Structural Mechanics (9 CFU)

Academic Year 2023/2024

Sustainable Building Engineering

Professor: Dr Andrea Arena (andrea.arena@uniroma1.it)

Location: room 11, Palazzo Aluffi, via Cintia 106, Rieti, Italy.

Two lectures on Monday from 2:00 pm to 6:00 pm

Two lectures on Wednesday from 9:30 am to 1:30 pm

Suggested book

Title: Fundamentals of Structural Mechanics

Authors: Alberto Taliercio, Umberto Perego

Other suggested materials

Lecture notes, taken by the student

Notes provided by the teacher via Google Classroom

Syllabus of the course

Lecture – 1 and 2

- Introduction to the course of Structural Mechanics.
- Recap of vector algebra: Euclidean space

Lecture - 3 and 4

- