllability and stabilizability – especially where such properties have not been available before. As a by-product we find that forward guidance severely limits the scope and incentives for time inconsistent behaviour in an economy whose policy goals are ultimately reachable. It can therefore add to the credibility of a set of policies.

Classification JEL: *E42, E58, E61, E63*
Keywords: *Managing Expectations, Stabilizability, Dynamic Controllability, Time Consistency, Two point boundary solutions*

1. INTRODUCTION

John Williams, President of the San Francisco Federal Reserve and member of the Fed's Open Market Committee, has argued that forward guidance and large scale asset purchases (popularly known as Quantitative Easing) are now the leading and most important forms of unconventional monetary policy (Williams, 2011). Both

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techniques were used extensively to engineer a recovery from the Great Financial Crisis of 2008-12.

But, whereas Quantitative Easing has been studied in some detail[1] and is comparatively well understood in terms of how it is supposed to work, if not in terms of how large impact it has had in practice, forward guidance has been equally widely used but with little understanding of how and in what circumstances it can work successfully, what the drawbacks might be, and what the impact might be. In short, we lack a proper analysis of the *strategic* value of forward guidance as a tool of monetary policy – both in general and in difficult circumstances.

The existing literature on forward guidance is, at this point, rather limited and restricted to a few specific problems or circumstances. A natural concern is whether forward guidance statements have had a perceptible impact on expectations in practice. Kool and Thornton (2012) argue that they have not, at least not in the context of monetary policy in four leading OECD economies; whereas Campbell et al (2012) argue that they did in the US over a longer sample period (1990-2011). However, Del Negro et al (2013) suggest that our standard models often overestimate the size of these impacts. Obviously the jury is still out on that question.

Unsurprisingly, given recent history, much of the theoretical work has been done in the context of interest rates being stuck at their zero lower bound. Here, for example, Gavin et al (2013) find forward guidance announcements lower interest rates, prompting consumption and output to recover, ***if*** private sector expectations adjust – more so, the longer the horizon. However, in an important qualification, Levin et al (2010) warn that the stability of the economy may be at risk. They also find that, given moderate negative shocks, forward guidance can be used to stabilize the system, but conjecture that large negative shocks (such as will appear in a big recession) will overwhelm the forward guidance effects and leave us unable to stabilize the economy. But whether that is a result of large shocks, or of insufficiently responsive policies/forward guidance, is a moot point – this is a point we return to below and find that it to be the latter. These results in turn prompt another line of thought; that the effectiveness of forward guidance may depend on the form of guidance offered – specifically whether it is Delphic (the expectations offered are in terms of outcomes, such as would be the case when trying to escape a serious recession), or Odyssean (the expectations offered are in terms of a policy rule, or how the authorities will react to changes in certain conditions)[2]. This corresponds to unconditional (or unconditioned) vs. conditional forward guidance with an exit strategy, in the language of Acocella and Hughes Hallett (2014).

In trying to construct a theory of forward guidance, any formal analysis must fit first within the general theory of policy announcements (Hughes Hallett and others 2012a,b; Acocella et al 2014)[3]. Any such a theory needs an understanding of the strategic advantage offered by forward guidance, its role in the policy arsenal, its value

[1] Williams (2011), Ugai (2007), Gagnon et al (2012), Joyce and others (2012, plus the papers referred to therein)
[2] See Raskin (2013), and Contessi and Li (2013) respectively.
[3] See also Amato, Morris and Shin (2002), or Brand, Buncic and Turunen (2010) for further evidence.

in terms of overall economic performance, and whether the expectations generated would be sustained or dissipated by time inconsistent revisions. That is the subject of this paper. We proceed by examining the advantages of policy rules which contain forward guidance, and then explore what is lost if those guidance terms are withdrawn.

