Tax evasion and confidence in institutions:  
A theoretical model

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Abstract

In this paper we present a theoretical model of tax evasion, which takes into account the confidence of the taxpayer in respect of institutions and his sense of social responsibility. At this aim, in addition to the utility of income, is attributed to the agent a utility (‘confidence’) of contributing to the collective welfare. Confidence is a function of declared income, tax rate and effectiveness of public expenditure.

The classical model of Allingham-Sandmo-Yitzhaki ([1] and [14]) does not explain the cases in which tax compliance is chosen despite an apparently convenient gamble. Furthermore, in the classical model tax compliance increases as tax rate increases. On these issues, the classical model is contradicted by empirical studies.

In our model, instead, because we take into account utility (confidence) of contributing to the collective welfare, there are citizens who choose tax compliance even in case of convenient gamble. Furthermore, in our model an increase in tax rate may lead to an increase in tax evasion. In regard to these issues, this model proposes a solution to the conflicts between the results of the classical model and the empirical findings.

1 Introduction

In this paper we present a theoretical model of tax evasion, which takes into account the confidence of the taxpayer in respect of institutions and his sense of social responsibility. To do this, in addition to the utility of income, is attributed to the agent a utility (‘confidence’) of contributing to the collective welfare. Confidence is a function of declared income, tax rate and effectiveness of public expenditure.

A model on tax evasion, based on the maximization of the expected utility (of income), is proposed by Allingham and Sandmo in [1]. In this model, the

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choice of evading is treated as a gamble, because if the evader is discovered he has to pay a fine (proportional to the concealed income). In particular, a citizen with (positive) income $W$ is called to pay taxes, at a (fixed) rate equal to $\theta$. If the citizen declares his entire income, therefore, he pays a tax of $\theta W$. In case the citizen declares an amount $X < W$, it is assumed that he can be recognized as evader with (perceived) probability equal to $p$. In such case, the evader has to pay tax on the concealed income $W - X$ at a penalty rate $\pi$ which is higher than $\theta$, i.e. he has to pay (overall) $\theta X + \pi(W - X)$. This model assumes that the taxpayer has marginal income utility which is positive and decreasing. Furthermore, the taxpayer chooses $X$ so as to maximize the expected (with respect to the probability $p$) utility.

Yitzhaki in [14] modifies the model of Allingham and Sandmo applying a fine proportional to the evaded tax (rather than to the undeclared income). Namely, if the citizen declares an amount $X < W$ and is discovered, then he has to pay a fine $f > 1$ on the evaded tax $\theta(W - X)$, i.e. he has to pay (overall) $\theta X + f\theta(W - X)$. In this way, the model fits to a widespread tax system. In the following, we denote the model of Allingham-Sandmo-Yitzhaki as the ‘classical model’.

The classical model does not take into account the factors that make evasion different from a gamble. They are due to social and psychological motivations and are also connected with functioning of services and confidence in institutions (on various aspects of the tax morale see, for instance, the work by Torgler [13]). Allingham and Sandmo themselves mention in [1] the possibility of using a more extensive form of utility, which takes into account a (binary type) variable of reputation as a citizen. However, they focus their analysis simply on the utility of income.

These social, psychological and structural factors explain, among other things, the cases in which tax compliance is chosen despite an apparently convenient gamble. Moreover, in the classical model, if one assumes that the absolute risk aversion is decreasing with income, compliance increases as tax rate increases. But this relationship, as well as being counterintuitive, is contradicted by empirical studies in this regard (see, for instance, the work by Clotfelter [3] and that by Pommerehne and H. Weck-Hannemann [10]).

In general, for the literature on the empirical evidences in contrast to the classical model, see the introduction of the work by Myles and Naylor [8] and the survey of Freire-Seren and Panades [5].
Various theoretical models have been developed to solve these contrasts. Cowell and Gordon in [4] propose a model in which utility depends (as well as on consumption) on public goods.

Gordon in [6] proposes a model in which, to the utility of consumption, is added a (linear) term which decreases as hidden income increases. This added term can be interpreted as a psychic cost of evasion.

The model of Bordignon [2] includes a fairness constraint, which constitutes a ceiling on evasion. The fairness constraint depends on tax rate, public goods and behavior of the other citizens.

The model of Myles and Naylor [8] distinguishes the utility of evading (constructed as in the classical model) from the utility of not evading (constructed taking into account income, social customs and behavior of the other citizens). The choice of evading occurs when the utility of evading is greater than the utility of not evading.

In the model of Sour [11], which is inspired by the models [6] and [8], the utility function takes into account (as well as income) psychic cost of evasion and behavior of the other citizens.

