Financial Safety Nets, Bailouts and Moral Hazard

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The Doctoral School of Economics

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ABSTRACT. The paper argues that policymakers bail out banks with financial problems to avoid the costs of financial repression. After financial liberalization and when risk is verifiable, in some circumstances policymakers can commit to policies that discipline banks ex-ante and ex-post, by providing bailout to conservative banks and threatening the takeover of risky banks. When these policies are time consistent, regulatory policies to deal with moral hazard ex-ante, like for example prudential regulation, become redundant and policymakers refrain from implementing them.

JEL Classification: G01, G21, G28.

1. INTRODUCTION

Although attention today is naturally concentrated on the present crisis, significant banking crises have occurred in the past both in developing and developed countries. These crises have been regularly preceded by a process of liberalization of the financial system without adequate efforts to improve regulation and supervision. If banks suffer a shock that renders them insolvent, governments tend to bail them out,\(^1\) which implies high costs for the taxpayers and serious problems of moral hazard.

This paper tries to answer the questions raised by these empirical regularities. Why do governments liberalize the banking sector without an adequate regulation and supervision in the first place? Why do governments bailout banks despite the negative moral hazard effects and the fiscal costs involved in such operations?

The main point made in the paper is that when banking crises are preceded by financial liberalization there is a bias to bailout banks and underinvest in bank supervision. Policymakers prefer bailouts because they want to avoid reintroducing financial repression, i.e. control and/or ownership of private banks by the state sector or a significant degree of regulation of the banking sector.

Some hold that it is justifiable to bailout banks when there are asymmetries of information between bankers and regulators regarding the quality of the banks’ assets (Aghion et al. 1999; Gorton and Huang 2002; Mitchell 1998 and Osano 2002). Regulators find it difficult to commit themselves to takeover banks because, as a response, bankers could hide their financial problems and misallocate credit. Therefore, a bailout

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\(^1\)For a cross-section analysis of several banking crises in developed and developing economies see Kaminsky and Reinhart (1999) and Reinhart and Rogoff (2009)
may be a second best alternative as it induces bankers to reveal their financial problems and correct their credit allocation.\footnote{It has also been argued that bailouts can be designed in such a way to avoid ex-ante moral hazard. Bailout designs are generally analyzed under the hypothesis of non-verifiable risk. For example, Cordella and Yeyati (1999) show that, under non-verifiability, a credible commitment to bailout banks as a reaction to ‘bad’ aggregate shocks eliminates bankers’ ex-ante moral hazard. Aghion et al. (1999) show that non-verifiability may create a problem of overcapitalization, which can be corrected if bailouts are designed to be conditional on the previous liquidation of the banks’ bad assets and the transfer scheme is non-linear. Gorton and Huang (2002) argue that bailouts are welfare increasing because they eliminate the costs of hoarding the liquidity necessary to make operative a market for the banks’ non-performing assets.}

In our opinion, this line of research leaves some questions unanswered. If asymmetries of information justifies bailouts, why are regulators not motivated to improve bank supervision and try to eliminate such asymmetries when the banking system is liberalized? Would the prospect of saving public funds not be a sufficient incentive to invest to improve bank supervision? To answer these questions, we develop a model of strategic interaction between bankers and regulators. The model explicitly considers the decision to liberalize banking activities. More precisely the model considers two cases of liberalization. As a benchmark case liberalization consists in bank privatization, and as an extension it consists in bank deregulation.\footnote{In many developed countries liberalization has taken such forms as the possibility for banks to operate in (riskier) sectors previously forbidden to them or the possibility to operate as banks for different financial institutions. In other cases, especially in less developed countries or transition economies, liberalization took the more specific form of privatization: banks, previously owned or controlled by the public sector, are sold to private bankers. See section 5, for some more details on this aspect.}

At the beginning of the game, the regulator decides whether to privatize the bank and whether to improve the quality of bank supervision; the banker decides whether to buy the bank. We assume that the investment in bank supervision affects the probability to detect the true net value of the bank, but it has no effect on the regulator’s ability to observe or control the banker’s investment choices. The bank operates with a full deposit guarantee and must make two consecutive investment choices in $t = 1$ and in $t = 2$.

Bank privatization is depicted in the model as the transition from financial repression to a situation in which the constraints to the bank’s investment possibilities are lifted. With financial repression, the bank can make only safe investments and there are no informational asymmetries between regulator and banker. There is, however, a problem of inefficiency represented by a low expected return to the bank’s investments.

When the bank privatization takes place the expected return of the safe investment increases. Privatization also allows the bank to make risky investments. Such investments are characterized by higher volatility of returns; moreover they give rise to informational asymmetries between regulator and banker.

The privatization of the bank creates a regulatory problem because the banker can now make a risky investment. The model shows that when the bank’s net value is (almost) depleted by a bad outcome of the first investment, the banker prefers to make a second risky investment because the possible bad outcome of the second investment can be transferred to the regulator since the deposits are fully guaranteed. This strategy is denominated ‘betting the bank for survival’ (Osano 2002 and Gorton and Huang
As a response to this strategy, the regulator can either bailout or takeover the bank. By affecting the payoff associated to the takeover, the investment in bank supervision influences the decision on how to tackle bad returns ex-post. The model assumes risk-verifiability therefore the regulator can design risk-contingent policies, i.e. policies dependent on the banker’s choice.

The model shows that financial liberalization creates a bias in favor of bailouts. When financial liberalization takes place, policies that include bailouts to benefit conservative bankers are always possible, independently of the cost to improve (‘upgrading’) bank supervision. However, the improvement of bank supervision, which improves the ability to detect bad returns, does not guarantee that a takeover will always take place because of the cost of financial repression associated to it. Bailouts are chosen when the deadweight cost of capitalizing the bank is lower than the reduction in the economic surplus (of the bank and debtors) associated to the takeover of the bank.

The banker’s ex-ante and ex-post moral hazard can be eliminated when the risk-contingent policies, which punish a risky behavior with takeover and benefit a conservative behavior with bailout, are time consistent. These policies can be implemented with or without investment in bank supervision. An investment in supervision is made only if the increase in the probability of detection compensates for the cost of financial repression, and allows the regulator to credibly threat the risky banker with a takeover in case of a bad return.

The efficiency properties of policies that benefit the conservative banker and punish the risky banker make cooperative equilibria possible in the case of bank privatization. The regulator refrains from investing in bank supervision in exchange for a higher bid price for the bank. When the regulator decides not to invest in supervision, the banker benefits from a bailout, instead of being punished by a takeover. The regulator, on the other hand, benefits from the banker being willing to share with him/her the higher surplus.

The time consistency feature of policies that benefit conservative bankers and punish risky bankers has also an impact on the regulator’s willingness to introduce ex-ante regulatory measures to deal with the problem of moral hazard. We show that the regulator does not introduce bank prudential regulations to control the banker’s ex-ante moral hazard, when the time consistent risk-contingent policy already disciplines the bank ex-ante (and possibly ex-post).

The paper is organized as follows. In section 2, the model’s assumptions on the returns to investment are presented together with the definition of the policies that the regulator can implement. Also the objective functions of the banker and the regulator are specified. Finally, the timing of the game played by the two agents is synthetically outlined. Section 3 is devoted to define and analyze the strategy of ‘betting the bank for survival’, i.e. the banker’s choice to always make a risky investment after experiencing a bad outcome in the previous period. Section 4 deals with the regulator’s responses, which are contingent on the banker’s actions in $t = 1$ and $t = 2$. Section 5 looks at the conditions under which financial liberalization takes place in $t = 0$. In section 5 the cooperative equilibria are characterized, and the extensions to the model are presented. Section 6 concludes. The proofs of propositions and results of the model are in the Appendix.
The model considers the strategic interaction of two agents, a banker and a regulator, over four periods \((t = 0, 1, 2, 3)\). The banker is risk-neutral and the free-risk interest rate is normalized to zero for simplicity. The banking system consists of only one bank, which has to make two consecutive investment decisions in two periods \((t = 1, t = 2)\).\(^4\)

### 2. The Model and the Timing of the Game

The model considers the strategic interaction of two agents, a banker and a regulator, over four periods \((t = 0, 1, 2, 3)\). The banker is risk-neutral and the free-risk interest rate is normalized to zero for simplicity. The banking system consists of only one bank, which has to make two consecutive investment decisions in two periods \((t = 1, t = 2)\).\(^4\)

### Financial Liberalization

In \(t = 0\), financial liberalization takes place: the regulator decides to privatize the bank and the banker must decide whether to bid for it. If the banker decides to bid, he/she makes a ‘take it or leave it’ offer.\(^5\) The regulator must then decide whether to accept the bid and, simultaneously, whether to invest in supervision. The cost of this investment is assumed to be \(g > 0\).

### Investment returns

The bank, in periods \(t = 1\) and \(t = 2\), underwrites a contract with depositors of a fixed value \(D\), paying a zero interest rate. The bank cannot issue equity capital in any period of the game. In \(t = 1\), the bank makes a long term investment that matures in the last period \(t = 3\). The bank chooses between a safe and a risky loan portfolio.

At the beginning of \(t = 2\) an ‘interim’ return \(R_1\) to investment in \(t = 1\) materializes. \(R_1\) is a random variable with the following characteristics.

\[
R_1 = \begin{cases} 
A - C_1 + r_1 & \text{if the owner chooses the risky investment in } t = 1 \\
A - C_1 + s_1 & \text{if the owner chooses the safe investment in } t = 1
\end{cases}
\]

It is \(A > D\) and \(r_1\) and \(s_1\) are discrete random variables denoting the ‘marginal’ returns to the investment. \(C_1\) represents the cost of financial repression in period \(t = 1\). When the bank is privatized and controlled by the private banker \(C_1 = 0\); when the bank is controlled by the regulator \(C_1 = c > 0\).\(^6\)

Both ‘marginal returns’ \(r_1\) and \(s_1\) can take only two possible values, a good and a bad one. \(r_1\) can take the values \(R_G\) or \(R_B\) and \(s_1\) the values \(S_G\) or \(S_B\).\(^7\) The probabilities of the good and bad outcomes are assumed to be the same for both the risky and safe investment. The probability of the good outcome is \(\delta\), while the probability of the bad outcome is \((1 - \delta)\); additionally, we assume that \(\delta > (1 - \delta)\).

The cost of financial repression \(c\) derives from the regulator’s inferior skill to screen investment projects. It is assumed that \(c\) cannot go above a threshold defined by,

\[
c \leq A - D + S_B + S_G
\]

In \(t = 2\), after the first outcome, the bank owner has the possibility to make again the same type of investment or to shift to the other type. At the beginning of \(t = 3\) the

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4Depositors and bankers are assumed to consume only in \(t = 3\).

5It is assumed that the banker purchases the bank with a fixed initial endowment of financial capital; the possibility to purchase the bank by using debt is excluded.

6The regulator is in control either when the bank is publicly owned or when the private bank is managed by the public sector.

7The subscript \(G\) denotes a good outcome and the subscript \(B\) a bad outcome.
asset portfolio matures and provides the bank owner with the final return, $R_2$.

$$R_2 = \begin{cases} R_1 - C_2 + r_2 & \text{if the owner chooses the risky investment in } t = 2 \\ R_1 - C_2 + s_2 & \text{if the owner chooses the safe investment in } t = 2 \end{cases}$$

The final return to the bank's asset portfolio, $R_2$, is equal to the interim value of the bank's assets, $R_1$, minus the potential cost of financial repression, $C_2$, plus the 'marginal' returns $r_2$ and $s_2$, which are discrete random variables identically distributed as $r_1$ and $s_1$ respectively.\(^8\)

It is assumed that the 'marginal' returns to the safe investment have higher means than those associated to the risky investment, but lower variance. In particular, the expected 'marginal' return to the safe investment is zero:

\begin{equation}
E(s_i) = \delta S_G + (1 - \delta) S_B = 0 \quad (i = 1, 2)
\end{equation}

Instead, the expected 'marginal' return to the risky investment is negative:

\begin{equation}
E(r_i) = \delta R_G + (1 - \delta) R_B < 0
\end{equation}

2.2 and 2.3 imply that the expected return to the risky investment is smaller than that to the safe investment. This poses the question of why a risk-neutral banker should ever choose the risky investment. The following assumption makes the risky investment a viable choice:

\begin{equation}
R_G > S_G
\end{equation}

that is to say that the risky investment's good return is larger than the safe investment’s good return.\(^9\)

However, the choice of the risky investment requires, that there exists some mechanism thanks to which the bank owner can translate the risky investment’s bad returns to third parties and, instead, enjoy the good returns of the investment. In the model, such mechanism is the fact that deposits with the bank are fully guaranteed.\(^10\)

**Asymmetries of information.**

When the bank is in the hands of the private banker there arise informational asymmetries between the banker and the regulator. We distinguish between two types of asymmetries:

1. the regulator cannot observe the banker’s investment choices (risky or safe) in any period;
2. the regulator cannot observe with precision the bank’s net value $E_2(R_2 - D)$ in period $t = 2$, when the interim return of the bank’s investment is revealed to the banker.

In period $t = 2$ the regulator may detect the bank’s net value either through audits or by offering a bailout. With audits the probability of detection is $\gamma$, which depends on the regulator’s investment in bank supervision in $t = 0$. With investment, the probability of detection is perfect and equal to one, $\gamma = 1$; without investment, it is $0 \leq \gamma_0 < 1$.

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\(^8\)The random variables $r_1$, $s_1$, $r_2$ and $s_2$ are independently distributed and their covariances are always zero.