2. ECONOMIC MODELS WITH FORWARD LOOKING EXPECTA

Without loss of generality, we can write the generic linear RE model in its reduced form for a single policy authority, as follows:

$$y_t = Ay_{t-1} + Bu_t + Cy_{t+1|t} + v_t \quad \text{for } t = 1,\ldots,T. \qquad (1)$$

where $y_{t+1|t} = E\left[y_{t+1} \mid \Omega_t\right]$ denotes the mathematical expectation of y_{t+1} conditional on Ω_t (a common information set available to all agents at *t*) and u_t is a vector of *m* control variables in the hands of the policymakers. The matrices *A*, *C* and *B* are constant and of order *S*, *S*, and $S \times m$, respectively, and have at least some elements which are nonzero. In this representation, y_0 is a known initial condition, and $y_{T+1|1}$ is some known, assumed or expected terminal condition (most probably one that describes the economic system's long run equilibrium state[4]); and both are part of each information set Ω_t. Note that the values of u_t are not part of Ω_t since they are determined by the policymakers.

Finally v_t is a vector of exogenous shocks and other influences on y_t, with a known mean but which comes from an unspecified probability distribution. Notice also that the policy authority may have only $q \leq S$ explicit targets, but the *m* instruments are assumed to be linearly independent. Thus, $y \in R^S$ and $u \in R^m$.

This model can now be solved from the perspective of any particular period, say *t* = 1, by putting it into its final form conditional on the information set available in that period:

$$\begin{pmatrix} y_{1|1} \\ : \\ : \\ : \\ y_{T|1} \end{pmatrix} = \begin{bmatrix} I & -C & 0 & . & 0 \\ -A & I & & . & . \\ 0 & & . & & 0 \\ . & . & & . & -C \\ 0 & . & 0 & -A & I \end{bmatrix}^{-1} \left\{ \begin{bmatrix} B & 0 & . & . & 0 \\ 0 & . & & & . \\ . & & . & & . \\ . & & & . & 0 \\ 0 & . & . & 0 & B \end{bmatrix} \begin{pmatrix} u_{1|1} \\ . \\ . \\ . \\ u_{T|1} \end{pmatrix} + \begin{pmatrix} v_{1|1} \\ . \\ . \\ . \\ v_{T|1} \end{pmatrix} + \begin{pmatrix} Ay_0 \\ 0 \\ . \\ . \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ . \\ . \\ 0 \\ Cy_{T+1|1} \end{pmatrix} \right\}$$

(2)

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[4]There is no indeterminacy here. The dynamic conditions which guarantee the existence of a solution are automatically satisfied, given any particular information set, ***if*** the inverse in (2) exists – which we show to be the case. Given such an inverse, Hughes Hallett and Fisher (1988) show that the saddle point property (that the system has the correct number of stable and unstable roots to ensure a solution; Blanchard and Khan, 1980) is satisfied. One implication is that it no longer matters what the value of the terminal condition is, or if none is specified, if the policy horizon is far enough away (if $T \to \infty$). Indeterminacy may follow if $y_{T+1/1}$ values cannot be specified.

Although equation (1) has been solved from the point of view of Ω_1, it must be understood that it could have been derived for each Ω_t, t = 1,...,T, in turn, where $y_{j|t} = E_t\left(y_j\right)$ if $j \geq t$, but $y_{j|t} = y_j$ if $j < t$; and similarly for *u* and *v*.

The equation to which (2) is the solution makes it clear that *neither* policymakers, *nor* the private sector are required to make expectation errors for the policies to work as planned. In fact, equation (5) below shows just the opposite in that those expectations are exactly consistent with what the private sector/policymakers expect the outcomes to be. It then only remains to determine if it is possible to shift expectations in such a way that the economy's outcomes can reach certain specified target values at certain points of time.

It is easy to show that this final form solution always exists since the inverse matrix in (2) is well defined provided the matrix product *AC* does not contain a unit root. To see this, define the Toeplitz matrix in (2) to be:

$$T_T = \begin{bmatrix} I & -C & 0 & . & 0 \\ -A & I & & . & . \\ 0 & & . & & 0 \\ . & . & & . & -C \\ 0 & . & 0 & -A & I \end{bmatrix}. \qquad (3)$$

This matrix is of order *ST*. Using the partitioning by time, the determinant of T_T is given by:

$$\left|T_{T-1}\right| . \left|I_n - \left(-C, 0.....0\right) T_{T-1}^{-1} \left(-A', 0.....0\right)'\right| \qquad (4)$$

However $\left|T_{T-1}\right| = \left|T_{T-2}\right| \left|I_n - \left(-C, 0.....0\right) T_{T-2}^{-1} \left(-A', 0.....0\right)'\right|$, and so on. Hence we can write: $\left|T_i\right| = \left|T_{i-1}\right| \left|I_n - \left(-C : O\right) T_{i-1}^{-1} \left(-A' : 0\right)'\right| = \left|T_{i-1}\right| \left|T_{i-2}\right| ... \left|T_3\right| \left|I_n - CA\right| \neq 0$ for *i* = 2,…,*T*.

These equalities follow from the partitioning in T_{i-j}^{-1} and repeated applications of the Woodbury formula for the inverse of a matrix sum; and the ***in***equality from the absence of a unit root in *AC*. But $\left|T_1\right| = \left|I_n\right|$. Hence the inverse always exists by induction, subject to the no unit root condition imposed on *AC*.

3. CONTROLLABILITY

Given that the inverse discussed above always exists, we can write the model in final form in the following way:

$$\begin{pmatrix} y_{1|1} \\ . \\ . \\ . \\ y_{T|1} \end{pmatrix} = \begin{bmatrix} R_{11} & . & . & . & R_{1T} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ R_{T1} & . & . & . & R_{TT} \end{bmatrix} \begin{pmatrix} u_{1|1} \\ . \\ . \\ . \\ u_{T|1} \end{pmatrix} + \begin{pmatrix} b_{1|1} \\ . \\ . \\ . \\ b_{T|1} \end{pmatrix}, \text{ or } y = Ru + b \quad (5)$$

where $R = T_T^{-1}(I \otimes B)$, $b = T_T^{-1}\left\{E(v\,|\,\Omega_1) + \left(A':0\right)' y_0 + \left(0:C'\right) y_{T+1|1}\right\}$, and $\otimes$ denotes a Kronecker product. In this representation, each $R_{t,j} = \partial y_{t|1} / \partial u_{j|1}$ is an $S \times m$ matrix of policy multipliers for t, j = 1,…,T. Notice that $R_{t,j} \neq 0$ even if $t < j$. Hence equation (5) implies that $R_{t,j}$ is a matrix of conventional policy multipliers between $y_{t|1}$ and $u_{j|1}$, with a delay of $t–j$ between implementation and realization if $t \geq j$. In other words, causality runs forwards. But if $t < j$, then $R_{t,j} \neq 0$ represents a matrix of anticipatory effects, on $y_{t|1}$, of an announced or anticipated policy change $u_{j|1}$ at some point in the future.[5]

Multi-period static controllability

Static controllability defines the set of conditions which must hold if an arbitrary set of target values can be achieved for the endogenous variables y_t in each period. Define those target values to be $\bar{y}_{t|1}$; and $\bar{y}$ the corresponding stacked vector of those desired values across time periods.

Static controllability, meaning the ability to reach desired values for the targets in each period, evidently requires the matrix R in (5) to possess an inverse:[6]

$$u = R^{-1}(\bar{y} - b) \qquad (6)$$

where y, u and b are all understood to be expectations conditioned on the current information set Ω_t, including the terminal condition, as specified in equation (5). Hence:

THEOREM 1 (*static controllability under REs*). Under REs, static controllability by a single player, as in any conventional backwards looking model, requires as many independent policy instruments as there are target variables in each time period.