In our model we assume for the tax system the same hypotheses made in the classical model. But unlike the classical model, the taxpayer does not choose to maximize the expected utility. He, instead, chooses the declared income $X$ so as to maximize the expected (with respect to the probability $p$) well-being.

In our model the ‘well-being function’ $B$ is in the form

$$B = U + C,$$

where $U$ is the classical utility of income and $C$ is the ‘confidence function’ (a utility of contributing to the collective welfare).

For the use of the concept of well-being in the economical literature, see the survey of Stutzer and Frey [12]. A well-being function in which there are an addend concerning the economic utility and one for the moral utility, is in the work by Parada Daza [9] (although in a form that is not compatible with the assumptions of our model, listed in the following at the section 2).

The confidence function $C(X, \theta, \alpha)$ can be interpreted as the social responsibility of the individual, or his confidence in institutions. $C$ depends, as well as on the declared income $X$, on $\theta$ (tax rate) and on $\alpha$ (perceived effectiveness of public expenditure).

For the role of trust in institutions in tax morale, see (Torgler, [13]).
With regard to the model of Gordon [6] (in which to the utility of consumption is added a linear term which decreases as hidden income increases), well-being in our model can be considered a generalization of Gordon’s utility, with the dependence on the hidden income that may be non-linear (and with tax rate and effectiveness of public expenditure, not present in the utility of Gordon).

Among the results of our model, we can observe that, because we take into account utility (confidence) of contributing to the collective welfare, there are citizens who choose tax compliance even in case of convenient gamble. Moreover, in our model an increase in tax rate may lead to an increase in tax evasion. In regard to these issues, therefore, this model proposes a solution to the conflicts between the results of the classical model and the empirical findings.

Within our model we can also describe different types of taxpayers, as the free rider, the honest citizen, the taxpayer sensitive to the effectiveness of public expenditure and the taxpayer sensitive to the tax rate. We present an example in this regard.

In section 2, our model is conceptually described and mathematically formalized. Section 3 deals with the analysis of the model. Section 4 offers some examples. In section 5 there are the conclusions.

2 The model

Let us imagine that a citizen with (positive) income $W$ is called to pay taxes, at a (fixed) rate equal to $\theta$, with $0 < \theta < 1$.

If the citizen declares his entire income, therefore, he pays a tax of $\theta W$. In case the citizen declares an amount $X < W$, it is assumed that he can be recognized as evader with (perceived) probability equal to $p$ (with $0 < p < 1$). In such case, the evader has to pay a fine $f > 1$ on the evaded tax $\theta(W - X)$, i.e. he has to pay $f\theta(W - X)$. For technical reasons, namely to avoid dealing with the utility of negative amounts, we put a ceiling on the fine, assuming $f < \frac{1}{p}$ (see in this regard appendix A). The latter hypothesis does not eliminate the interest of the model. For example, it is consistent with tax systems characterized by frequent checks and fines not too high.

According to our assumptions, the undiscovered evader ends up with an effective income $Y$ equal to
\[ Y = W - \theta X. \]  
(1)

The discovered evader, instead, ends up with an effective income \( Z \) equal to

\[ Z = W - \theta X - f\theta(W - X). \]  
(2)

As yet, our model follows the model of Allingham-Sandmo-Yitzhaki (except for the ceiling on the fine, not present in the classical model). Now, rather than limit ourselves to consider the utility of income (classical model), we assume that the citizen has its own ‘well-being function’ \( B \) of the form

\[
B(y_1, y_2, \theta, \alpha) = U(y_1) + C(y_2, \theta, \alpha),
\]

where \( y_1 \in [0, \infty[ \) is the effective income, \( y_2 \in [0, \infty[ \) is the declared income, \( \alpha \in [0, 1[ \) is a parameter of (perceived) efficacy of public expenditure, the (real-valued) function \( U \) is the classical utility of income and the (real-valued) function \( C \) is the ‘confidence’ (a utility of contributing to the collective welfare).

In this way we are modelling the fact that choices in terms of tax compliance are not a simple gamble (in which who evade wins if he is not discovered and loses otherwise), as in the classical model. They are also linked to the confidence of the citizen in respect of institutions and his sense of social responsibility and, in this way, to the well-being of the citizen (and therefore to social and psychological factors and to functioning of services).

The parameter \( \alpha \) is subjective, since it refers to perceived effectiveness. It can be seen how a perception of quality of public services or policies (or even, for example, of control of corruption). One can obtain an indicator of \( \alpha \) by statistical methods, such as in (Kaufmann-Kraay-Mastruzzi [7]). The parameter \( \alpha \) may also be interpreted as a perception of equity, or of distributive efficiency (for example, Pareto efficiency).