\(^9\)Equations 2.2, 2.3, and 2.4 guarantee that the variance of the returns of the risky investment is higher than the variance of returns of the safe investment, i.e. $R_G - R_B > S_G - S_B$

\(^10\)We shall return to this issue in the next section.
The investment in supervision can eliminate only the second type of asymmetry, but it has no effect on the ability to observe the banker’s investment choices. Consequently, investment in supervision has an impact on how to deal with bad returns ex-post, but cannot allow the regulator to observe and control the banker’s behavior ex-ante.\footnote{In section 5 it will be assumed that the regulator can invest in ex-ante bank regulation in order to observe and control the banker’s investment choices in \( t = 1 \).} With bailouts the regulator can observe the bank’s true net value.

**Financial safety net**

When the bank remains under public ownership the financial safety net consists only of a public deposit guarantee to reimburse deposits at par value in \( t = 3 \) (when the bank closes). If the bank is privatized in \( t = 0 \), the full deposit guarantee stays in place, but the regulator introduces a risk-contingent policy to deal ex-post with the potential bad outcome of the bank’s first investment. At the beginning of \( t = 2 \) the regulator announces a risk-contingent policy, which is expressed as a function that associates every possible banker’s investment behavior in \( t = 1 \) to an action of the regulator to tackle the potential bad outcome. The risk-contingent policy is defined below.

**Definition 1** (Risk-contingent policy). The regulator’s risk contingent policy is defined as a pair of actions \( \{ a_R, a_S \} \), one for each banker’s investment choice in \( t = 1 \).\footnote{The subscripts \( R \) and \( S \) represent the banker’s choice of the risky and safe investment respectively.} The actions \( a \) are of three possible kinds: forbearance (F), takeover (T) and bailout (B).

The regulator can play this policy even though it doesn’t observe the banker’s actions because it is able to infer them ex-post when detecting the bank’s true net value in \( t = 2 \).

With forbearance, the regulator does nothing. When the bank experiences a bad outcome, the regulator allows the banker to retain the ownership and control of the bank and simultaneously denies any capitalization of the bank.

With takeover, the regulator takes control of the bank’s management. If, for any possible investment choice in \( t = 2 \), the bank has a negative net value, \( E_2(R_2 - D) < 0 \), it is either closed or nationalized. The banker bears expected private costs equal to \( \zeta \geq 0 \) (legal prosecution). It is assumed that, if the bank is closed in \( t = 2 \), the liquidation price of its assets is zero. If the regulator discovers that the bank experienced a bad return but its net value would be still positive if the banker chooses the safe investment in \( t = 2 \), the regulator can assume only the management of the bank to make it sure that the safe investment is chosen. The banker retains the ownership of the bank.

With the bailout the banker retains the control of the bank and is offered a capitalization \( \Delta A \). When the banker accepts the bailout, the regulator observes the bank’s true net value. It is assumed that the regulator can only announce capitalization offers to which he/she can credibly commit after observing the bank’s true net value. Bailouts do not give the regulator either a stake in the bank’s ownership or any influence on the bank’s decision making. The capitalization offers cannot include gifts, which means that the injection of resources cannot be larger than the bad shock. This is formalized as follows.

\[
\begin{align*}
0 & < \Delta A_S \leq -S_B \\
0 & < \Delta A_R \leq -R_B
\end{align*}
\]
From 2.2, 2.3, and 2.4 it follows that $R_B$ and $S_B$ are negative. The subscripts $R$ and $S$ indicate that the constraints on the capitalization offers depend on the type of investment played by the banker in $t = 1$.

The bailout takes place in $t = 2$; it takes the form of a non-tradable public bond (with maturity at $t = 3$) transferred to the bank. In $t = 3$ the bank asset portfolio matures (loans and the public bond in case of bailout). With the proceeds of the loan portfolio, depositors are compensated for their investment. If the proceeds are insufficient to repay the deposits, the regulator covers the difference (full deposit guarantee).

**Objective Functions**

The regulator seeks to maximize the expected value of an objective function $\Omega$. $\Omega$ is increasing in the value of the bank’s assets in $t = 3$ (excluding the public bond), represented by $R_2$, and it is decreasing in the amount of public funds used to deal with the bank’s insolvency:

$$\Omega = R_2 + \lambda F$$

with $\lambda < 0$ that denotes the ratio of the deadweight cost\(^{14}\) of using one dollar of public funds to the value generated by one dollar of loans.\(^{15}\) The parameter $F$ includes the public funds used for the capitalization of the bank, the payment of the guarantee on deposits, and the cost of the investment to improve bank supervision, net of the proceeds from the bank’s privatization and the net value (if positive) of the nationalized bank.

The banker’s objective is to maximize the expected value of the investment in the bank, $\pi$, minus the price $P$ paid for it, $E(\pi - P)$. The value of the investment is

$$\pi = l_{nac}\zeta + (1 - l_{nac})[\text{Max}(0, R_2 + l_B\Delta A - D)]$$

$l_{nac}$ and $l_B$ are functions that take the value of 1 when the bank is nationalized or bailed out and zero otherwise.

**Consistency condition**

After privatization, the value of the bank at the beginning of $t = 1$ (before the banker’s first investment) must be positive:

$$E_1(R_2 - D) > 0.$$  \hspace{1cm} (2.7)

The term $E_1(R_2)$ is the expected value in $t = 1$ of the bank’s assets excluding the public bond that would be transferred to the bank with a bailout policy. If condition 2.7 is not fulfilled, the bank is technically insolvent even before making any investment choice, and the regulator would be induced to take over the bank in $t = 1$. But the regulator may have the incentive to takeover the bank even if condition 2.7 is satisfied. This would be the case when the regulator presumes that the banker is playing risky in $t = 1$.\(^{16}\)

\(^{13}\)This ensures that the depositors are willing to renew their deposits in $t = 1$ and $t = 2$.

\(^{14}\)The deadweight cost derives from the distorting effects of raising taxes and increasing fiscal deficits.

\(^{15}\)Ω is a reduced form. The extended form of the objective function is $\omega = (R_2 - D) + \beta R_2 + \theta F$, where $(R_2 - D)$ is the net value of the bank, while $\beta R_2$ is the economic surplus generated by the bank’s debtors (assumed to be a proportion $\beta$ of the bank’s loans). The term $\theta$ stands for the deadweight costs of a dollar of fiscal funds. Dividing by $(1 + \beta)$ we have that $\frac{\omega}{1+\beta} = R_2 - \frac{D}{1+\beta} + \lambda F$, where $\lambda = \frac{\theta}{(1+\beta)}$.

Maximizing $\frac{\omega}{1+\beta}$ yields the same results than maximizing $\Omega$; in fact, $\frac{\omega}{1+\beta}$ is fixed and independent of the adopted policy.

\(^{16}\)Such presumption is possible because the objective functions and investment returns are common knowledge.
The model is first solved by assuming that there is no ex-ante intervention.\textsuperscript{17} In the extensions (section 5), this assumption is removed and the model is solved for the case in which ex-ante interventions are possible.

**Timing of events**

The complete game takes place over four periods, but it reduces to a three stage game. The first stage consists in a simultaneous-move game where it is decided whether privatization takes place and whether the regulator invests in bank supervision. If privatization does not take place the regulator makes the two investment decisions before the bank is closed and the game ends. If the bank is privatized, in the second stage the regulator and the private banker engage in another simultaneous-move game: the banker decides the first investment and the regulator announces a risk-contingent policy to tackle the possible bad outcomes of the bank’s investment. In the last stage of the game, the banker (or the regulator if in control of the bank) makes the second investment choice. A summary of the structure of the game and timing of events is presented in table 1.

<table>
<thead>
<tr>
<th>( t = 0 )</th>
<th>The regulator offers the bank for privatization</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 1 )</td>
<td>First investment decision of the bank</td>
</tr>
<tr>
<td>( t = 2 )</td>
<td>The outcome of the first investment is revealed to the bank owner</td>
</tr>
<tr>
<td>( t = 3 )</td>
<td>The outcome of the second investment revealed to the bank owner</td>
</tr>
<tr>
<td></td>
<td>The public bond matures</td>
</tr>
<tr>
<td></td>
<td>The bank is closed</td>
</tr>
<tr>
<td></td>
<td>The regulator pays the guarantee on deposits (if required)</td>
</tr>
</tbody>
</table>

### Table 1. Timing of events

3. **BETTING THE BANK FOR SURVIVAL**

The rationale of the regulator’s intervention is to stop the banker’s strategy of ‘betting the bank for survival’ (BBFS) in the last stage of the game. This strategy is defined as follows.

\textsuperscript{17}It is assumed that there are institutions that stop the regulator from taking over the bank in \( t = 1 \) if the audit cannot prove that the banker is playing risky, which is always the case because by definition the banker’s actions are not observable.
Definition 2 (Betting the bank for survival). *Betting the bank for survival (BBFS) is a strategy according to which, in* \( t = 2 \), *the banker always makes a risky investment after a bad return to the first investment in* \( t = 1 \). *Instead, the banker always makes a safe investment after a good return to the investment in* \( t = 1 \).

When the banker plays BBFS, the regulator is induced to intervene because risky investments are socially inefficient, as their expected returns are inferior to the returns that can be obtained when playing safe (equations 2.2 and 2.3).

The intuition that explains the banker’s willingness to play BBFS is that with a low (or zero) net value of the bank, due to the first bad return, the regulator bears the losses from a subsequent bad outcome because of the commitment to fully guarantee deposits. In other words, the full guarantee of deposits eliminates the depositors’ incentive to discipline the banker by demanding risk premia. As bad returns are transferred to the regulator, the decision about which investment to make, risky or safe, depends exclusively on the magnitude of the good returns. As the good returns of the risky investment are assumed to be higher, the banker will always choose the risky investment.

BBFS, however, requires some additional conditions in order to be chosen by the banker. First, the bad outcome in the first period must reduce the bank’s net value to such an extent that a subsequent bad return must be mainly borne by the regulator. Second, after a bad return, a subsequent good return must allow the bank to recover and realize a positive net value. All this is formalized in the following proposition.

**Proposition 1.** Given the set of possible results \( \{R_G, R_B, S_G, S_B, A, D\} \), the following conditions are sufficient to ensure that the banker would adopt the strategy BBFS if the regulator does not intervene (forbearance).

1. In \( t = 2 \), the bank becomes insolvent, \( E_2(R_2 - D) < 0 \), only after a bad return of the risky investment.
2. In \( t = 3 \), the bank is insolvent, \( R_2 - D < 0 \), only after two consecutive bad returns regardless of the type of investment chosen.

**Proof.** See Appendix

Conditions 1 and 2 of proposition 1 ensure that a bad return, regardless of the type of investment made in the first period, reduces the bank’s net value to such an extent that the next bad return must be partially or totally absorbed by the regulator. Condition 2 also ensures that a good return to the second investment provides the banker with a positive net value at the end of \( t = 3 \).

In these conditions, the banker has the incentive to make a risky investment in \( t = 2 \) after a bad return to the first investment. In fact, the banker would bear limited costs from an additional bad return but would enjoy entirely the benefits of a good return.

When it is known that the banker will play BBFS, the regulator can intervene in two ways: takeover or bailout. A takeover prevents the bank from making a second risky investment because the investment (or the closing down of the bank) is now decided

\[ \text{Obviously, the bank must be privately owned. Only in this case can the banker transfer bad returns to the regulator. A public bank would never choose the risky investment because the regulator internalizes the inefficiencies of this type of investment.} \]
by the regulator. A bailout disciplines the banker by increasing the bank’s net value; therefore the inefficiency of the risky investment is internalized by the bank.

The effectiveness of the takeover depends on the probability of detection $\gamma$. In fact, the regulator can take the bank over only if an audit detects the bad outcome of the investment. The effectiveness of the bailout depends on the size of the capitalization offer, which can affect the banker’s behavior only if it is above a certain threshold. All this is formalized in proposition 2 below.

**Proposition 2.** The regulator’s intervention in $t = 2$ prevents the banker from playing BBFS in the following two cases:

1. If the audit detects, with probability $\gamma$, a bad outcome of the investment in $t = 1$ (takeover).
2. If the capitalization offer to the banker is sufficiently high (bailout). More precisely, it must be:
   
   (a) $\Delta A_S \geq -S_B + \frac{\delta}{1-\delta} R_G - (A - D)$ when the banker has played safe in $t = 1$.
   
   (b) $\Delta A_R \geq -R_B + \frac{\delta}{1-\delta} R_G - (A - D)$ when the banker has played risky in $t = 1$.

**Proof.** See Appendix

The bailout policy that disciplines the banker is feasible only if the capitalization offer satisfies the no-gift constraints of equations 2.5 and 2.6. Formally,

$$
\frac{\delta}{1-\delta} R_G < A - D
$$

4. Bailouts and the ex-ante and ex-post discipline of the banker

In $t = 1$, the banker makes the first investment choice and, simultaneously, the regulator announces the risk-contingent policy.\(^{19}\) The regulator can announce only time-consistent policies because he/she cannot pre-commit. Therefore the regulator’s selected risk-contingent policy must consist of a pair of best responses to every banker’s choice in $t = 1$. As a consequence, the regulator implements a strictly dominant policy, and the banker reacts to it.

The intervention of the regulator and the convenience to bailout are justified when the banker plays the BBFS strategy in $t = 2$ and when the disciplining capitalization specified in proposition 2 is feasible.\(^{20}\) The game is solved only for a feasible set as defined below.

**Definition 3** (feasible set). The feasible set $\Psi=\{R_G, R_B, S_G, S_B, A, D, \delta\}$ that comply with proposition 1 and equation 3.1.

\(^{19}\)The game is depicted in this way even though chronologically the banker’s decision is made before the regulator’s announcement. The rationale for considering the two decisions as simultaneous is the existence of informational asymmetries that force the regulator to make his/her choice without knowing the banker’s previous move.