PROOF:

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[5] A conventional *backwards looking* model will have $R_{t,j}$ =0 for all $t<j$; *and* constant multipliers $R_{t,j} = R_{t-j}$ for $t–j$ = 0,...,T–1 if the model at (2) is linear. Neither of these things is necessarily true in (5).

[6] For convenience we have assumed that the number of instruments and targets does not vary over time.

From (5), $R = T_T^{-1} B_T$ where $B_T = I_T \otimes B$. Hence $R_T^{-1} = (T_T^{-1} B_T)^{-1} = B_T^{-1} T_T$ exists if and only if $B_T^{-1} = I_T \otimes B^{-1}$ exists, since we already know that T_T^{-1} always exists. But the coefficient matrix, *B*, can only possess an inverse if *S* = *m* ***and*** if it has full rank: i.e., has rows and columns that are linearly independent. But those are exactly the same conditions as would provide period-by-period static controllability in a backward looking dynamic model (*C*=0).■

Multi-period dynamic controllability

A model is said to be dynamically controllable if a sequence of instrument values $u_1,...,u_t$ can be found that will reach any arbitrary values, $\bar{y}_t$, for the target variables in period *t* (in expectation) given an arbitrary starting point y_0. In this case we are not concerned with the period-by-period controllability of the target variables between periods 1 and *t* - 1. Starting from period 1, dynamic controllability therefore requires a sequence of intended instrument values, $u_{1|1},.....,u_{T|1}$, that guarantee $\bar{y}_{t|1}$ is reached in period *t*=*T*. Given an initial state y_0 and terminal condition $y_{T+1|1}$, this is possible only if the sequence of policy multipliers and anticipatory effects in the *t*-th row block of (5), [$R_{t,1}.....R_{t,T}$], is of full rank. That is, if $\mathrm{r}[R_{t,1}....R_{t,T}] = S$.

THEOREM 2 (*sufficient conditions for dynamic controllability with REs*). The economy represented here by (1) is dynamically controllable over the sub-interval (1, *t*), when *T* ≥*S* and when *t*<*T,* if $\mathrm{r}[R_{t,1}....R_{t,S}] = S$.[7]

PROOF:

$\bar{y}_{t/1} = (R_{t,1}....R_{t,T})u + b_{t|1}$ is reachable over (1, *t*), using a Moore-Penrose generalized left inverse in $u = \left(R_{t,1}...R_{t,T}\right)^{+}\left[\bar{y}_{t|1} - b_{t|1}\right]$, if $\mathrm{r}[R_{t,1}....R_{t,T}] = S$. But if *T* ≥ *S*, then $\mathrm{r}[R_{t,1}...R_{t,T}] =$ $\mathrm{r}[R_{t,1}...R_{t,S}] = S$, which provides the result.■

Comment 1**.** It is important to see why time inconsistency will not appear here. Controllability at period *t* means that, barring unforeseen shocks, the policymaker will be able to reach his desired values for y_t in expectation. Hence, $y_{t|t}$= $y_{t|1}$= $\bar{y}_t$ are fixed or at least known quantities. But $y_{1|t}$ = $y_{1|1}$ is fixed by history; and $u_{1|t}$=$u_{1|1}$ likewise. It is then easy to see that, if nothing else changes, $u_{t|t}$= $u_{t|1}$. The policymaker is of course free to set $u_{t|t} \neq u_{t|1}$. But he would never do so because $\bar{y}_t$ is his first best value and is reachable given no information changes or unforeseen shocks. Policy makers have no incentive, still less a strategic interest, in choosing to make themselves worse off than

[7] This theorem provides a sufficient condition for dynamic controllability. The corresponding necessary condition may involve a smaller subset of R_{tj} having full rank depending on how many policy instruments are available.

they need to be. Hence, to assert time inconsistency is to claim that rational policymakers would choose to make themselves worse off: a contradiction.