We assume that \( U \) has continuous second order derivative. We also assume that \( C \) has continuous second order derivatives with respect to all variables.

Implicitly, we can define \( C(0, \theta, \alpha) \) and \( C_{y_2}(0, \theta, \alpha) \) as limits (eventually not finished) for \( y_2 \) that tends to 0 from the right.

In order to have \( U \) increasing and strictly concave (risk aversion), we assume for all \( y_1 \)

\[ U'(y_1) > 0, \]
\[ U''(y_1) < 0. \]

For \( C \), with respect to the declared income, it seems quite reasonable assuming similar hypotheses (marginal confidence of positive and decreasing type). We admit also the cases of null sign (in this way the classical model can be seen as a particular case of our model, that is, the case of null \( C' \)), then for all \( y_2, \theta, \alpha \) we have

\[ C_{y_2}(y_2, \theta, \alpha) \geq 0, \]

\[ C_{y_2y_2}(y_2, \theta, \alpha) \leq 0. \]

We also assume that there exists \( \theta \in [0, \frac{1}{f}] \) such that for \( \theta > \bar{\theta} \) (and for all \( y_2, \alpha \)) we have

\[ C_{y_2\theta}(y_2, \theta, \alpha) \leq 0, \]

namely, the marginal confidence of declared income is monotonically non-increasing with respect to \( \theta \), at least above \( \bar{\theta} \). This assumption seems reasonable, because we imagine that if taxes seem too high, citizens lose confidence in institutions and are less motivated to give their fiscal contribution.

Furthermore, we assume that for all \( y_2, \theta, \alpha \) we have

\[ C_{y_2\alpha}(y_2, \theta, \alpha) \geq 0, \]

that is, the marginal confidence of declared income is monotonically non-decreasing with respect to \( \alpha \). Also this assumption seems reasonable, because we imagine that if public expenditure is perceived as effective, the citizen acquires confidence in institutions and he is more motivated to give his fiscal contribution to welfare of the community.

3 Analysis and solution of the model

The expected well-being of a citizen who chooses to declare \( X \) is therefore equal to

\[
E[B] = (1 - p)B(W - \theta X, X, \theta, \alpha) + pB(W - \theta X - f\theta(W - X), X, \theta, \alpha) =
\]
= (1 - p)U(W - \theta X) + pU(W - \theta X - f\theta(W - X)) + C(X, \theta, \alpha).

We assume that the citizen declares an amount \( \overline{X} \) that maximizes the expected well-being.

In case \( \overline{X} \) is an interior maximum of \([0, W]\), it has to satisfy the following first order condition

\[
\frac{\partial E[B]}{\partial X} = 0,
\]

that is

\[-\theta(1-p)U'(W - \theta X) - (\theta - f\theta)pU'(W - \theta X - f\theta(W - X)) + C_y(X, \theta, \alpha) = 0,\]

that is

\[-\theta(1-p)U'(Y) + \theta(f - 1)pU'(Z) + C_y(X, \theta, \alpha) = 0,\]

where \( Y \) and \( Z \) are defined, respectively, in (1) and (2).

The second order condition (sufficient for a relative interior maximum, once verified that of the first order) is given by

\[
\frac{\partial^2 E[B]}{\partial X^2} < 0,
\]

that is

\[\theta^2(1-p)U''(Y) + \theta^2(1 - f)^2pU''(Z) + C_{y2y2}(X, \theta, \alpha) < 0.\]

This condition, thanks to the assumptions on \( U \) and \( C \), is verified for each \( X \in [0, W] \). It implies that \( \frac{\partial E[B]}{\partial X} \) is decreasing with respect to \( X \). Then \( \frac{\partial E[B]}{\partial X} \) is null at a (unique) interior point of \([0, W]\) if and only if both the following conditions are verified

\[
\frac{\partial E[B]}{\partial X} \big|_{X=0} > 0, \tag{6}
\]

\[
\frac{\partial E[B]}{\partial X} \big|_{X=W} < 0. \tag{7}
\]
Therefore these constitute a necessary and sufficient condition for the existence of a (unique) interior point $X$ of absolute maximum (figure 1, case 1).

If condition (6) is false, instead, the expected well-being is maximized for $X = 0$ (because $\frac{\partial E[B]}{\partial X} < 0$ in $]0, W[$ and, thus, $E[B]$ is decreasing) (figure 1, case 2). If condition (7) is false, the expected well-being is maximized for $X = W$ (because $\frac{\partial E[B]}{\partial X} > 0$ in $]0, W[$ and, thus, $E[B]$ is increasing) (figure 1, case 3).