\(^{20}\)The regulator can only commit to bailout with capitalizations that are equal to the minimum required to prevent the banker from playing BBFS, $\Delta A_S = -S_B + \frac{\delta}{1-\delta} R_G - (A - D)$ and $\Delta A_R = -R_B + \frac{\delta}{1-\delta} R_G - (A - D)$. A larger capitalization would be redundant; while a bailout with a lower capitalization is always strictly dominated by forbearance, which is equivalent to a bailout with a zero capitalization offer.
The nationalization of the bank always dominates its closure when the regulator takes over the bank and detects a bad return to a risky investment. This follows from the assumption of a zero liquidation price of the assets in $t = 2$, and the commitment to guarantee deposits.\footnote{If we assume that the regulator can refuse to guarantee the deposits, then, even if the liquidation price of the assets is zero, it may be advantageous to close the bank if $\lambda$ is sufficiently large, so that the benefits of not paying the deposit guarantee outweigh the costs of the asset liquidation. In order that the nationalization is dominant, it must be $E_2(\Omega_{\text{nationalization}}) > E_2(\Omega_{\text{closure}})$, which amounts to $A + R_B - c + \lambda(D - A - R_B + c) > 0$. Solving for $\lambda$, we obtain $\lambda > -\frac{A - D}{1}$. The right hand side of this inequality is negative and decreases with the leverage of the bank, for any given $A - D$. Thus, for this constraint on $\lambda$ not to be binding, it is sufficient to assume large values of $A$ and $D$ for a given $(A - D)$.}

The regulator’s best response to the banker’s investment choice in $t = 1$ requires balancing the costs of BBFS, financial repression and fiscal funds to capitalize the bank. With forbearance, BBFS takes place and reduces the economic value of the bank’s assets; consequently, there arise positive expected deadweight costs for the payment of the deposit guarantee. Takeover reduces the probability of BBFS (in proportion to the probability $\gamma$ of detecting bad returns) but implies the costs associated to financial repression. Bailout eliminates BBFS and the expected payment of the deposit guarantee, but it increases the deadweight costs of the public funds allocated to the capitalization of the bank.

The best response to the banker’s investment in $t = 1$, therefore, depends on $\{\psi, c, \lambda, \gamma\}$. Any option (forbearance, bailout or takeover) can be the best response. If any policy can be the best response, it must be true that there exists a set $\{\psi, c, \lambda, \gamma\}$ that supports strictly dominant risk-contingent policies that include bailout.

Proposition 3 shows that, when risk is verifiable, the regulator can implement policies that discipline the banker ex-ante and ex-post. Such policies are characterized by bailouts to the benefit of the conservative banker. The proposition also shows that even when the regulator cannot pre-commit to any policy, there still exists a subset of $\{\psi, c, \lambda, \gamma\}$ for which the regulator can announce these policies.

**Proposition 3.** When there is a pure strategy equilibrium the following holds:

1. The regulator implements a strictly dominant policy $a = \{a_R, a_S\}$, which depends on $\{\psi, c, \lambda, \gamma\}$. $a_i, (i = R, S)$ is the best response function.\footnote{Each $a$ depends on $\psi, c, \lambda, \gamma$; with $\psi \in \Psi$.}

2. When $\psi \in \Psi$ there exist sets $\{\psi, c, \lambda, \gamma\}$ that support risk-contingent policies that discipline the banker in $t = 1$ and $t = 2$. These policies imply a bailout in favor of the conservative banker when a bad return occurs: $\{T_R, B_S\}$ and $\{F_R, B_S\}$.

**Proof.** See Appendix

The set of all possible $\{\psi, c, \lambda, \gamma\}$ can be partitioned in three subsets, each associated to a different best response to the banker’s investment in $t = 1$ (either safe or risky). Figure 1 represents, in the space($c, \lambda$), the locus that separates the set into these three
subsets. Each area is the image in the \((c, \lambda)\) space of the intersection of sets in which a policy dominates the other two. The separating locus itself is formed by the indifference curves between pairs of policies, in which the third policy is strictly dominated. There always exists a best response because the indifference loci between pairs of policies all intersect in the same point \((c^*_i, \lambda^*_i)\), in which the regulator is indifferent between bailout, forbearance, and takeover.

Result 2 of proposition 3 states that risk-contingent policies that imply a bailout are always possible, and a subset of such policies will achieve ex-ante and ex-post discipline of the banker’s behavior. This can be illustrated by overlapping the \(a_R(\psi, \ c, \ \lambda, \ \gamma)\) and \(a_S(\psi, \ c, \ \lambda, \ \gamma)\) functions. By doing so, the feasible risk-contingent policies can be determined. This is presented in figure 2 in the \((c, \lambda)\) space. The best response function to the banker making a risky investment in \(t = 1\) has the indifference locus shifted to the right and upwards compared to the locus associated to the best response when the banker plays safe. The explanation is that the fiscal cost of stopping BBFS is higher when the banker has played risky and higher for bailout compared to takeover or forbearance.

The two policies \(\{T_R, B_S\}\), and \(\{F_R, B_S\}\), which include the bailout of the conservative banker, discipline the banker in \(t = 1\) and \(t = 2\). They discipline the banker in \(t = 1\) because they provide the conservative banker with subsidies and the risky banker with punishment. These policies also prevent BBFS because the bad returns are tackled with bailouts (as the banker always plays safe in the first investment). Figure 2 exemplifies condition (2) of proposition 3. It shows that there are always sets \(\{\psi, \ c, \ \lambda, \ \gamma\}\) that support strictly dominant \(\{T_R, B_S\}\) and \(\{F_R, B_S\}\) policies, for which the banker is disciplined in \(t = 1\) and \(t = 2\). It is so because the best response function to the risky

\[\frac{\delta}{1 - \delta} R_G + (1 - \delta)R_B - \delta S_B - \delta(A - D) > 0\]

which implies that it is feasible for the regulator to commit to forbearance \((F)\) when the banker played the safe investment in \(t = 1\).

This is explained by the fact that the regulator’s payoff to playing takeover is a linear combination of the payoff to playing forbearance and the payoff to playing takeover with perfect detection \((\gamma=1)\):

\[E_2(\Omega_T^1) \equiv (1 - \gamma)E_2(\Omega_F^0) + \gamma E_2(\Omega_T^{\gamma=1})\].

This has two consequences. First, the indifference locus between takeover and forbearance is determined by \(E_2(\Omega_T^0) = E_2(\Omega_T^{\gamma=1})\) and, second, the indifference locus between bailout and takeover is determined by \(E_2(\Omega_T^0) - E_2(\Omega_B^0) = \gamma(\{E_2(\Omega_F^0) - E_2(\Omega_T^{\gamma=1})\})\).

Then, for the set \(\{\psi, \ c, \ \lambda, \ \gamma\}\) where the regulator is indifferent between bailout and forbearance \((E_2(\Omega_F^0) = E_2(\Omega_B^0))\) and indifferent between takeover and forbearance \((E_2(\Omega_T^0) = E_2(\Omega_T^{\gamma=1}))\), he/she must also be indifferent between takeover and bailout.

An additional assumption is required, that is to say that \((1 - 2\delta)R_B - \delta(A - D) < (R_B - S_B)\), which ensures that a move from playing safe to playing risky in \(t = 1\) increases the fiscal costs of forbearance more than in the case of takeover. This rules out the possibility that the banker plays \(\{F_R, T_S\}\), a policy that benefits the risky banker relatively to the conservative banker.

Risk-contingent policies like \(\{B_R, B_S\}\) and \(\{T_R, T_S\}\) cannot discipline the banker. Pure bailout, \(\{B_R, B_S\}\), cannot discipline the banker ex-ante because the regulator cannot credibly commit to give premia to the conservative banker in the capitalization offer. Once the banker accepts the bailout offer, the regulator can only provide the minimum capitalization that eliminates BBFS. In the case of pure takeover, \(\{T_R, T_S\}\) the banker is disciplined ex-ante when the private costs \((\zeta)\) imposed to the (detected) risky banker are high enough. However, the banker is not disciplined ex-post as he/she will play BBFS with probability proportional to \(\gamma\).
investment shifts to the right and upwards compared to the best response to the safe investment.

Figure 2. The regulator’s feasible risk-contingent policies in $t = 2$
The constraints required to obtain the results of propositions 1, 2, and 3 are compatible with the consistency constraint, which requires that the net value of the bank is positive after the privatization. This is expressed in the following proposition.

**Proposition 4.** When the model is solved for all $\psi \in \Psi$ that comply with $(1 - 2\delta)R_B - \delta (A - D) < R_B - S_B$ and $\frac{\delta}{1 - \delta} R_G - (1 - \delta)R_B - \delta S_B - \delta (A - D) > 0$, and for costs of financial repression $c \leq A - D + S_B + S_G$, then:

1. The net value of the bank in $t = 1$ is positive, $E_1(R_2 - D) > 0$.
2. The banker's evaluation in $t = 0$ of the investment in the bank is positive, $E_0(\pi) > 0$.

**Proof.** See Appendix

5. **FINANCIAL LIBERALIZATION**

In the initial stage the banker must decide whether to bid or not for the bank and, simultaneously, the regulator must decide whether to invest or not in bank supervision. The following assumptions are made in order to solve this game:

(i) In the case of bidding, the banker must offer a non-negative price for the bank, $P \geq 0$.

(ii) There is no competition in the privatization process as there is only one bidder ('the banker') who makes a ‘take it or leave it’ offer.

(iii) The commitment to invest in bank supervision is mandatory only if the bank is privatized.

(iv) The regulator transfers the ownership of the bank when the bid is larger than the reservation price, $P_R$. Formally

$$P_R^j = \{ P : E_0(\Omega_{\text{priv}}^j(P, \psi, c, \lambda, \gamma_0, g, \zeta)) = E_0(\Omega_{\text{nopriv}})^{28} \}$$

where $E_0(\Omega_{\text{priv}}^j(P, \psi, c, \lambda, \gamma_0, g, \zeta))$ is the regulator’s expected payoff in $t = 0$ when playing ‘$j$’, and $E_0(\Omega_{\text{nopriv}})$ is the expected payoff in $t = 0$ when no privatization takes place.$^{29}$

With bank privatization and positive costs of financial repression, risk-contingent policies that discipline the banker and include bailouts in favor of the conservative banker, e.g. $\{T_R, B_S \}$, can be part of the equilibrium solution even if the cost $g$ of the investment in supervision is close to zero. It is so because the cost of financial repression associated to the takeover, if sufficiently high, can make the bailout dominate the takeover as the best response to any of the banker’s investment decisions for any level of $\gamma$.

**Proposition 5.** If there is a privatization process and the cost of financial repression is different from zero ($c > 0$), there exists a set $\{\psi, c, \lambda, \gamma_0, g, \zeta\}$ that generates equilibria with privatization, risk-contingent policies that include a bailout in favor of the conservative banker, i.e. $\{T_R, B_S \}$ and $\{F_R, B_S \}$. The banker plays safe both in $t = 1$ and $t = 2$. Additionally,

$^{27}$The possibility of transfers from the regulator to the banker during privatization is ruled out by constraining the banker’s bid price to be non-negative.

$^{28}$The value of $P_R^j(\psi, c, \lambda, \gamma_0, g, \zeta)$ can be positive or negative.
(1) The price paid for the bank is smaller than the value of the bank in private hands. In fact, the banker in equilibrium bids $P = \max(0, P^j_R)^+$, where $P^j_R$ is the regulator’s reservation price when $j$ is the policy played in $t = 0$.

(2) The set of all possible equilibria with $\{T_R, B_S\}$ is divided into a subset that supports investment in supervision and a subset in which investment in supervision never takes place.

Proof. See Appendix

The results of proposition 5 are illustrated in Figure 3, which shows the image in the $(c, \lambda)$ space of the regulator’s best response functions and how they change for the two possible values of $\gamma$. The effect of the increase from $\gamma_0$ to $\gamma = 1$ is to shift to the left and upwards the indifference locus between bailout and takeover, while leaving unaffected the other indifference loci. The shift is sufficient to make $(0, 0)$ the origin of the indifference locus.

By proposition 3 we know that $\{T_R, B_S\}$ and $\{F_R, B_S\}$ are always feasible, i.e. there exists a set $\{\psi, c, \lambda, \gamma_0, g, \zeta\}$ that supports such policies. When $\lambda \{\psi, c, \lambda, \gamma_0, g, \zeta\}$ is part of the set $A$ in Figure 3, the equilibrium with $\{T_R, B_S\}$ is associated to no investment in bank supervision. In this case the benefits of investing in supervision are zero because the increase in $\gamma$ cannot change any of the regulator’s best responses or the banker’s investment choice in $t = 1$, which is to play safe.

If $\{\psi, c, \lambda, \gamma_0, g, \zeta\}$ is part of the set $C$, the investment in bank supervision changes the best response to the banker playing risky in $t = 1$: from bailout to takeover. By investing in supervision the regulator is able to commit to the policy $\{T_R, B_S\}$ instead of $\{B_R, B_S\}$, simultaneously inducing a change in the banker’s investment choice in $t = 1$: from risky to safe. When this is the case, the regulator has positive benefits from investing in supervision; the investment is made if the cost $g$ is sufficiently low.