4. STABILIZABILITY UNDER RES

The theoretical analysis

We can apply the reasoning underlying Theorem 2 to show that any economy can be stabilized to an arbitrary degree under rational, forward looking expectations if it is also dynamically control-able in the sense of Theorem 2. An arbitrary degree of stabilization means that policy rules can be found to make the economy follow an arbitrarily stable path, based on an arbitrary set of eigenvalues, such that it returns to the original path following a shock. Theorem 3 below gives the Rational Expectations analogue of the standard stabilizability theorem for backward looking, or physical systems.[8]

THEOREM 3 (*stabilizability and REs*)**.** For any economy represented by (1), with arbitrary coefficient matrices *A*, *B* and *C*, we can always find a series of dynamic but forward-looking policy rules, $u_{t|1} = \sum_{j=1}^{T} K_{tj} y_{j-1|1} + k_{t|1}$,[9] such that the controlled economy is stabilizable up to an arbitrary set of eigenvalues, if that economy is dynamically controllable as defined in Theorem 2.

PROOF:
Equation (1), with arbitrary coefficient matrices *A*, *B* and *C*, can be reduced to its final form (2). Substituting the policy rule $u_{t|1} = \sum_{j=1}^{T} K_{tj} y_{j-1|1} + k_{t|1}$ for each *t* = 1,…,*T* shows that the controlled economy will behave as, similarly to (5):

$$\begin{pmatrix} y_{1|1} \\ . \\ . \\ . \\ y_{T|1} \end{pmatrix} = \begin{bmatrix} R_{11} & . & . & . & R_{1T} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ R_{T1} & . & . & . & R_{TT} \end{bmatrix} \begin{bmatrix} K_{1,1} & K_{1,2} & . & . & K_{1,T} \\ K_{2,1} & . & & & . \\ . & & . & & . \\ . & & & . & K_{T-1,T} \\ K_{T,1} & . & . & K_{T,T-1} & K_{T,T} \end{bmatrix} \begin{pmatrix} y_0 \\ y_{1|1} \\ . \\ . \\ y_{T-1|1} \end{pmatrix} + \begin{pmatrix} c_{1|1} \\ . \\ . \\ . \\ c_{T|1} \end{pmatrix} \quad (7)$$

where $y_{0|1} = y_0$ and $c_{t|1} = b_{t|1} + T_T^{-1} B k_{t|1}$ ($b_{t|1}$ was defined in (5)). Rewriting (7) in obvious notation, we now have

$$\tilde{y}_t = RK\tilde{y}_{t-1} + c \quad (8)$$

[8] See Wonham (1974).
[9] Note that this control rule, for use in period *t*<*T*, employs actions and anticipated actions up to the end of period *T*.

where $\tilde{y}_t$ is the stacked vector on the left of (7).

For an economy to be stabilizable at *t*, it must possess the property that it would return to the initially expected path, whatever the initial conditions and shocks experienced up to that point, given no further shocks or changes in expectations appear (Wonham, 1974). This property will exist if the iteration matrix, *RK*, has its roots inside the unit circle. But we can go further. Any particular $y_{t|1}$ will follow an arbitrarily stable path if we can pick $K_{t1}.....K_{tT}$ to generate an arbitrary set of eigenvalues for that matrix for each *t*. Suppose we want to choose iteration matrix $D = Z\Lambda Z^{-1}$, where Λ is a diagonal matrix of the chosen eigenvalues, and *Z* is a matrix of the corresponding eigenvectors. Then, as long as *T*>*S* and the matrix *R* has full rank *ST* (i.e., *m*≥*S*, so that static controllability applies), we can calculate the required *K* from $K = R^{-1}Z\Lambda Z^{-1}$. But if *m*<*S* and dynamic controllability applies (as in the theorem), then we can use a generalized left-inverse instead: $K = R^{+}Z\Lambda Z^{-1}$, with $R^{+} = \left(R'R\right)^{-1}R'$ as one obvious possibility. This generalized inverse always exists, given dynamic controllability, since $R'R$ has full rank with r[*R*]= *mT* by Sylvester's inequality. To see this, recall $R = T_T^{-1}\left(I \otimes B\right)$ where T_T^{-1} is a square *ST*x*ST* matrix of full rank and $\left(I \otimes B\right)$ is a block diagonal matrix with rank *mT*. Hence, by Sylvester's inequality, r[*R*]≥ *ST*+*mT*-*ST* = *mT*. But if *m*<*S,* then $\mathrm{r}\left[I \otimes B\right]$ cannot be greater than *mT* by definition. Hence r[*R*]=*mT*, which means that $\left(R'R\right)^{-1}$ exists and this value of R^{+} is always available. ■