![Figure 1: The expected well-being as a function of X.](image)

Condition (6) may be explicitly written as follows (consider the left-hand side of (3) for $X = 0$)

$$-\theta (1 - p) U'(W) - (\theta - f \theta) p U'(W - f \theta W) + C_{y_2}(0, \theta, \alpha) > 0,$$

that is equivalent (dividing by $-\theta U'(W - f \theta W)$ ) to

$$(1 - p) \frac{U'(W)}{U'(W - f \theta W)} - (f - 1)p - \frac{C_{y_2}(0, \theta, \alpha)}{\theta U'(W - f \theta W)} < 0,$$

that is

$$\frac{U'(W)}{U'(W(1 - f \theta))} < \frac{p(f - 1)}{1 - p} + \frac{C_{y_2}(0, \theta, \alpha)}{\theta U'(W(1 - f \theta))(1 - p)}. \quad (8)$$
Condition (7) may be explicitly written as follows (consider the left-hand side of (3) for \( X = W \))

\[-\theta(1 - p)U'(W - \theta W) - (\theta - f\theta)pU'(W - \theta W) + C_{y_2}(W, \theta, \alpha) < 0 ,\]

that is equivalent to

\[-\theta U'(W - \theta W)[1 - p + (1 - f)p] + C_{y_2}(W, \theta, \alpha) < 0 ,\]

that is

\[[1 - p + p - pf] - \frac{C_{y_2}(W, \theta, \alpha)}{\theta U'(W - \theta W)} > 0 ,\]

that is equivalent to

\[pf < 1 - \frac{C_{y_2}(W, \theta, \alpha)}{\theta U'(W(1 - \theta))} .\quad \text{(9)}\]

In the classical model of Allingham-Sandmo-Yitzhaki [14], instead, the two conditions (8), (9) (obtained simply maximizing expected \( U \)) may be explicitly written, respectively, as

\[\frac{U'(W)}{U'(W(1 - f\theta))} < \frac{p(f - 1)}{1 - p} ,\]

and

\[pf < 1 .\]

Comparing the conditions of our model with those of the classical model, we realize that condition (8) is weaker than the first classical condition, while (9) is stronger than the second classical condition.

Thus if null income is declared in our model, namely condition (8) is false, then the first classical condition is even false and therefore also in the classical model null income is declared.

On the other hand, if the entire income is declared in the classical model, namely the second classical condition is false, then condition (9) is even false and therefore also in our model the entire income is declared.

In the following there is an example (see section 4, example 1) in which in our model the entire income is declared, whereas in the classical model there is tax evasion.
Now we examine how the parameter $\theta$ affects the interior point of absolute maximum.

Let $\overline{X}(\theta)$ be the solution of the first order condition (4), or (equivalently) of

$$(1 - p)U'(Y) - (f - 1)pU'(Z) - \frac{C_{y2z}(X, \theta, \alpha)}{\theta} = 0. \quad (10)$$

Varying $\theta$ (and the relative $\overline{X}(\theta)$) in a suitable neighborhood, condition (10) remains verified (for details see appendix B) and the first member of (10) is a constant function (equal to 0) of $\theta$, thus it has null derivative with respect to $\theta$. On the basis of this property, differentiating the first member of (10) respect to $\theta$ and setting

$$\overline{Y} = W - \theta \overline{X},$$
$$\overline{Z} = W - \theta \overline{X} - f\theta(W - \overline{X}),$$

we obtain

$$(1 - p)\left(-\overline{X} + \theta \frac{\partial \overline{X}}{\partial \theta} \right)U''(Y) + (1 - f)p\left(-\overline{X} + \theta \frac{\partial \overline{X}}{\partial \theta} - f(W - \overline{X}) + f\theta \frac{\partial \overline{X}}{\partial \theta}\right)U''(Z) +$$

$$+ \frac{1}{\theta^2} \left[ - \theta \left( C_{y2}\overline{X}(\theta, \alpha) \frac{\partial \overline{X}}{\partial \theta} + C_{y2\theta}(\overline{X}, \theta, \alpha) \right) + C_{y2}(\overline{X}, \theta, \alpha) \right] = 0$$

(for the existence of $\frac{\partial \overline{X}}{\partial \theta}$ see appendix B), that is equivalent to

$$\frac{\partial \overline{X}}{\partial \theta} = -\frac{\theta}{D} \left[ \overline{X} \left( (1 - p)U''(Y) - p(f - 1)U''(Z) \right) - pf(f - 1)(W - \overline{X})U''(Z) +$$

$$+ \frac{C_{y2\theta}(\overline{X}, \theta, \alpha)}{\theta} - \frac{C_{y2}(\overline{X}, \theta, \alpha)}{\theta^2} \right] \quad (11)$$