$\text{These equilibria can occur only if the private valuation of the bank is positive and larger than the regulator’s reservation price. Only in this case can the banker offer a bid that the regulator is willing to accept:} \tag{5.1}$

$$E_0(\pi^j(\psi, c, \lambda, \gamma_0, g, \zeta)) > \max(0, P^j_R(\psi, c, \lambda, \gamma_0, g, \zeta))$$

[30]To clarify these results it is convenient to compare them with those that would arise under the alternative hypothesis that there is no liberalization and the cost of financial repression is zero ($c = 0$). In this case investment in supervision would take place when $g$ is sufficiently low and $\{T_R, T_S\}$ would be the only dominant risk contingent policy, ruling out bailouts from the equilibrium path of the game. In terms of Figure 3, the sets $\{\psi, c, \lambda, \gamma_0, g, \zeta\}$ would be represented only as a point on the vertical axis, e.g. point $H$, and the indifference loci reduced to a point on the vertical axis (the intersection of the loci with the vertical axis). Forbearance would be strictly dominated by takeover, and therefore there would be only three possible dominant risk-contingent policies. When investment in supervision is made, takeover will be a best response for any of the banker’s choices in $t = 1$ and for $\lambda < 0$, making $\{T_R, T_S\}$ the dominant policy. Any point on the vertical axis would be below the indifference point $(0, 0)$. As a consequence, investment in supervision takes place when the cost $g$ is so small that the expected benefits outweigh it.
of risk-contingent policy played in equilibrium by the regulator, or the type of investment chosen by the banker in $t = 1$ and $t = 2$ (see proposition 4).

When the reservation price is positive (the cost of financial repression is sufficiently low) it is possible that privatization does not take place as the condition 5.1 might not be fulfilled. In this case, 5.1 reduces to:

$$ (5.2) \quad (1 - \frac{1}{\lambda})(E_0(R_2(\psi, c, \lambda_0, \gamma, \zeta, g)) - E_0(R_2^{NP})) - 1_{\text{inv}}g > 0. $$

The evaluation of the bank by the banker is larger than the regulator’s reservation price only when the expected value of the bank’s assets ($E_0(R_2(\psi, c, \lambda, \gamma_0, \zeta, g))$) is higher under private ownership than under public ownership ($E_0(R_2^{NP})$). Only in this case can the banker compensate the regulator and still make a profit by purchasing the bank.

5.2 is always satisfied when, in the equilibrium of the complete game for set $A$, the regulator makes no investment in supervision in $t = 0$ and the risk-contingent policy $\{T_R, B_S\}$ is played. In fact, $\{T_R, B_S\}$ disciplines the banker in $t = 1$ and $t = 2$ and therefore the value of the bank’s assets is maximum and $1_{\text{inv}} = 0$, as no investment in supervision is made.

In the case of policies $\{T_R, B_S\}$ and with investment in bank supervision, set $C$ (see Figure 3), the feasibility of the equilibrium requires that 5.2 with $l_{\text{inv}} = 1$ is fulfilled. For the highest levels of $g$ that still allow investment in bank supervision, 5.2 can only be satisfied by a subset of the elements of set $C$.\footnote{The proof of this statement is included in the proof of proposition 5.}

**Figure 3.** The regulator’s best response functions in $t = 2$ with $\gamma_0$ and $\gamma = 1$

**Cooperative equilibria and no investment in bank supervision**

When the banker and the regulator are allowed to cooperate, $\{T_R, B_S\}$ can be part of the
equilibrium solution even if in the non-cooperative game, the regulator has an incentive to invest in supervision and then, as a result, play \( \{T_R, T_S\} \). This is the case of set \( B \) in Figure 3. In a non-cooperative scenario, if the cost \( g \) is sufficiently low, the set \( B \) leads to \( \{T_R, T_S\} \), investment in supervision, and the minimum price \( P = \max(0, P_R^{WI}) \) paid for the bank.

In the cooperative scenario, the banker pays a higher price for the bank, \( P > \max(0, P_R^{NI}) \), in exchange for a low-quality bank supervision, i.e. the regulator makes no investment in supervision in \( t = 0 \). As a consequence the regulator plays \( \{T_R, B_S\} \) (instead of \( \{T_R, T_S\} \)), the banker is disciplined in \( t = 1 \) and benefits from a bailout in case of bad returns. The banker then bids a higher price for the bank. The condition for this equilibrium to be possible is:

\[
E_0(\pi^{NI}) - E_0(\pi^{WI}) > P_R^{NI} - P_R^{WI}
\]

where the superscript \( WI \) and \( NI \) respectively denote whether there is or not an investment in bank supervision.

In order for the compensation transfer to be possible, the resulting increase in the private evaluation of the bank must be larger than the increase in the regulator’s reservation price. This inequality can be simplified to:

\[
(1 - \frac{1}{\lambda})(E_0(R_2(\ldots)^{WI}) - E_0(R_2(\ldots)^{NI})) + g > 0
\]

which means that the cooperative equilibrium is possible when the value of the bank’s assets without investment in supervision is larger than the value of the assets with investment. This is always true because \( \{T_R, B_S\} \) disciplines the banker ex-ante and ex-post, while \( \{T_R, T_S\} \) cannot avoid the efficiency costs of BBFS and financial repression. Cooperative equilibria in set \( C \) are not viable because in that case the value of the bank’s assets is higher when an investment in supervision is made.

**Time inconsistency of ex-ante regulatory solutions**

In some circumstances, ex-ante regulatory solutions are not time consistent. So far it was assumed that the investment in bank supervision affects only the regulator’s options to tackle the bank’s insolvency ex-post. It can be shown that if the investment affects the regulator’s ability to detect the banker’s actions ex-ante, this does not ensure that the regulator always invests in supervision.

The introduction of ex-ante regulation requires the following modifications of the model:

(i) The regulator has the option to invest in bank prudential regulation in \( t = 0 \) and be able to observe the banker’s action in \( t = 1 \) and, therefore, force him/her to play safe in \( t = 1 \).

(ii) The cost of the investment in prudential regulation must be \( g_1 > 0 \).

(iii) The regulator can detect the net value of the bank in \( t = 2 \) with probability \( \gamma = 1 \).

\(^{32}\)Assumption (iii), which means perfect detection by the regulator, simplifies the analysis.
The regulator has an incentive to invest in bank prudential regulation in \( t = 0 \) only when the dominant risk-contingent policy in the continuation game (in \( t = 2 \)) induces the banker to play risky in \( t = 1 \). When \( \{ \psi, c, \lambda, \gamma = 1, g, \zeta \} \) supports policies \( \{ T_R, B_S \} \) and \( \{ F_R, B_S \} \), where the banker is disciplined ex-ante, the regulator never invests in bank prudential regulation. For example, this occurs within sets \( A \) and \( C \) in Figure 3. Within these sets, investment in prudential regulation is superfluous because the regulator tackles a bad return with bailout, regardless of having previously invested or not in prudential regulation. This is expressed by the following proposition.\(^{33}\)

**Proposition 6.** For equilibria with pure strategies in the complete game, when \( \{ \psi, c, \lambda, \gamma = 1, g, \zeta \} \) supports risk contingent policies that discipline the banker in \( t = 1 \), privatization takes place without any investment in bank prudential regulation.

*Proof.* The key assumption for this result is that the investment in bank prudential regulation is only effective in \( t = 1 \), but not in \( t = 2 \) after that the bad outcome materializes and the regulator cannot induce the banker to play safe unless it decides to bailout or takeover the bank.

**Bank deregulation**

Most of the model’s results hold also for the case in which financial liberalization takes the form of deregulation. The main difference is that, in the case of deregulation, the banker cannot compensate the regulator for the liberalization by bidding for the bank; therefore the set \( \{ \psi, c, \lambda, \gamma_0, g, \zeta \} \) that supports bank deregulation is a subset of the set that supports liberalization with privatization.

To allow the model to deal with the deregulation case the following changes must be made:

(i) The banker owns the bank during all four periods (except when the bank is nationalized). Therefore, in \( t = 0 \) the banker’s bidding price is zero (\( P = 0 \)), and the regulator agrees to deregulate when the reservation price is negative \( P_R^R < 0 \). The reservation price must also take into account that the banker owns the bank.\(^{34}\)

(ii) With financial repression the bank has access only to the safe investment and there exists a cost \( c \) that reduces the investments’ average returns. The cost \( c \) represents the negative effects of limiting the banker’s access to investment technologies.

\(^{33}\)An alternative (when there is no investment in bank prudential regulation) is to allow the regulator to take the bank over in \( t = 0 \). If the regulator infers that the banker plays risky in \( t = 1 \) then he/she has an incentive to take the bank over in \( t = 0 \) in order to avoid the banker’s moral hazard. Takeover is rational in \( t = 0 \) when the regulator cannot observe the banker’s actions and therefore is not able to control the banker’s behavior. If the banker expects that the regulator intervenes in \( t = 0 \), he/she may not bid for the bank and privatization does not take place. The introduction of this feature however does not modify our results. The risk-contingent policies that support the banker’s discipline in \( t = 1 \) and \( t = 2 \), such as \( \{ T_R, B_S \} \) or \( \{ F_R, B_S \} \), where bailouts are played in equilibrium to benefit the conservative banker, by definition do not elicit the takeover by the regulator in \( t = 0 \).

\(^{34}\)The definition of the reservation price changes to \( P_R^R = \{ P : E_0(\Omega(P, \psi, c, \lambda, \gamma_0, g, \zeta)) = E_0(\Omega_{\text{nodereg}}) \} \). The difference from the previous definition is that the objective function with no deregulation (\( E_0(\Omega_{\text{nodereg}}) \)) is now:

\[
E_0(\Omega_{\text{nodereg}}) = \begin{cases} 
A - 2c - (1 - \delta)^2 \lambda(A - D + 2S_b - 2c) & \text{if } c < \frac{1}{4}(A - D + 2S_G + S_B) \\
A - 2c - \lambda(A - D - 2c) + \lambda \delta^2 (A - D + 2S_G - 2c) & \text{if } c > \frac{1}{4}(A - D + 2S_G + S_B)
\end{cases}
\]
Deregulation, by allowing the banker to have access to all investment technologies, increases the safe investment’s average return (by making \( c = 0 \)), provides the bank with access to the risky investment, and creates asymmetries of information between the banker and the regulator.

(iii) When the regulator takes over the bank it reintroduces financial repression because he/she does not have the knowhow to operate with the new technologies.\(^{35}\)

With deregulation, the liberalization stage is exclusively determined by the regulator’s decision to deregulate and invest in bank supervision. Deregulation depends entirely on the regulator because the banker bids zero and always prefers deregulation.

Assumptions (ii) and (iii) guarantee that the results of stages two and three of the game are the same as in the case of privatization; therefore, also they can be illustrated by using Figure 3.

The main results of proposition 5 hold also for the case of deregulation, i.e. there is always a set \( \{\psi, c, \lambda, \gamma_0, g, \zeta\} \) that supports equilibria with deregulation and \( \{TR, BS\} \) as the regulator’s dominant policy in \( t = 2 \). Figure 3 shows that the policy \( \{TR, BS\} \) is the regulator’s strictly dominant policy in \( t = 2 \) for some \( \{\psi, c, \lambda, \gamma_0, g, \zeta\} \).

In such cases some of these \( \{\psi, c, \lambda, \gamma_0, g, \zeta\} \) comply with \( P_R(\psi, c, \lambda, \gamma_0, g, \zeta) < 0 \) making \( \{TR, BS\} \) part of the equilibria with deregulation in the complete game. Since bankers cannot bid for deregulation, the set \( \{\psi, c, \lambda, \gamma_0, g, \zeta\} \) that supports deregulation must be a subset of the set that supports privatization. This is expressed in the following proposition.

**Proposition 7.** When liberalization takes the form of deregulation under assumptions (i)-(iii) above, the following holds:

1. There exists a set \( \{\psi, c, \lambda, \gamma_0, g, \zeta\} \) that supports equilibria with deregulation, \( \{TR, BS\} \), and the banker playing safe in \( t = 1 \) and \( t = 2 \).
2. This set is only a subset of the elements that support \( \{TR, BS\} \) with privatization.

**Proof.** See Appendix.

The absence of transfer mechanisms reduces the set of elements that support liberalization and also rules out the possibility of cooperative equilibria of the kind described above. This feature of the deregulation process has policy implications. Deregulation takes place only for cases in which \( c \) is sufficiently high, while privatization admits lower levels of the cost of financial repression. It is evident that deregulation is identical to privatization if the banker is allowed to bid for the deregulation.

It is possible to extend the model to consider a case of ‘regulatory capture’, where the banker has an incentive to provide direct informal monetary transfers to the regulator and these transfers do not generate revenues for the public sector. Deregulation through cronyism may be costly in terms of efficiency if \( \lambda \) takes on a negative value sufficiently large. It would therefore be advantageous to eliminate the phenomenon by introducing formal mechanisms through which the banker can bid for deregulation.

Assumptions (ii) and (iii) make proposition 6 valid also for the case of deregulation. The regulator does not invest in ex-ante prudential regulation when the time consistent

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\(^{35}\)If the regulator could manage efficiently the new investment technologies then it would be convenient for him/her to purchase (nationalize) the bank from the banker and afterwards deregulate.
policy already disciplines the banker in $t = 1$. By proposition 7 we know that policies that include bailouts to conservative bankers such as $\{T_R, B_S\}$ are feasible, and discipline bankers in $t = 1$. By proposition 6 we know that in that case the regulator would never invest in prudential regulation to control the banker’s ex-ante risk taking.

6. CONCLUSIONS

The present model was developed to explain how governments react to solvency problems of banks. The model assumes that the banker’s risk is verifiable, that the regulator could not pre-commit and that the liberalization process (bank privatization) takes place before a potential solvency shock.

In the model, the rationale of bailouts is to avoid the costs of financial repression associated to the takeover of the bank. Under certain assumptions, equilibria with bailouts are always feasible. This result differs from the result that is obtained when the privatization process is not part of the game and the cost of financial repression is zero. In this latter case, equilibria always includes takeover and bailouts are unfeasible if the costs of investment in bank supervision are sufficiently low.