Comment 2. Note that the policy rules described in Theorem 3 are both forward and backward looking in that they react to expected future developments, including to the effects of these rules applied in the future, *and* to feedback from past outcomes (past "failures") – in exactly the same way as the private agents in the economy have been assumed to do.

Comment 3. Thus we can infer that a RE model which is dynamically controllable at *t* = 1 in the sense of Theorem 2, is also stabilizable from *t* = 1. Hence Theorem 3 generalizes on Wonham's original theorem, where stabilizability can be achieved for the first time only in period *S*. Notice that it is *not*, in general, possible to dispense with the feed-forward part of the policy rule for the obvious reason that it has to control both the feed-forward and the feedback behavior of the private agents. However it would be possible to apply a block diagonalization to the matrix *K* in Theorem 3, and transform the policy instruments to match, in order to remove any dependence on expected future outcomes. Such a block diagonalization exists since $R^{+}D$ is square. But, such a transformation typically involves a Jordan canonical form whose off-diagonal elements will still cross time intervals.

Comment 4. The key lesson therefore is that, in models with forward looking behavior, the closed loop (as opposed to feedback) characteristics of our policy rules are of special importance. Closed loop means reacting to changes in expectations of future events as they appear, in addition to past outcomes as they deviate from plan. In backwards looking models, future events are represented by future exogenous variables. In such models, future variables do not influence current behavior (although economic performance might improve if they did). As a result, the distinction between closed loop and feedback rules is traditionally ignored as being unimportant. But given forward looking markets where current behavior and outcomes depend directly on expectations of the future, and where expectations of the future depend on the current outcomes, the distinction can be large (see Hughes Hallett, Acocella and Di Bartolomeo, 2012a).

5. AN ILLUSTRATION: STABILIZABILITY, WITH AND WITHOUT FORWARD LOOKING POLICIES.

We now construct two simple examples of this stabilizability result to illustrate the importance of using forward looking policies and forward guidance given forward looking behavior by the private sector. The examples are constructed to explain a paper by Cochrane (2011) which claims that the Taylor rule in a New Keynesian model will produce results that are typically unstable. The claim is correct, but not for the reason Cochrane supposes. The correct reason is, no rational policymaker would ever attempt to use a backward looking policy rule to manage an economy with forward looking behavior or anticipations. Forward guidance is needed as well.
For the purposes of illustration, consider a one equation RE model with dynamics:

$$y_t = ay_{t-1} + by^e_{t+1} + cx_t + \varepsilon_t \qquad (9)$$

Such a model can be derived from a conventional New Keynesian model of the type used by Mishkin (2002), say, to assess the effectiveness of Taylor rules for controlling inflation and the output gap. That is, we can start from:

$$\pi_t = (1-\lambda)\beta E_t\pi_{t+1} + \lambda\pi_{t-1} + \kappa z_t + \phi f_t + v_t \qquad (10)$$

$$z_t = \varsigma E_{t-1}z_t - \sigma(i_t - E_t\pi_{t+1}) + \chi f_t + \eta_t \qquad (11)$$

where (10) is an aggregate supply equation with dynamics, π_t is the rate of inflation, z_t the output gap; f_t is the stance of fiscal policy[10], i_t the interest rate (monetary policy instrument), and v_t and η_t are random shocks. Equation (11) is therefore a forward looking IS curve. Eliminating z_t between (10) and (11) now yields:

$$\pi_t = (1-\lambda)\beta E_t\pi_{t+1} + \lambda\pi_{t-1} - \kappa\sigma(i_t - E_t\pi_{t+1}) + \varepsilon_t \qquad (12)$$

where $\varepsilon_t = \kappa E_{t-1}z_t + (\kappa\chi + \phi) f_t + \kappa\eta_t + v_t$ represents a composite term of "shocks" exogenous to both monetary policy and the economy. This definition of ε_t involves a natural approximation in that $E_{t-1}z_t$ should, strictly speaking, be an endogenous

[10] Many New Keynesian models specify marginal costs, mc_t, as the push factor in inflation, in place of f_t. In this case (12) would have have $\kappa\chi f_t+\phi mc_t$ in ε_t in place of the $(\kappa\chi+\phi)f_t$ term. This alternative specification would lead to a model of the identical form to that specified here.

(rational) expectation of the output gap. So classifying it as part of the composite error term is to recognize the reality that the private sector either does not have full REs for the output gap; or cannot measure them properly; or that the private sector is no more able to separate cyclical from structural changes in the trend of output on time than the policymakers – the typical delay needed for the data to do so accurately being up to 4 years[11]. Agents therefore typically use a simple forecasting or extrapolation device for $E_{t-1}z_t$ instead. Given that, we can now recast (12) as a particular case of (9) with $a=\lambda\,;b=(1-\lambda)\beta+\kappa\sigma$; and $c=-\kappa\sigma$. In this specification, x_t is the policy instrument (interest rate); and y_t is the policy target (inflation).

Stabilization in expectation, based on our notion of controllability, can now be investigated using Theorem 3. Consider now two different decision rules for managing (9):

a) with no forward looking elements,

$$x_t = k_1 y_{t-1} + k_2\,; \quad \text{and} \qquad (13)$$

b) with an added forward looking element,

$$x_t = k_1 y_{t-1} + d y^e_{t+1} + k_2 \text{ say.} \qquad (14)$$

We substitute (13) and (14) into (9) to see the behavior of the economy under control in each case:

$$y_t = (a + ck_1) + by^e_{t+1} + \varepsilon^*_t;\text{ and} \qquad (15)$$

$$y_t = (a + ck_1)y_{t-1} + (b + cd)y^e_{t+1} + \varepsilon^*_t \qquad (16)$$

respectively, where $\varepsilon^*_t = \varepsilon_t + ck_2$. Renormalizing (15) and (16) on their lead terms, then taking expectations conditional on information available in period *t*, and dropping the superscript "*e*" for simplicity, leaves us with two alternative models to be stabilized:

$$y_{t+1} = b^{-1}y_t + b^{-1}\left(a+ck_1\right)y_{t-1} + b^{-1}\varepsilon^*_t \qquad (17)$$

$$y_{t-1} = \left(b+cd\right)^{-1} y_t + \left(b+cd\right)^{-1}\left(a+ck_1\right)y_{t-1} + \left(b+cd\right)^{-1}\varepsilon^*_t \qquad (18)$$

It is already obvious that (17) presents us with only one opportunity, via the choice of k_1, to choose the coefficients and thus the roots of the economy under control. Yet there are two roots. In fact, it will not be possible to stabilize this economy with a simple feedback rule at all, unless *b>1*, let alone do so up to an arbitrary pair of eigenvalues.

Equation (18), by contrast, gives us the opportunity to choose two coefficients, and hence both characteristic roots in the economy under control, given our freedom to choose both k_1 and *d*. Hence, this system is stabilizable; and stabilizable up to an arbitrary set of eigenvalues.