(for the complete proof see appendix B) where $D$ is the first member of the second order condition (5) evaluated at $\overline{X}$, that is

$$D = \theta^2(1 - p)U''(Y) + \theta^2pf(f - 1)^2U''(Z) + C_{y2z}(\overline{X}, \theta, \alpha).$$
Notice that the form of $\frac{\partial X}{\partial \theta}$ in the classical model of Allingham-Sandmo-Yitzhaki is the one reported in the first line of equation (11). Assuming that the absolute risk aversion ($R_A(y_1) = \frac{U''(y_1)}{U(y_1)}$) is decreasing with income, one has $\frac{\partial X}{\partial \theta} > 0$ (see [14], p.202), in contrast to the empirical results. In our case, the terms of the classical model (see first line of equation (11) ) may assume different values, and the terms in $C_{y_2}$ and in $C_{y_2}$ of equation (11) are negative or null (at least for $\theta > \bar{\theta}$). For these reasons, in our model $\frac{\partial X}{\partial \theta}$ can take negative values, consistently with the empirical results. In the following there is an example (see section 4, example 2) in this regard. Already in (Gordon [6]) the addition of a psychic cost changes the classical result (at the end of example 2 it is shown as the utility of Gordon is a special case of our well-being function).

Proceeding as before, we get also the expression of $\frac{\partial X}{\partial \alpha}$. Differentiating the first member of (10) with respect to $\alpha$ (and setting null the derivative) we obtain

\[
(1 - p)\left(-\theta \frac{\partial X}{\partial \alpha} u''(Y) - (f - 1)p\left(-\theta \frac{\partial X}{\partial \alpha} + f\theta \frac{\partial X}{\partial \alpha}\right)u''(Z) + \right.
\]

\[
-\frac{\left(C_{y_2y_2}(X, \theta, \alpha) \frac{\partial X}{\partial \alpha} + C_{y_2\alpha}(X, \theta, \alpha)\right)}{\theta} = 0,
\]

that is

\[
\frac{\partial X}{\partial \alpha} \left(-\theta(1 - p)u''(Y) - \theta p(f - 1)^2u''(Z) - C_{y_2y_2}(X, \theta, \alpha)\right) = C_{y_2\alpha}(X, \theta, \alpha),
\]

that is equivalent to

\[
\frac{\partial X}{\partial \alpha} = -\frac{C_{y_2\alpha}(X, \theta, \alpha)}{D}.
\]

Therefore, as it was reasonably expected, it is always

\[
\frac{\partial X}{\partial \alpha} \geq 0
\]

(namely, tax compliance is monotonically non-decreasing with respect to the effectiveness of public expenditure).
4 A few examples

Example 1.
Let us see now an example in which in the classical model there is tax evasion and in our model, instead, the entire income is declared. In this way, our model is consistent with the fact that there are citizens who do not evade despite tax evasion may arise as an apparently convenient gamble.

Let us consider a utility of income and a confidence function of the form

\[
\begin{align*}
U(y_1) &= \ln y_1, \\
C(y_2, \theta, \alpha) &= \frac{\alpha}{\theta} \ln y_2.
\end{align*}
\tag{12}
\]

The assumptions on $U$ and $C$ are, therefore, all verified.

Assigning, in addition, the values

\[W = 1, \ p = \frac{1}{5}, \ f = 3, \ \theta = \frac{1}{5}, \ \alpha = \frac{1}{2},\]

it results $f \theta < 1$ (the assumption about $f$ is verified). Moreover $pf < 1$, namely, as we have seen previously, in the classical model there is tax evasion. Let us remember that condition (7) is explicitly written in condition (9)

\[pf < 1 - \frac{C_{y_2}(W, \theta, \alpha)}{\theta U'(W(1-\theta))}\]

and, by using (12), it becomes

\[pf < 1 - \frac{\alpha}{\frac{\partial W}{\partial \theta}} W(1-\theta),\]

that is

\[pf < 1 - \frac{\alpha(1-\theta)}{\theta^2} .\]

Since

\[1 - \frac{\alpha(1-\theta)}{\theta^2} = -9 \leq \frac{3}{5} = pf ,\]

condition (7) is not verified and therefore, for this configuration of functions and parameters, in our model the entire income is declared.

Example 2.
As we have seen previously, in the classical model if the citizen has absolute risk aversion decreasing with income, then $\frac{\partial X}{\partial \theta} > 0$, in contrast to the empirical studies.

Let us see now an example in which, instead, in our model a citizen has absolute risk aversion decreasing with income and, for certain values of the parameters, it is $\frac{\partial X}{\partial \theta} < 0$ (thus, as taxes increase, declared income decreases).