In some circumstances, the regulator can commit to policies that punish the risky banker with takeover and benefit the conservative banker with bailout. This policy disciplines the banker ex-ante and ex-post and the value of the bank’s assets takes its maximum value.

From this result several implications follow. First, there is a set of cooperative equilibria where the regulator finds convenient not to invest in bank supervision in exchange for a higher bidding price for the bank. Second, the regulator does not introduce prudential regulation ex-ante if the time consistent policies to tackle bad returns ex-post also discipline the banker ex-ante. This result depends on the regulator’s inability to pre-commit and the fact that prudential regulation can affect only the banker’s first investment, while it has no effect on the banker’s second investment.

Finally, the main results of the model also apply to a case in which liberalization takes the form of deregulation. There are, however, some differences between the two cases of liberalization. With deregulation, liberalization is less likely to occur because the banker cannot compensate the regulator for the liberalization. Deregulation takes places only when the cost of financial repression is so high that a monetary compensation is unnecessary. In the absence of a formal channel to bid for liberalization, there can be informal mechanisms of bidding, such as the case of ‘regulatory capture’.
REFERENCES


Appendix A

A.1. Proof of proposition 1. First, assume that the banker played risky in $t = 1$ and realized a bad return. If the banker plays risky again in $t = 2$ the expected pay-off is

$$E_2(\pi_F^{RR} - P) = \delta(\text{Max}(0, A + R_B + R_G - D) - P) + (1 - \delta)(\text{Max}(0, A + 2R_B - D) - P)$$

If the banker plays safe in $t = 2$, the expected pay-off is

$$E_2(\pi_F^{RS} - P) = \delta(\text{Max}(0, A + R_B + S_G - D) - P) + (1 - \delta)(\text{Max}(0, A + R_B + S_B - D) - P)$$

Since it is $\text{Max}(0, A + 2R_B - D) = 0$ and $\text{Max}(0, A + R_B + S_B - D) = 0$, the banker will always play risky because

$$E_2(\pi_F^{RR} - P) - E_2(\pi_F^{RS} - P) = R_G - S_G > 0$$

Second, assume that the banker played risky in $t = 1$ and realized a good return. If the banker plays risky again in $t = 2$ his expected payoff is

$$E_2(\pi_F^{RR} - P) = \delta(\max(0, A + 2R_G - D) - P) + (1 - \delta)(\max(0, A + R_G + R_B - D) - P)$$

If he plays safe in $t = 2$ his expected payoff is

$$E_2(\pi_F^{RS} - P) = \delta(\max(0, A + R_G + S_G - D) - P) + (1 - \delta)(\max(0, A + R_G + S_B - D) - P)$$

Since it is $\text{Max}(0, A + R_G + R_B - D) = A + R_G + R_B - D$ and $\text{Max}(0, A + R_G + S_B - D) = A + R_G + S_B - D$, the banker will always play safe in $t = 2$ because $E_2(\pi_F^{RR} - P) - E_2(\pi_F^{RS} - P) = \delta R_G + (1 - \delta)R_B < 0$.

Second, assume that the banker played safe in $t = 1$, the same result applies. After a bad result the banker always plays risky because

$$E_2(\pi_F^{SR} - P) - E_2(\pi_F^{SS} - P) = R_G - S_G > 0$$

and after a good result he/she always plays safe in $t = 2$ because

$$E_2(\pi_F^{SR} - P) - E_2(\pi_F^{SS} - P) = \delta R_G + (1 - \delta)R_B < 0.$$

A.2. Proof of proposition 2. Assume that the set $\{R_G, R_B, S_G, S_B, A, D\}$ fulfills conditions (1) and (2) of proposition 1.

A.2.1. Case 1 of proposition 2 (takeover). The proof is two parts: (i) it is shown that if the regulator takes the bank over, in $t = 2$ a safe investment is always made; (ii) it is shown that when the regulator chooses to take the bank over, the banker hides bad returns and, therefore, BBFS can be stopped only with probability $\gamma$, i.e. the probability to detect the bank’s true net value.

i. Assume that the regulator detected a bad return to the risky investment in $t = 1$ and has taken the bank over. If the regulator plays risky in $t = 2$, the expected payoff is

$$E_2(\Omega_T^{RR}) = \delta(A + R_B + R_G - c) + (1 - \delta)(A + 2R_B - c) + \lambda[-(\delta(A + R_B + R_G - c - D) + (1 - \delta)(A + 2R_B - c - D))] + \lambda(g - P)$$

If the regulator plays safe in $t = 2$, the expected payoff is

$$E_2(\Omega_T^{RS}) = \delta(A + R_B + S_G - c) + (1 - \delta)(A + R_B + S_B - c) + \lambda[-(\delta(A + R_B + S_G - c - D) + (1 - \delta)(A + R_B + S_B - c - D))] + \lambda(g - P)$$
The regulator will always play safe because
\[ E_2(\Omega_T^{R^S}) - E_2(\Omega_T^{R^R}) = (\lambda - 1)(\delta R_G + (1 - \delta)R_B) > 0 \]

Now assume that the regulator has detected a bad return to the safe investment and has taken the bank over. If the regulator plays risky in \( t = 2 \), the expected payoff is
\[ E_2(\Omega_T^{S^R}) = \delta(A + S_B + R_G - c) + (1 - \delta)(A + S_B + R_B - c) \\
+ \lambda M \max(0, (A + S_B + R_G - c)) \\
+ (1 - \delta) M \max(0, (A + S_B + R_B - c - D))] + \lambda(g - P) \]

If the regulator prefers to play safe because 
\[ E_2(\Omega_T^{S^S}) = \delta(A + S_B + S_G - c) + (1 - \delta)(A + 2S_B - c) \\
+ \lambda M \max(0, (A + S_B + S_G - c)) \\
+ (1 - \delta) M \max(0, (A + 2S_B - c - D))] + \lambda(g - P) \]

Notice that, from equation 2.1, \( M \max(0, D - (A + S_B + R_B - c - D)) = 0 \) and \( M \max(0, D - (A + S_B + S_G - c)) = 0 \).

These payoffs differ from those associated to the previous case when the banker plays risky in \( t = 1 \). The expected pay-off in \( t = 1 \), the expected pay-off is
\[ E_2(\Omega_T^{S^S}) - E_2(\Omega_T^{S^R}) = -\delta R_G + (1 - \delta)R_B + \lambda(1 - \delta)(R_B - S_B) > 0 \]

ii. Assume that the banker played risky in \( t = 1 \) and realized a bad return. The regulator chooses to take the bank over. If the regulator reveals the true net value of the bank, the expected pay-off is
\[ E_2(\pi_{T,hide}^R - P) = -\gamma \zeta + (1 - \gamma) \delta (A + R_B + R_G - D) - P \]

The banker will always hide the true net value because
\[ E_2(\pi_{T,hide}^R - P) - E_2(\pi_{T,reveal}^R - P) = (1 - \gamma) \zeta + (1 - \gamma) \delta (A + R_B + R_G - D) > 0 \]

Assume now that the banker played safe in \( t = 1 \) and realized a bad return. If the banker reveals the true net value, the expected pay-off is
\[ E_2(\pi_{T,reveal}^S - P) = \delta(A + S_B + S_G - c - D) - P \]

If the banker hides the true net value, the expected pay-off is
\[ E_2(\pi_{T,hide}^S - P) = \gamma(\delta(A + S_B + S_G - c - D) - P) + (1 - \gamma)(\delta(A + S_B + R_G - D) - P) \]

The banker will always hide the true net value because
\[ E_2(\pi_{T,hide}^S - P) - E_2(\pi_{T,reveal}^S - P) = (1 - \gamma) \delta (R_G - S_G + c) > 0 \]
A.2.2. Case 2 of proposition 2 (bailout). The proof is in two parts: (i) it is shown that if the banker accepts the bailout offer, he/she will play safe in $t = 2$ only if the capitalization is larger than the threshold indicated in the proposition; (ii) it is shown that the bailout offer is always accepted by the banker.

i. Assume that the banker played the risky investment in $t = 1$; a bad return is revealed; the bailout offer is accepted and the true net value of the bank is revealed. If the banker plays risky again in $t = 2$, the expected pay-off is

$$E_2(\pi^{RR}_B - P) = \delta (\text{Max}(0, A + R_B + R_G + \Delta A_R - D) - P) +$$

$$+(1 - \delta) (\text{Max}(0, A + 2R_B + \Delta A_R - D) - P)$$

Conditions 1 and 2 of proposition 1 and equation 2.6 ensure that $\text{Max}(0, A + 2R_B - D + \Delta A_R) = 0$ and $\text{Max}(0, A + R_B + R_G + \Delta A_R - D) = A + R_B + R_G + \Delta A_R - D$, therefore, if the banker plays safe in $t = 2$, the expected pay-off is

$$E_2(\pi^{RS}_B - P) = \delta (\text{Max}(0, A + R_B + S_G + \Delta A_R - D) - P) +$$

$$+(1 - \delta) (\text{Max}(0, A + R_B + S_B + \Delta A_R - D) - P)$$

From condition 2 of proposition 1, it is

$$\text{Max}(0, A + R_B + S_G + \Delta A_R - D) = A + R_B + S_G + \Delta A_R - D$$

If $\Delta A_R > -(A + R_B + S_B - D)$, the banker plays safe only if

$$E_2(\pi^{RS}_B - P) - E_2(\pi^{RR}_B - P) = (A + R_B + \Delta A_R - D - P) +$$

$$-(\delta (A + R_B + R_G + \Delta A_R - D) - P) \geq 0$$

which can be simplified to

$$E_2(\pi^{RS}_B - P) - E_2(\pi^{RR}_B - P) = (1 - \delta)(\Delta A_R - (-R_B + \frac{\delta}{1 - \delta} R_G - (A - D)))$$

Thus, the banker plays safe only if

$$\Delta A_R \geq -R_B + \frac{\delta}{1 - \delta} R_G - (A - D)$$

In this case it is always true that $\Delta A_R > -(A + R_B + S_B - D)$. Assume now that the banker made a safe investment in $t = 1$, and realized a bad return. Applying the same methodology we know that the banker plays safe in $t = 2$ when

$$E_2(\pi^{SS}_B - P) - E_2(\pi^{SR}_B - P) = (1 - \delta)(\Delta A_S - (-S_B + \frac{\delta}{1 - \delta} R_G - (A - D))) \geq 0$$

Thus, the banker plays safe only if

$$\Delta A_S \geq -S_B + \frac{\delta}{1 - \delta} R_G - (A - D).$$

ii. The banker always accepts a bailout offer and reveals the bank’s net value because the capitalization is non-negative. If the capitalization were zero, the banker would be indifferent between accepting and rejecting the bailout, as a bailout with zero capitalizations is equivalent to forbearance.
A.3. **Proof of proposition 3.** The proof is in two parts. First it is shown that the regulator plays a strictly dominant policy \(\{a_R(\psi, c, \lambda, \gamma), a_S(\psi, c, \lambda, \gamma)\}\). The second part shows that there exists a set \(\{\psi, c, \lambda, \gamma\}\) for which the strictly dominant risk-contingent policy implies a bailout in the equilibrium path of the game, and simultaneously disciplines the banker in \(t = 1\) and \(t = 2\).

A.3.1. **Proof of part 1 of proposition 3.**

(1) **Definitions**

a. \(E_2(\Omega_R), E_2(\Omega_B), \) and \(E_2(\Omega_T)\), are the expected payoffs of playing \(F, B, \) or \(T\) as a response to the banker playing \(i\) in \(t = 1\) (\(i = R, S\)).

b. The payoffs \(E_2(\Omega_F^R), E_2(\Omega_B^R), \) and \(E_2(\Omega_T^R)\) are defined as:

\[
E_2(\Omega_F^R) = A + (2 - \delta)C_e + \lambda(1 - \delta)^2(D - A - 2R_B) + \lambda(g - P)
\]

\[
E_2(\Omega_B^R) = A + C_e + \lambda(1 - \delta)(-R_B + \frac{\delta}{1 - \delta}R_G - (A - D)) + \lambda(g - P)
\]

\[
E_2(\Omega_T^R) = A + C_e + (1 - \delta)((1 - \gamma)C_e - \gamma c) + \lambda\gamma(1 - \delta)(D - A - R_B + c) + \lambda(1 - \gamma)(1 - \delta)^2(D - A - 2R_B) + \lambda(g - P)
\]

The payoffs \(E_2(\Omega_F^S), E_2(\Omega_B^S), \) and \(E_2(\Omega_T^S)\) are defined as:

\[
E_2(\Omega_F^S) = A + (1 - \delta)C_e + \lambda(1 - \delta)^2(D - A - S_B - R_B) + \lambda(g - P)
\]

\[
E_2(\Omega_B^S) = A + C_e + \lambda(1 - \delta)(-S_B + \frac{\delta}{1 - \delta}R_G - (A - D)) + \lambda(g - P)
\]

\[
E_2(\Omega_T^S) = A + (1 - \delta)((1 - \gamma)C_e - \gamma c) + \lambda(1 - \delta)^2(D - A - S_B - \gamma(S_B - c) - (1 - \gamma)R_B) + \lambda(g - P)
\]

c. Define the following functions:

\[
\Phi_{FB}^i = E_2(\Omega_F^i) - E_2(\Omega_B^i)
\]

\[
\Phi_{FT}^i = E_2(\Omega_F^i) - E_2(\Omega_T^i)
\]

\[
\Phi_{BT}^i = E_2(\Omega_B^i) - E_2(\Omega_T^i)
\]

where \(i\) is the type of investment played by the banker in \(t = 1\).