We can demonstrate these two claims as follows. The roots of equation (17) are:

$$\lambda_{1,2} = \frac{1}{2}b^{-1}\left\{1 \pm \left[1+4b\left(a+ck_1\right)\right]^{\frac{1}{2}}\right\} \qquad (19)$$

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[11] See Hughes Hallett, Kattai and Lewis (2012).

We can therefore only choose to have real roots (by choosing k_1>-(1+4ab)/4ac, if $b,c>0$); or to have the product of the roots less than one (by setting $k_1<(b-a)/c$ in under the same conditions). But we cannot choose the size of the roots individually. In fact to minimize the larger of the two, the best we can do is set k_1=- a/c. That will give us $\lambda_{1,2}=\left\{0,b^{-1}\right\}$; which means that we can stabilize the system with a simple feedback rule *only* if the system is stable already – that is, if and only if b>1; otherwise never.

Notice that stabilizability is, in this case, determined by the coefficients of the model – not by the policy rule. So Cochrane (2011) is right to say that a forward looking New Keynesian model *may* not be stabilized by a Taylor rule. But the correct inference is: a New Keynesian economy can be stabilized by a Taylor rule, but only when that model is already stable. So if instability follows, the fault lies not with the rule, but with the model. Taylor rules do not, in themselves, destabilize the economy.

By contrast, if we use a forward looking rule like (14), the controlled economy will behave as in equation (18). The roots of this system are:

$$\lambda_{1,2}=\frac{1}{2}(b+cd)^{-1}\left\{1\pm[1+4(b+cd)(a+ck_1)]^{1/2}\right\} \qquad (20)$$

which can be set to be arbitrarily close to zero – for example, by selecting i) $d=-b/c+\omega$ where $\omega>1$; and ii) k_1=- a/c. This implies roots of $\lambda_{1,2}=\frac{1}{2}\left(\omega^{-1}\pm\omega^{-1}\right)=\left\{0,\omega^{-1}\right\}$ for (20), both of which lie within the unit circle. Stability is therefore assured in *all* possible circumstances. Indeed these roots are arbitrarily small if ω is made large enough. In this case, therefore, an arbitrary degree of stability can be conferred on any model, including those that are unstable to start with. Moreover, the stabilization is done by the policy rule, not by the model. The key difference is that, in this case, the forward looking component in the rule, d, supplies the roots of the *stabilized* economy.

6. CONCLUSIONS

In this paper we have outlined a general theory of forward guidance in economic policymaking. We have used entirely conventional assumptions about the working of an economy under rational expectations, on the model used to represent it, and the way private sector expectations are formed and can be exploited by the policymaker. We have thus provided a framework to explain the role and strategic advantages of including forward guidance as an explicit policy tool, and to underline the adverse consequences for performance when that guidance is withdrawn. What this paper shows is that forward guidance is an essential component of any policy rule in an economy which is subject to forward looking anticipations of future behavior.

i) Given the rank condition discussed in theorem 2, forward guidance is necessary to secure stabilizability and controllability (the ability to reach and stabilize around specified values for the target variables) – unless the economy is already stable, in which case forward guidance is only necessary for stabilizing around the specified target values.

ii) Without forward guidance, which provides the private sector with information about the policymaker's future intentions, the economy may not be stabilizable; and will not, in general, be controllable with respect to any given target values.

iii) The stabilizability and controllability properties conferred by forward guidance take effect immediately, from period $t=1$, rather than after t periods delay as would be the case in an econ-omy without anticipations effects. Forward guidance therefore creates an acceleration of the required policy impacts. It offers the policymaker the opportunity to control the economy from any date, as the private sector anticipates his future behavior and knows that he can control the economy under the rank conditions assumed.

iv) Time inconsistency is not a problem under forward guidance *unless* there are insufficient policy instruments *and* policymakers are impatient and have very short horizons. To be specific, there are no grounds to suppose that policymakers acting in their own interest will show time inconsistent behavior unless both $m<S$ and $S/m<t$, where t is the date by which stability and/or the desired target values are to be attained.

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