We assume the same utility and confidence functions as in the previous example (12).

Notice that the absolute risk aversion of $U$ is

$$R_A(y_1) = -\frac{U''(y_1)}{U'(y_1)} = -\frac{1}{y_1} = \frac{1}{y_1},$$

therefore $R_A(y_1)$ is decreasing with income. We assign the values

$$W = 1, \ p = \frac{1}{5}, \ f = 2, \ \theta = \frac{9}{20}, \ \alpha = \frac{1}{5}.$$ 

It results $f\theta < 1$ (the assumption about $f$ is verified). In addition, condition (8) takes the form

$$\frac{1}{W} < p(f - 1) + \lim_{y_2 \to 0^+} \frac{\alpha}{y_2},$$

then it is verified since the left-hand side is finite and $\lim_{y_2 \to 0^+} \frac{\alpha}{y_2} = +\infty$.

Condition (9) takes the form

$$pf < 1 - \frac{\alpha(1 - \theta)}{\theta^2},$$

and, for the configuration of parameters considered, it becomes equivalent to

$$1 < \frac{185}{162}.$$ 

Then there exists a unique $X$ of interior maximum, obtainable as solution of the first order condition (in the equivalent form (10))

$$(1 - \frac{1}{5}) \cdot \frac{1}{1 - \theta X} + (1 - 2) \cdot \frac{1}{5} \cdot \frac{1}{1 - \theta X - 2\theta(1 - X)} + \frac{1}{5} \cdot \frac{1}{-\theta} = 0,$$

that is equivalent to
\[4\left(1-\theta X -2\theta(1-X)\right)\theta^2 X -(1-\theta X)\theta^2 X - (1-\theta X)\left(1-\theta X -2\theta(1-X)\right) = 0.\]

For \(\theta = \frac{9}{20}\) the solution is
\[
\bar{X} = \frac{2}{585}(117 + \sqrt{26689}).
\]

The sign of the derivative (11) coincides with that of the factor
\[
\bar{X}\left((1-p)U''(\bar{Y}) - p(f-1)U''(\bar{Z})\right) - pf(f-1)(W - \bar{X})U''(\bar{Z}) +
\]
\[
+ \frac{C_{y_2\theta}(\bar{X}, \theta, \alpha)}{\theta} - \frac{C_{y_2}(\bar{Y}, \theta, \alpha)}{\theta^2},
\]
that is equal to \(-6.21297\) (with numerical calculation), and therefore \(\frac{\partial \bar{X}}{\partial \theta} < 0\).

Gordon [6] provides another example of \(\frac{\partial \bar{X}}{\partial \theta} < 0\) for a suitable choice of the parameters. This example may be cast in our setting by choosing a well-being function of the type
\[
B(y_1, y_2) = U(y_1) - v(W - y_2)
\]
where \(v\) is a non-negative constant. Notice that the confidence function is, in this case, independent of \(\theta\) and \(\alpha\).

Figure 2 shows the curves of declared income as \(\theta\) varies in the interval \([0.401, 0.499]\), in the classical model (lower curve) as well as in our model (upper curve), with the choice of functions and parameters of example 2. We can observe, in particular, that for \(\theta = \frac{9}{20} = 0.45\) in the classical model it has positive derivative and in our model negative derivative, as we have seen in example 2. In addition, in the first part of the interval, we note that in our model the entire income is declared despite in the classical model there is tax evasion (convenient gamble), as it happens in example 1 (but with a different choice of parameters).
Example 3.
In this example we describe four types of taxpayers: the free rider, the honest citizen, the taxpayer sensitive to the effectiveness of public expenditure and the taxpayer sensitive to the tax rate.

a) The free rider.
Within our model, we can represent the free rider as a taxpayer that, in the decision on the income to declare, does not take into account his social responsibility. In other words, he maximizes the utility of income $U$, while his confidence function $C$ is identically zero. In this way, the choice of the income to declare $X$ matches exactly with that of the classical model.

To obtain an example of free rider, we assign, as in the previous example, the values

$$W = 1, \ p = \frac{1}{5}, \ f = 2, \ \alpha = \frac{1}{5},$$

and we assume

$$\begin{cases} U(y_1) = \ln y_1 , \\ C(y_2, \theta, \alpha) = 0. \end{cases}$$

Figure 3 shows the values of $X$, as $\theta$ varies in the interval $[0.001, 0.499]$. Naturally, the curve of the classical model and that of our model are coincident.
b) The honest citizen.
Within our model, we can represent the honest citizen as a taxpayer that always chooses to declare the entire income.
In other words, his utility function $U$ and his confidence function $C$ are such that $\mathcal{X} = W$, for all $W$, $p$, $f$, $\theta$, $\alpha$.