Solving the \(\Phi\) functions renders for \(i = R\),

\[
\Phi_{FB}^R = C_e - \lambda(X + B)
\]

\[
\Phi_{FT}^R = C_e + c - \lambda(B + c)
\]

\[
\Phi_{BT}^R = \lambda(X + B(1 - \gamma) - \gamma c) - (1 - \gamma)C_e + \gamma c
\]

and for \(i = S\),

\[
\Phi_{FB}^S = C_e - \lambda(D + B')
\]

\[
\Phi_{FT}^S = C_e + c - \lambda(1 - \delta)(E + c)
\]

\[
\Phi_{BT}^S = \lambda(D + B' - (1 - \delta)\gamma(E + c)) - (1 - \gamma)C_e + \gamma c
\]

Where,

\[
X = \frac{\delta}{1 - \delta}R_G
\]

\[
B = (1 - 2\delta)R_B - \delta(A - D)
\]

\[
E = (R_B - S_B)
\]

\[
B' = (1 - \delta)R_B - \delta S_B - \delta(A - D)
\]
\( C_e = \delta R_G + (1 - \delta) R_B \)

d. From proposition 1 and equation 2.6 we have \( X > 0, B < 0, E < 0, X + B > 0, B < C_e, \) and \( E < C_e. \)

e. Additionally, we assume that \( B < E \) and \( X + B' > 0. \)

f. Define \( \{ \psi, \gamma, c^*, \lambda^*_i \} \) as the elements that make the regulator indifferent between the three possible responses to the banker's first investment \( (E_2(\Omega^*_F) = E_2(\Omega^*_R) = E_2(\Omega^*_T)). \)

If the banker plays risky in \( t = 1 \) and for any given \( \{ \psi, \gamma \}, \) it is true that \( E_2(\Omega^*_F) = E_2(\Omega^*_R) = E_2(\Omega^*_T) \) when \( \lambda = \lambda^*_R \) and \( c = c^*_R, \) with

\[
\lambda^*_R = \frac{C_e}{X + B}
\]

\[
c^*_R = -C_e \frac{X}{X + B - C_e}
\]

If the banker plays safe in \( t = 1 \) and for any given \( \{ \psi, \gamma \}, \) it is true that \( E_2(\Omega^*_F) = E_2(\Omega^*_B) = E_2(\Omega^*_T) \) when \( \lambda = \lambda^*_S \) and \( c = c^*_S, \) with

\[
\lambda^*_S = \frac{C_e}{X + B'}
\]

\[
c^*_S = -C_e \frac{X + B' - (1 - \delta)E}{X + B' - (1 - \delta)C_e}
\]

We know that \( \lambda^*_R > \lambda^*_S. \)

(2) \textit{Definition of best response function when the banker plays risky in } t = 1

\textbf{Forbearance} is a best response when \( \Phi^R_{FB} > 0 \) and \( \Phi^R_{FT} > 0, \) which boils down to:

- If \( 0 < c < c^*_R, \) forbearance is never a best response.
- If \( c^*_R < c < -B, \) forbearance is a best response when \( \lambda^*_R > \lambda > \frac{c + c}{B + c}. \)
- If \( -B < c, \) forbearance is a best response when \( \lambda < \lambda^*_R. \)

\textbf{Bailout} is a best response when \( \Phi^R_{FB} < 0 \) and \( \Phi^R_{BT} > 0, \) which boils down to:

- If \( 0 < c < c^*_R, \) bailout is a best response when \( \lambda > \frac{(1 - \gamma)C_e - \gamma c}{X + B(1 - \gamma) - \gamma c}. \)
- When \( c^*_R < c, \) bailout is a best response when \( \lambda > \lambda^*_R. \)

\textbf{Takeover} is a best response when \( \Phi^i_{FT} < 0 \) and \( \Phi^i_{BT} < 0, \) which boils down to:

- When \( 0 < c < c^*_R, \) takeover is a best response when \( \lambda < \frac{(1 - \gamma)C_e - \gamma c}{X + B(1 - \gamma) - \gamma c}. \)
- When \( c^*_R < c < -B, \) takeover is a best response when \( \lambda < \frac{c + c}{B + c}. \)
- When \( -B < c \) then takeover is never a best response.
Putting everything together the best response function can be defined as:

$$a_R(\psi, c, \lambda, \gamma) = \begin{cases} 
0 < c < c_R^a : & \begin{cases} 
\lambda > \frac{(1-\gamma)C_e - \gamma c}{X + B(-1(1-\gamma)) - \gamma c} : \text{Bailout} \\
\lambda < \frac{(1-\gamma)C_e - \gamma c}{X + B(-1(1-\gamma)) - \gamma c} : \text{Takeover} 
\end{cases} \\
c_R^b < c < -B : & \begin{cases} 
\lambda > \lambda_R^\psi : \text{Bailout} \\
\lambda < \text{Takeover} 
\end{cases} \\
-B < c : & \begin{cases} 
\lambda > \lambda_R^\psi : \text{Bailout} \\
\lambda < \lambda_R^\psi : \text{Forbearance} 
\end{cases}
\end{cases}$$

(3) Definition of the best response function to the banker playing safe in \(t = 1\)

With the same methodology used for the previous case, the best response function can be defined as:

$$a_S(\psi, c, \lambda, \gamma) = \begin{cases} 
0 < c < c_S^* : & \begin{cases} 
\lambda > \frac{(1-\gamma)C_e - \gamma c}{X + B(-1(1-\gamma)) - \gamma c} : \text{Bailout} \\
\lambda < \frac{(1-\gamma)C_e - \gamma c}{X + B(-1(1-\gamma)) - \gamma c} : \text{Takeover} 
\end{cases} \\
c_S^* < c < -E : & \begin{cases} 
\lambda > \lambda_S^\psi : \text{Bailout} \\
\lambda < \text{Takeover} 
\end{cases} \\
-E < c : & \begin{cases} 
\lambda > \lambda_S^\psi : \text{Bailout} \\
\lambda < \lambda_S^\psi : \text{Forbearance} 
\end{cases}
\end{cases}$$

A.3.2. Proof of part 2 of proposition 3. First it will be shown that risk contingent policies \(\{T_R, B_S\}\) and \(\{F_R, B_S\}\) discipline the banker in \(t = 1\) and \(t = 2\), and that in the event of a bad return bailout will be played in the equilibrium path of the game. Second, it will be shown that the there exists a set \(\{\psi, c, \lambda, \gamma\}\) that supports \(\{T_R, B_S\}\) or \(\{F_R, B_S\}\) as the regulator’s strictly dominant risk contingent policy.

i. Let us assume that the regulator’s strictly dominant policy is \(\{F_R, B_S\}\). If the banker plays safe in \(t = 1\) then its expected payoff will be

$$E_1(\pi^R_{\{F_R, B_S\}} - P) = \delta\delta(A + 2S_G - D) + (1 - \delta)(A + S_G + S_B - D) + (1 - \delta)[\delta\max(0, A + S_B + S_G + \Delta A_S - D)] - P = A - D + (1 - \delta)\Delta A_S - P.$$ 

When the banker plays risky in \(t = 1\) its expected return is

$$E_1(\pi^R_{\{F_R, B_S\}} - P) = \delta\delta(A + R_G + S_G - D) + (1 - \delta)(A + R_G + S_B - D) + (1 - \delta)[\delta\max(0, A + R_B + R_G - D) + (1 - \delta)\max(0, A + 2R_B - D)] - P = \delta(A + R_G - D) + (1 - \delta)\delta(A + R_B + R_G - D) - P.$$ 

Then

$$E_1(\pi^R_{\{F_R, B_S\}} - P) = -S_B - \delta(A - D + R_B + R_G) > 0$$

for all \(\psi \in \Psi\).

Now let’s assume that the regulator’s strictly dominant policy is \(\{T_R, B_S\}\). It will always be true that

$$E_1(\pi^R_{\{T_R, B_S\}} - P) > E_1(\pi^R_{\{F_R, B_S\}} - P)$$

because it is true that

$$E_1(\pi^R_{\{T_R, B_S\}} - P) > E_1(\pi^S_{\{T_R, B_S\}} - P).$$

This is sufficient to prove that the banker will always play safe, as it was shown above that

$$E_1(\pi^S_{\{T_R, B_S\}} - P) > E_1(\pi^R_{\{F_R, B_S\}} - P),$$

and it is straightforward that

$$E_1(\pi^R_{\{T_R, B_S\}} - P) < E_1(\pi^R_{\{F_R, B_S\}} - P).$$

As both \(\{T_R, B_S\}\) and \(\{F_R, B_S\}\) discipline the banker in \(t = 1\) to play safe, and both have bailout as a best response to a bad return of the safe investment, it is true that the
regulator will always play bailout in the equilibrium path of the game when a bad return occurs.

ii. There exists a set \( \{\psi, c, \lambda, \gamma\} \) that supports \( \{T_R, B_S\} \). The proof for \( \{F_R, B_S\} \) is analogous. The conditions for \( \{T_R, B_S\} \) to be strictly dominant are:

1. When \( 0 < c < c^*_{SR} : \lambda < \frac{(1-\gamma)C_e-\gamma c}{X+B(1-\gamma)-\gamma c} \)
2. When \( c^*_{SR} < c < -B : \lambda < \frac{C_e/c - \gamma c}{B(c+c)} \)
3. When \( 0 < c < c^*_{SR} : \lambda > \frac{(1-\gamma)C_e-\gamma c}{X+B(1-\gamma)(E+c)} \)
4. When \( c^*_{SR} < c : \lambda > \lambda_S^s \)

Conditions (1) and (2) are the conditions for takeover to be the best response to the banker’s risky investment in \( t = 1 \), and conditions (3) and (4) are the conditions for bailout to be the best response to the banker’s safe investment in \( t = 1 \). Thus, it must be shown that there exists a set \( \{\psi, c, \lambda, \gamma\} \) that satisfies simultaneously the above inequalities.

When \( c^*_{SR} > c^*_{SR} \), conditions (1)-(4) can be reduced to the three conditions presented below:

1. For \( 0 < c < c^*_{SR} : \frac{(1-\gamma)C_e-\gamma c}{X+B(1-\gamma)-\gamma c} > \lambda > \frac{(1-\gamma)C_e-\gamma c}{X+B(1-\gamma)(E+c)} \)
2. For \( c^*_{SR} < c < c^*_{SR} : \frac{(1-\gamma)C_e-\gamma c}{X+B(1-\gamma)-\gamma c} > \lambda > \lambda_S^s \)
3. For \( c^*_{SR} < c : \frac{C_e/c - \gamma c}{B(c+c)} > \lambda > \lambda_S^s \)

It will be sufficient here to show that there always exists a \( \lambda \) to comply with (1’). (although there is always a \( \lambda \) that complies with (2’) and (3’), but the proof will not be presented here.) We know this because it is true that \( \frac{(1-\gamma)C_e-\gamma c}{X+B(1-\gamma)-\gamma c} < 0 \) and \( \frac{(1-\gamma)C_e-\gamma c}{X+B(1-\gamma)(E+c)} < 0 \), and \( \frac{(1-\gamma)C_e-\gamma c}{X+B(1-\gamma)-\gamma c} > \frac{(1-\gamma)C_e-\gamma c}{X+B(1-\gamma)(E+c)} \). If the last inequality is solved for \( c \) then it renders \( -\frac{B}{\delta} + E(1 + \frac{\delta - \gamma}{\delta}) > c \). We know that if \( 0 < c < c^*_{SR} \) then it will always be true that \( -\frac{B}{\delta} + E(1 + \frac{\delta - \gamma}{\delta}) > c \). The left side of the last inequality cannot be lower than \( -\frac{1+\gamma}{\delta} E \), which is the case if we substitute \( E \) for \( B \), that is \( -\frac{B}{\delta} + E(1 + \frac{\delta - \gamma}{\delta}) \) \( > -\frac{B}{\delta} + E(1 + \frac{\delta - \gamma}{\delta}) \) \( = -\frac{1+\gamma}{\delta} E \). On the other hand the value of \( c \) must be lower than \( c^*_{SR} \). It is easy to show that \(-\frac{1+\gamma}{\delta} E > c^*_{SR} \) because by definition \( c^*_{SR} < -E \) (see the best response function to the banker playing safe in \( t = 1 \)).


1. Proof of result 1

It is sufficient to calculate the net value of the bank when the banker is expected to play risky in \( t = 1 \) and the regulator takeover in \( t = 2 \), with the maximum possible level of costs of financial repression \( c = A + S_B + S_G - D \). If the net value is positive it will also be positive for any other action by the banker and policy choice by the regulator.

The bank’s net value in \( t = 0 \) right after liberalization is:

\[
E_{t=0,T_R,T_S} = A + C_e + (1 - \delta)(-\gamma c + (1 - \gamma)C_e) - D
\]
Because \( C_\epsilon > -(A + S_B + S_G - D) \), then it is sufficient to show that the net value is positive for \( \gamma = 1 \):

\[
E_{0, \{T_R, T_S\}}(R_2(\gamma = 1) - D) = A + C_\epsilon - (1 - \delta)c - D > 0
\]

Which can be simplified to:

\[
\delta R_G + (1 - \delta)R_B + \delta(A - D) + (2\delta - 1)S_G > 0.
\]

To prove that this term is always positive, it must be showed that if \( R_B, S_G \) and \( R_G \) are substituted by their lowest possible values then the term is still positive. What are the lowest possible values of \( R_B, S_G, \) and \( R_G \)? When we solve the model for the set of all \( \psi \in \Psi \) that comply with \( B < E \) and \( X + B' > 0 \), then the set \( (R_G, R_B, S_G, S_B, \delta, A, D) \) is constrained to the following:

1. \(-\delta((A - D) + 2R_B) < -S_B < \frac{\delta}{1-\delta}R_G\)
2. \(-(1 - \delta)(A - D + 2R_B) < R_G < \frac{1 - \delta}{\delta}(A - D)\)
3. \(-(1 - \delta)(A - D + 2R_B) < S_G < R_G\)
4. \((A - D) < -R_B < (A - D)(\frac{1 + \delta}{2\delta})\)
5. \(\delta^2 + \delta - 1 > 0\).

Then \( E_0(R_2 - D) \rightarrow (A - D)(-\frac{3 + 8\delta - 3\delta^2}{2\delta}) \) a term that will always be positive for \( \delta \) which comply with \( \delta \in (\frac{1}{2}, 1) \) and \( \delta^2 + \delta - 1 > 0 \). When

\[
\begin{align*}
R_G &\rightarrow -(1 - \delta)(A - D + 2R_B) \\
S_G &\rightarrow -(1 - \delta)(A - D + 2R_B) \\
R_B &\rightarrow -(A - D)
\end{align*}
\]

Then \( E_0(R_2 - D) \rightarrow (A - D)(-\frac{2 + 6\delta - 3\delta^2}{2\delta}) \) a term that will always be positive for \( \delta \) which comply with \( \delta \in (\frac{1}{2}, 1) \) and \( \delta^2 + \delta - 1 > 0 \).

2. **Proof of result 2** If \( E_0(R_2 - D) > 0 \) and there is a probability zero that the bank is nationalized (where the banker suffers \( \zeta \) private costs) then it will always be true that \( E_0(\pi) > 0 \) because the banker benefits from limited liability in the bank’s capital. When there is a positive probability of bank nationalization, the banker plays risky in \( t = 1 \) an the regulator’s best response is takeover, then it must be true that \( E_1(\pi^R_{(T_R, T_S)}) > E_1(\pi^S_{(T_R, T_S)}) \), and we know that \( E_1(\pi^S_{(T_R, T_S)}) = \delta(A + S_G - D) + (1 - \delta)\gamma(S_G - c) + (1 - \gamma)R_G \) is always positive. Therefore, when risky is played, \( E_1(\pi^R_{(T_R, T_S)}) \) must be positive too, guaranteed by low enough private cost \( \zeta \), such that:

\[
\zeta < -\frac{\delta(S_G - R_G) + (1 - \delta)(1 - \gamma)(S_B - R_B)}{1 - \delta}(A + S_B + S_G - c - D)
\]

A.5. **Proof to proposition 5.** It is proven that with bank privatization and \( c > 0 \), there exists a set \( \{\psi, c, \lambda, gamma, 0, g, \zeta\} \) that supports \( \{T_R, B_S\} \), and the banker playing safe both in \( t = 1 \) and \( t = 2 \), in the complete game. It is also shown that only
a subset of these equilibria implies investment in supervision. In any case, the banker will always bid the price $P = \max(0, P_R^j)^\dagger$. The case of $\{F_R, B_S\}$ will not be proven here.

Case 1. Equilibrium with $\{T_R, B_S\}$ and no investment in bank supervision. In this case $\{T_R, B_S\}$ is the regulator’s strictly dominant policy for both $\gamma_0$ and $\gamma = 1$.

Step 1. There exists a set $(p, c, \lambda, \gamma_0, \zeta)$ that complies with:

1. $\Phi_{F_T^R}(\gamma, c) = C_e + c - \lambda(B + c) < 0$
2. $\Phi_{B_T^R}(\gamma_0, c) = \lambda(X + B(1 - \gamma_0) - \gamma_0c) - (1 - \gamma_0)C_e + \gamma_0c < 0$
3. $\Phi_{B_T}^R(\gamma = 1, c) = \lambda(X - c) + c < 0$
4. $\Phi_{F_B}^S(\gamma, c) = C_e - \lambda(X + B') < 0$
5. $\Phi_{B_T}(\gamma = 1, c) = \lambda(X + B' - (1 - \delta)(E + c)) + c > 0$
6. $\Phi_{B_T}^S(\gamma_0, c) = \lambda(X + B' - (1 - \delta)\gamma_0(E + c)) + \gamma_0c - (1 - \gamma_0)C_e > 0$

Inequalities (1), (2), and (3) define the conditions for bailout to be the best response to the banker playing risky in $t = 1$ with $\gamma_0$ and $\gamma = 1$. Inequalities (4), (5), and (6) are the conditions for bailout to be the best response to bankers playing safe in $t = 1$ with both $\gamma_0$ and $\gamma = 1$. Let us assume that $c_S^* < c_R^*$. These inequalities lead to the following conditions:

1. If $0 < c < c_S^*$: then $\frac{(1 - \gamma_0)C_e - \gamma_0c}{X + B(1 - \gamma_0) - \gamma_0c} > \lambda > -\frac{c}{X + B' - (1 - \delta)(E + c)}$
2. If $c_S^* < c < c_R^*$: then $\frac{(1 - \gamma_0)C_e - \gamma_0c}{X + B(1 - \gamma_0) - \gamma_0c} > \lambda > \lambda_S^*$
3. If $c_R^* < c < -B$: then $\frac{c + c}{B + c} > \lambda > \lambda_S^*$

It is sufficient here to prove that a $\lambda$ exists to comply with condition (1') (even though there also exists $\lambda$ to comply with conditions (2') and (3')).

If the inequality $\frac{(1 - \gamma_0)C_e - \gamma_0c}{X + B(1 - \gamma_0) - \gamma_0c} > -\frac{c}{X + B' - (1 - \delta)(E + c)}$ is solved for $c$ it renders $c > -C_e c(1 - \gamma_0)(X + B' - (1 - \delta)E) - (B_0 + (1 - \gamma_0)c_e - (1 - \delta)E)$ or $\lambda = \lambda_S^*$.

This last inequality is sufficient to prove that condition (1') is met because it is true that $\frac{(1 - \gamma_0)(X + B' - (1 - \delta)E) - (B_0 + (1 - \gamma_0)c_e - (1 - \delta)E)}{\delta} < c_S^*$. For the assumption $c_S^* > c_R^*$, the proof is analogous.

Step 2. The regulator will play $\{T_R, B_S\}$ for both $\gamma_0$ and $\gamma = 1$ and the banker plays safe in $t = 1$. In case of a bad return the regulator plays bailout with a capitalization offer $\Delta A_S = -S_B + \frac{\delta}{1 - \delta}R_G - (A - D)$, both when $\gamma_0$ and $\gamma = 1$. It is therefore obvious that $E_0(\Omega(\Phi, \psi, c, \lambda, \gamma, \gamma = 1, g = 0, \zeta)) - E_0(\Omega(\Phi, \psi, c, \lambda, \gamma_0, g = 0, \zeta)) < -\lambda g$, the reservation price associated to the regulator playing investment is higher to the one associated to not playing investment: $P_R(\psi, c, \lambda, \gamma = 1, g, \zeta) > P_R(\psi, c, \lambda, \gamma_0, g = 0, \zeta)$, and vice versa. The proof of this statement is presented below.
By definition, reservation prices are those that make equal the regulator’s payoff with and without privatization. Formally:

1) \( E_0(\Omega(P, g, \gamma = 1, \ldots)) = E_0(\Omega_{\text{no priv}}) \)

2) \( E_0(\Omega(P, g = 0, \gamma_0, \ldots)) = E_0(\Omega_{\text{no priv}}) \)

The reservation price is the \( P \) that guarantees the above equalities: \( P_R^{WI} \) stands for the \( P \) that guarantees the equality when investment is played, and \( P_R^{NI} \) stands for the price \( P \) that guarantees the second equality, when no investment is played. We can write the left side of the equalities in the following way:

1') \( E_0(\Omega(P, g, \gamma = 1, \ldots)) = E_0(\Omega(P = 0, g = 0, \gamma = 1, \ldots)) + \lambda(g - P) \)

2') \( E_0(\Omega(P, g = 0, \gamma_0, \ldots)) = E_0(\Omega(P = 0, g = 0, \gamma_0, \ldots)) + \lambda(-P) \)

By introducing (1') and (2') into (1) and (2) respectively, and then solving for \( P \) we can define the reservation prices as:

\[
P_R^{WI} = \frac{1}{\lambda}[E_0(\Omega(P = 0, g = 0, \gamma = 1, \ldots)) - E_0(\Omega_{\text{no priv}})] + g
\]

\[
P_R^{NI} = \frac{1}{\lambda}[E_0(\Omega(P = 0, g = 0, \gamma_0, \ldots)) - E_0(\Omega_{\text{no priv}})]
\]

Then, the difference \( P_R^{NI} - P_R^{WI} \) can be expressed in the following terms:

\[
P_R^{NI} - P_R^{WI} = \frac{1}{\lambda}[E_0(\Omega(P = 0, g = 0, \gamma_0, \ldots)) - E_0(\Omega(P = 0, g = 0, \gamma = 1, \ldots))] - g.
\]

When \( P_R^{NI} - P_R^{WI} < 0 \), then it must be true that:

\[
E_0(\Omega(P = 0, g = 0, \gamma = 1, \ldots)) - E_0(\Omega(P = 0, g = 0, \gamma_0, \ldots)) < -\lambda g.
\]

Additionally we know that

\[
E_0(\Omega(P = 0, g = 0, \gamma = 1, \ldots)) - E_0(\Omega(P = 0, g = 0, \gamma_0, \ldots)) = E_0(\Omega(P, g = 0, \gamma = 1, \ldots)) - E_0(\Omega(P, g = 0, \gamma_0, \ldots)),
\]

therefore the difference \( P_R^{NI} - P_R^{WI} < 0 \) implies that:

\[
E_0(\Omega(P, g = 0, \gamma = 1, \ldots)) - E_0(\Omega(P, g = 0, \gamma_0, \ldots)) < -\lambda g.
\]

This is the condition that guarantees that the investment in bank supervision reduces the regulator’s expected payoff.

Step 4. Here it is proven that if \( \{T_R, B_S\} \) is strictly dominant for \( \gamma_0 \) and \( \gamma = 1 \), then in the first stage of the game the Nash equilibrium will consist in \{No investment, \( P = \max(0, P_R^j) \}\}, and privatization will take place.

The banker’s objective is to maximize \( E(\pi^j) - P \), where \( j \) represents the policy of investment or no investment in bank supervision in \( t = 0 \). If the banker decides not to bid or bids \( P < P_R^j \) then there is no privatization in equilibrium and the banker’s payoff is 0. If the banker’s bid \( P > P_R^j \) then privatization takes place and the banker’s benefits are \( E(\pi^j) - P \). So when \( E(\pi^j) < P_R^j \), the banker will not bid and his payoff will be 0, and when \( E(\pi^j) > P_R^j \), with the objective of maximizing his payoff, the banker will bid the minimum price that induces the regulator to accept and privatize the bank \( P = \max(0, P_R^j) \). The regulator’s objective is to maximize \( E_0(\Omega(\ldots)) \), which is equivalent to maximizing the difference between the bid and the reservation price \( P - P_R \). When the banker doesn’t bid or bids a price inferior to both reservation prices, then the privatization doesn’t take place and the regulator’s extraction is 0. For any given bidding price \( P \), that is superior to at least one of the reservation prices, the regulator will chose to invest or not invest in bank supervision in order to minimize the reservation price \( P_R \) and maximize the extraction \( P - P_R \). By step
2 we know that \( E_0(\Omega(P, g = 0, \gamma = 1, \ldots)) - E_0(\Omega(P, g = 0, \gamma_0, \ldots)) < -\lambda g \), and therefore, by step 3, \( P_{R}^{NI} < P_{R}^{WI} \). Therefore, the option of no investment weakly dominates that of investment in bank supervision. When this is the case \{No investment, \( P = \max(0, P_{R}^{NI})^{+} \)\} is the only possible equilibrium in \( t = 0 \). The regulator never deviates because playing investment would mean an inferior extraction, and possibly negative, in which case privatization would not take place. The banker never deviates because \( P = \max(0, P_{R}^{NI})^{+} \) maximizes \( E(\pi^{NI}) - P \). The pair \{ Investment, \( P = \max(0, P_{R}^{WI})^{+} \)\} is never a Nash equilibrium of the stage game because the regulator benefits from deviating to "no investment" as that increases the extraction \( P - P_{R} \) because \( P_{R}^{NI} < P_{R}^{WI} \).

Step 5. The equilibrium \{No investment, \( P = \max(0, P_{R}^{NI})^{+} \)\} requires that the evaluation of the bank by the private banker is larger than the minimum bid the regulator would accept for the bank:

\[
E_0(\pi^{NI}) - \max(0, P_{R}^{NI})^{+} > 0
\]

This term can be simplified to \((1 - \frac{1}{\lambda})(E_0R_2(\psi, c, g = 0, \lambda, \gamma_0, \zeta) - E_0R_{2,\text{no priv}}) > 0\). When \( \{T_{R}, B_{S}\}\) is part of the equilibrium, the banker is disciplined in \( t = 1 \) and \( t = 2 \) and therefore the economic value of the bank’s assets is the maximum. Formally,

\[
E_0(\pi^{NI}) - \max(0, P_{R}^{NI})^{+} = (1 - \frac{1}{\lambda})2c > 0
\]

and therefore privatization always takes place.

Case 2. Equilibrium with \( \{T_{R}, B_{S}\}\) and investment in bank supervision. In this case \( \{T_{R}, B_{S}\}\) is the regulator’s strictly dominant policy for both \( \gamma = 1 \) and \( \{B_{R}, B_{S}\}\) is strictly dominant for \( \gamma_0 \).