To obtain an example of honest citizen, we assume

$$
\begin{align*}
U(y_1) &= \ln(1 + y_1), \\
C(y_2, \theta, \alpha) &= 2y_2.
\end{align*}
$$

In this case, in fact, condition (9) becomes

$$
pf < 1 - \frac{2}{\theta \frac{1}{1+W(1-\theta)}}.
$$

In order to show that the condition is false for all choices of the parameters, it suffices to prove that

$$
2 \geq \frac{\theta}{1 + W(1 - \theta)},
$$

that is true, because $\theta < 1$ and $1 + W(1 - \theta) > 1$.

Figure 4, obtained with $W = 1$, $p = \frac{1}{5}$, $f = 2$, $\alpha = \frac{1}{5}$, shows the values of $\mathcal{X}$, as $\theta$ varies in the interval $[0.001, 0.499]$. In this case, in our model we have $\mathcal{X} = W$, despite in the classical model (where the taxpayer only maximizes $U$) we have $\mathcal{X} = 0$ in the whole interval.
c) The taxpayer sensitive to $\alpha$.

Figure 5, obtained with

$$\begin{aligned}
U(y_1) &= \ln(y_1), \\
C(y_2, \theta, \alpha) &= \frac{1}{4} \theta \ln y_2,
\end{aligned}$$

and

$$W = 1, \quad p = \frac{1}{5}, \quad f = 2, \quad \theta = \frac{9}{20},$$

shows the values of $\overline{X}$, as $\alpha$ varies in the interval $[0.001, 0.999]$.

It describes an example of taxpayer that is sensitive to $\alpha$, namely a taxpayer that varies considerably his choice on the percentage of declared income depending on the perceived effectiveness of public expenditure.

In particular, he passes from a percentage lower than 30% (as in the classical model) for $\alpha$ near to 0, to a percentage equal to 100% (very far from that of the classical model) when $\alpha$ assumes high values.
d) *The taxpayer sensitive to $\theta$.*

Figure 6, obtained with

\[
\begin{align*}
U(y_1) &= \ln(y_1), \\
C(y_2, \theta, \alpha) &= \frac{1}{2} \theta \ln y_2,
\end{align*}
\]

and

\[
W = 1, \ p = \frac{1}{5}, \ f = 2, \ \alpha = \frac{1}{5},
\]

shows the values of $X$, as $\theta$ varies in the interval $[0.001, 0.499]$. It describes an example of taxpayer that is sensitive to $\theta$, namely a taxpayer that varies considerably his behavior depending on the tax rate. In particular, the difference between the declared income in our model and the one declared in the classical model passes from a percentage equal to 100% for $\theta$ near to 0, to a percentage lower than 30% when $\theta$ assumes high values.
5 Conclusions

Among the results of our model, we have seen that, because we take into account utility (confidence) of contributing to the collective welfare, there are citizens who choose tax compliance even in case of convenient gamble. Moreover, in our model an increase in tax rate may lead to an increase in tax evasion. In regard to these issues, therefore, this model proposes a solution to the conflicts between the results of the classical model and the empirical findings.

We have also seen that within our model we can describe different types of taxpayers, as the free rider, the honest citizen, the taxpayer sensitive to the effectiveness of public expenditure and the taxpayer sensitive to the tax rate.

Naturally, the model that we have proposed is a simplification of a reality much more various and complex. It is also affected by the methodological problems that are common to all models based on the maximization of a form of expected utility.

However, keeping in mind these limitations, it has the advantage of describing the choice of a taxpayer taking into account aspects not covered by the classical model, such as the confidence in institutions, the sense of social responsibility, the (perceived) fairness of tax rate and the (perceived) effectiveness of public expenditure.

Possible extensions of the model may capture other aspects of tax systems
and fiscal choices. For example, the assumption of fixed tax rate may be replaced by the assumption of progressive taxation. Or, the choice of a taxpayer may be influenced by the choices of other taxpayers.

The model that we have presented is static. Among the possible developments there are its dynamical extensions (discrete or continuous time, with deterministic or stochastic variables), also in view of an inclusion in a general equilibrium model.

Furthermore, the assumptions and the results of the model should be compared with the data through econometric verifications.

**Appendix**

**A Hypothesis on f**

The hypothesis $f < \frac{1}{\theta}$ ensures that even if the evader is discovered his effective income $Z$ remains positive (in this way we avoid dealing with the utility of negative amounts, with respect to which the hypothesis of concavity is problematic). Indeed (since $0 < \theta < 1$, $W > 0$ and $0 \leq X \leq W$) we have

$$-\theta X \geq -X$$

and

$$-f\theta(W - X) \geq -(W - X)$$

(where at least one of the two inequalities is strict), therefore

$$Z = W - \theta X - f\theta(W - X) > W - X - (W - X) = 0.$$  

The hypothesis is also necessary to have $Z > 0$ for all $X$. Indeed if we assume $f \geq \frac{1}{\theta}$, for $X = 0$ we have

$$Z = W - \theta X - f\theta(W - X) = W - f\theta W = (1 - f\theta)W \leq 0.$$
B Derivative of $\overline{X}$ with respect to $\theta$

**Theorem 1** Let $\overline{X}$ be the interior solution that maximizes $E[B]$ for a certain value $\overline{\theta}$ of the tax rate (fixed the other parameters). Then there exists a neighbourhood $I$ of $\overline{\theta}$ in which the maximization problem still admits interior solution (thus defining the maximum function $\overline{X}(\theta)$ in $I$) and, furthermore, the maximum function is continuously differentiable and one has (11).

**Proof.**

The pair $(\overline{\theta}, \overline{X})$ meets, with respect to the first order condition (4), the assumptions of the implicit function theorem (thanks to the regularity of the functions and to the sign of the second order condition (5)). Therefore, in a suitable neighbourhood $I$ of $\overline{\theta}$ (with $I \subseteq ]0,1[),$ is defined the implicit function $\overline{X}(\theta),$ continuously differentiable in $I.$ Choosing $I$ suitably small (so that the function $\overline{X}(\theta),$ by continuity, takes values in $]0,W[,$ $\overline{X}(\theta)$ is the maximum function sought.

Notice that the implicit function theorem gives directly the form of $\frac{\partial \overline{X}}{\partial \theta}.$ Nevertheless here we get it explicitly. Let us observe that the points $(\theta, \overline{X}(\theta))$ meet the condition of the first order, also in the equivalent form (10). Therefore the first member of (10), for $X = \overline{X}(\theta),$ is a constant function (equal to 0) of $\theta$ in $I,$ then it has null derivative. Differentiating the first member of (10) with respect to $\theta$ and setting

$$
\overline{Y} = W - \theta \overline{X},
$$

we obtain

$$(1-p)\left(-\overline{X} - \theta \frac{\partial \overline{X}}{\partial \theta} \right) U''(\overline{Y}) + (1-f)p\left(-\overline{X} - \theta \frac{\partial \overline{X}}{\partial \theta} - f(W-\overline{X}) + f\theta \frac{\partial \overline{X}}{\partial \theta} \right) U''(\overline{Z}) +$$

$$
-\theta \left(C_{y_2y}(\overline{X},\theta,\alpha) \frac{\partial \overline{X}}{\partial \theta} + C_{y_2}(\overline{X},\theta,\alpha) \right) + C_{y_2}(\overline{X},\theta,\alpha)
$$

that is equivalent to

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\[ \frac{\partial X}{\partial \theta} \left( (p-1)\theta U''(Y) + (f-1)p\theta U''(Z) - f(f-1)p\theta U''(Z) - \frac{C_{y_2 y_2}(X, \theta, \alpha)}{\theta} \right) + \\
+ (p-1)XU''(Y) + (f-1)pXU''(Z) + f(f-1)p(W-X)U''(Z) - \frac{C_{y_2}(X, \theta, \alpha)}{\theta} + \frac{C_{y_2}(X, \theta, \alpha)}{\theta^2} = 0, \]

that is

\[ \frac{\partial X}{\partial \theta} \left( (p-1)\theta U''(Y) + p\theta(f-1 - f^2 + f)U''(Z) - \frac{C_{y_2 y_2}(X, \theta, \alpha)}{\theta} \right) = \\
= -X \left( (p-1)U''(Y) + (f-1)pU''(Z) \right) - f(f-1)p(W-X)U''(Z) + \frac{C_{y_2}(X, \theta, \alpha)}{\theta} - \frac{C_{y_2}(X, \theta, \alpha)}{\theta^2}. \]

Recalling that the amount

\[ D = \theta^2(1-p)U''(Y) + \theta^2 p(f-1)^2 U''(Z) + C_{y_2 y_2}(X, \theta, \alpha) \]

is non-zero (being the first member of the second order condition), we obtain (11)

\[ \frac{\partial X}{\partial \theta} = \frac{-\theta}{D} \left[ X \left( (1-p)U''(Y) - p(f-1)U''(Z) \right) - pf(f-1)(W-X)U''(Z) + \\
+ \frac{C_{y_2}(X, \theta, \alpha)}{\theta} - \frac{C_{y_2}(X, \theta, \alpha)}{\theta^2} \right]. \]

**References**