Step 1. It must be proven that it is feasible for \( \{T_{R}, B_{S}\}\) to be strictly dominant for \( \gamma = 1 \) and \( \{B_{R}, B_{S}\}\) for \( \gamma_0 \) in \( t = 2 \). For this to be true the set \( \{\psi, c, \lambda, \gamma_0, g, \zeta\} \) that respects the following inequalities must be different from an empty set \( \emptyset \):

1. \( \Phi_{FB}^{R}(\gamma, c) = C_{e} - \lambda(X + B)0 < 0 \)
2. \( \Phi_{FB}^{S}(\gamma, c) = C_{e} + c - \lambda(B + c) < 0 \)
3. \( \Phi_{BT}^{R}(\gamma, c) = \lambda(X + B(1 - \gamma_0) - \gamma_0c) - (1 - \gamma_0)C_{e} + \gamma_0c > 0 \)
4. \( \Phi_{BT}^{S}(\gamma = 1, c) = \lambda(X - c) + c < 0 \)
5. \( \Phi_{FB}^{R}(\gamma = 1, c) = C_{e} - \lambda(X + B') < 0 \)
6. \( \Phi_{BT}^{R}(\gamma = 1, c) = \lambda(X + B' - (1 - \delta)(E + c)) + c > 0 \)
7. \( \Phi_{BT}^{S}(\gamma, c) = \lambda(X + B' - (1 - \delta)\gamma_0(E + c)) + \gamma_0c - (1 - \gamma_0)C_{e} > 0 \)

Conditions (1)-(4) are required to secure that bailout is the best response with \( \gamma_0 \) to the banker playing risky in \( t = 1 \), and takeover is the best response with \( \gamma = 1 \) to the banker playing risky in \( t = 1 \). Conditions (5)-(7) are required to guarantee that bailout is always the best response to the banker playing safe for \( \gamma = 1 \) and \( \gamma_0 \). These seven conditions can be summarized in the following two:

Bailout is dominant for \( \gamma = 1 \) and \( \gamma_0 \) as a response to the banker playing safe in \( t = 1 \) when:

(a) If \( 0 < c < c_{S}^{\ast} \) then \( \lambda > -\frac{X + B' - (1 - \delta)(E + c)}{X + B' - (1 - \delta)\gamma_0(E + c) + \gamma_0c - (1 - \gamma_0)C_{e}} \)
(b) If } c^*_S < c \text{: then } \lambda > \lambda^*_S. \text{ Bailout is dominant for } \gamma_0 \text{ and takeover is dominant for } \gamma = 1 \text{ as a response to the banker playing risky in } t = 1 \text{ when: }

\begin{align*}
(1') \text{ If } 0 < c < c^*_B: \text{ then } \frac{c}{X - c} > \lambda > \frac{(1 - \gamma_0)C_e - \gamma_0 c}{X + B(1 - \gamma_0) - \gamma_0 c}.
\end{align*}

Let us assume that } c^*_S < c^*_B. \text{ It is sufficient to show that,}

\begin{align*}
- \frac{c}{X - c} > - \frac{c}{X + B' - (1 - \delta)(E + c)}
\end{align*}

to prove that the existence of a } \lambda \text{ that complies with above inequalities.

By solving for } c \text{ it renders } c < - \frac{B' - (1 - \delta)E}{\delta}. \text{ The right side of the inequality is positive and therefore the inequality will be true for at least a subset of the interval } 0 < c < c^*_S.

\text{Step 2. The regulator will play } \{T_R, B_S\} \text{ for } \gamma = 1 \text{ and the banker plays safe in } t = 1. \text{ In case of a bad return the regulator plays bailout with a capitalization offer } \Delta A_S = -S_B + \frac{\delta}{1 - \delta} R_G - (A - D), \text{ when } \gamma = 1. \text{ The regulator will play } \{B_R, B_S\} \text{ for } \gamma_0 \text{ and the banker plays risky in } t = 1. \text{ In case of a bad return the regulator plays bailout with a capitalization offer } \Delta A_R = -R_B + \frac{\delta}{1 - \delta} R_G - (A - D). \text{ Then:}

\begin{align*}
E_0(\Omega^{Safe}_{(T_R, B_S)}) - E_0(\Omega^{Risky}_{(B_R, B_S)}) = -C_e + \lambda(1 - \delta)E > 0,
\end{align*}

and

\begin{align*}
E_0(\Omega^{Safe}_{(T_R, B_S)}) - E_0(\Omega^{Risky}_{(B_R, B_S)}) > -\lambda g \text{ when } -\lambda g \text{ is sufficiently low, in which case investment in bank supervision increases the expected value of the regulator’s payoff.}
\end{align*}

\text{Step 3. When the benefits of investing in bank supervision are higher than the costs, } E_0(\Omega(P, \psi, c, \lambda, \gamma = 1, g = 0, \zeta)) - E_0(\Omega(P, \psi, c, \lambda, \gamma_0, g = 0, \zeta)) > -\lambda g, \text{ the reservation price associated to the regulator playing investment is lower than the one associated to not playing investment: } P_R(\psi, c, \lambda, \gamma = 1, g, \zeta) < P_R(\psi, c, \lambda, \gamma_0, g = 0, \zeta), \text{ and viceversa (see the proof above for the previous case).}

\text{Step 4. Here it will be proven that for the set } (\psi, c, \lambda, \gamma_0, g, \zeta) \text{ that supports } \{T_R, B_S\} \text{ as strictly dominant for } \gamma = 1, \text{ and } \{B_R, B_S\} \text{ for } \gamma_0, \text{ then for a subset it will be true that in the first stage of the game the Nash equilibrium will consist in } \{\text{Investment, } P = max(0, P_{RJ}^W)\}, \text{ and privatization will take place.}

\text{When } E(\pi^j) < P_{RJ}^j, \text{ the banker will not bid and his payoff will be 0, and when } E(\pi^j) > P_{RJ}^j, \text{ with the objective of maximizing his payoff, the banker will bid the minimum price that induces the regulator to accept and privatize the bank } P = max(0, P_{RJ}^W)^+.

\text{By step 2 we know that } E_0(\Omega(P, g = 0, \gamma = 1, \ldots)) - E_0(\Omega(P, g = 0, \gamma_0, \ldots)) > -\lambda g, \text{ and therefore, by step 3, } P_{RJ}^{NI} > P_{RJ}^W. \text{ Therefore, the option of investment weakly dominates that of no investment in bank supervision. When this is the case } \{ \text{investment, } P = max(0, P_{RJ}^W)^+ \} \text{ is the only possible equilibrium in } t = 0. \text{ The regulator never deviates because playing no investment would mean an inferior extraction } P - P_R, \text{ and possibly negative, in which case privatization}
would not take place. The banker never deviates because $P = \max(0, P^W_R)$ maximizes $E(\pi^W) - P$. The pair $\{ \text{No investment}, P = \max(0, P^N_R) \}$ is never a Nash equilibrium of the stage game because the regulator benefits from deviating to "investment" as that increases the extraction $P - P_R$ because $P^N_R > P^W_R$.

Step 5. The equilibrium $\{ \text{investment}, P = \max(0, P^W_R) \}$ requires the compliance with two additional constraints:

\begin{enumerate}
\item $(1 - \frac{1}{\lambda})(E_0R_2(\psi, c, g, \lambda, \gamma = 1, \zeta) - E_0R_{2, no \ priv}) > 0$
\item $E_0(\Omega^\text{Safe}_{TR, BS}) - E_0(\Omega^\text{Risky}_{BR, BS}) > -\lambda g$
\end{enumerate}

Condition (1) represents the constraint that the banker’s private valuation of the bank must be superior to the minimum bid the regulator is willing to accept, and condition (2) represents the condition for investment in supervision to improve the regulator’s expected payoff. These conditions can be simplified respectively to:

\begin{enumerate}
\item $(1') (1 - \frac{1}{\lambda})2c - g > 0$
\item $(2') -C_e + \lambda(1 - \delta)E > -\lambda g$
\end{enumerate}

When $g$ approaches zero then the equilibrium is always possible. When $g$ is just enough to encourage investment in bank supervision, $g \to \frac{C_e}{\lambda} - (1 - \delta)E$, the constraints $(1')$ and $(2')$ can be presented in a single constraint:

$c > \frac{\lambda}{\lambda - 1}(\frac{C_e}{2\lambda} - (\frac{1 - \delta}{2})E)$.

The proof of the feasibility of privatization requires the proof that the intersection between set $C$ (see figure 3 of main text) and the above condition is non-empty. Set $C$ represents the set where $\{ TR, BS \}$ is dominant with $\gamma = 1$, and $\{ BR, BS \}$ with $\gamma_0$. First, it must be noted that for any $\{ \psi, \gamma \}$, set $C$ is bounded by $(c^*, \lambda^*_R)$. Taking this into consideration it is clear that to prove that privatization can take place, it is sufficient to show that

$\lambda < \frac{C_e}{2X + (1 - \delta)E}$.

The intersection is non-empty because $\frac{C_e}{2X + (1 - \delta)E} > \lambda^*_R$.


A.6.1. Equilibria with deregulation, $\{ TR, BS \}$, and no investment in bank supervision. By step 1, result 1 of proposition 5 we know that there exists a set of elements $(\psi, c, \lambda, \gamma_0, g, \zeta)$ for which the policy $\{ TR, BS \}$ is strictly dominant for $\gamma_0$ and $\gamma = 1$, i.e. set $A$ in figure 3 of the main text. On the other hand, the set of elements that supports deregulation is determined by a negative reservation price $P_R(\psi, c, \lambda, \gamma_0, g, \zeta) < 0$, which
can be presented as a constraint on \( c \): \( c > f(\psi, c, \lambda, \gamma_0, g, \zeta) \), equivalent to,

\[
\frac{(1 - \delta)\lambda}{(1 - \delta)^2 \lambda - 1} \left( \frac{X + B' - (1 - \delta)E}{2} \right) < c,
\]

where the left hand side of the inequality is always inferior to \( \frac{1}{2} (A - D + S_B + S_G) \). This means that for \( c > \frac{1}{2} (A - D + S_B + S_G) \) deregulation always takes place. For all the set \( \{\psi, c, \lambda, \gamma_0, g, \zeta\} \) that support \( \{T_R, B_S\} \) as strictly dominant for \( \gamma_0 \) and \( \gamma = 1 \), the constraint \( c > f(\psi, c, \lambda, \gamma_0, g, \zeta) \) is satisfied. This is true because,

\[
\frac{(1 - \delta)\lambda}{(1 - \delta)^2 \lambda - 1} \left( \frac{X + B' - (1 - \delta)E}{2} \right) < \frac{\lambda}{\lambda(1 - \delta) - 1} \left( \frac{X + B' - (1 - \delta)E}{2} \right)
\]

where for a given set \( \{\psi, \lambda, \gamma_0, g, \zeta\} \) the left hand side represents the minimum value of \( c \) that makes deregulation convenient for the regulator, and the right hand side is the minimum \( c \) that guarantees that bailout (and not takeover) is the regulator’s best response to the banker playing safe in \( t = 1 \) with perfect detection (\( \gamma = 1 \)). Solving the inequality we have \( 1 + \delta - \lambda(1 - \delta)^2 > 0 \), which is always true.

A.6.2. Equilibria with deregulation, \( \{T_R, B_S\} \), and investment in bank supervision. It must be shown that for the set \( \{\psi, c, \lambda, \gamma_0, g, \zeta\} \) that supports \( \{T_R, B_S\} \) as strictly dominant for \( \gamma = 1 \) and \( \{B_R, B_S\} \) for \( \gamma_0 \), in other words set \( C \) of figure 3 in the main text, if there is deregulation, it will only be a subset of the \( \{\psi, c, \lambda, \gamma_0, g, \zeta\} \) that support privatization.

First, it must be noted that for any \( \{\psi, \lambda, \gamma_0, g, \zeta\} \), set \( C \) is bounded in the \((c, \lambda)\) space by \( (\psi_R, \lambda_R^*) \). If \( c < \frac{1}{2} (A - D + S_B + S_G) \), and assuming the highest value of \( g \) that still provides the regulator with the incentive to invest in bank supervision, \( g \rightarrow \frac{C_e}{X} - (1 - \delta)E \), deregulation takes place when,

\[
\frac{(1 - \delta)\lambda}{(1 - \delta)^2 \lambda - 1} \left( \frac{X + B' - (1 - \delta)E}{2} \right) + \frac{(1 - \delta)\lambda}{(1 - \delta)^2 \lambda - 1} \left( \frac{C_e}{2\lambda} - \frac{(1 - \delta)E}{2} \right) < c.
\]

For the proof it is sufficient to show that for \( c < \frac{1}{2} (A - D + S_B + S_G) \) the intersection of set \( C \) of figure 3 of the main text and the set of elements supporting deregulation is non empty. Formally, the compliance with the following inequality is required:

\[
\frac{(1 - \delta)\lambda}{(1 - \delta)^2 \lambda - 1} \left( \frac{X + B' - (1 - \delta)E}{2} \right) + \frac{(1 - \delta)\lambda}{(1 - \delta)^2 \lambda - 1} \left( \frac{C_e}{2\lambda} - \frac{(1 - \delta)E}{2} \right) < X \frac{\lambda}{\lambda - 1},
\]

where the right hand side of the inequality consists in the maximum value of \( c \) (given \( (\psi, \lambda, \gamma_0, g, \zeta) \)) for which the regulator prefers takeover over bailout as a response to the banker playing risky in \( t = 1 \) with perfect detection (\( \gamma = 1 \)).

The constraint for deregulation is more binding because \( \frac{(1 - \delta)\lambda}{(1 - \delta)^2 \lambda - 1} \left( \frac{X + B' - (1 - \delta)E}{2} \right) \) is positive and because \( \frac{\lambda}{\lambda - 1} < \frac{\lambda}{(1 - \delta)^2 \lambda - 1} \). In proposition 5 case 2 it was shown that privatization
takes place in a subset of \( C \). In the present case deregulation may take place or not, but if it does it must be a subset of the case with privatization as the constraint associated to deregulation is tighter.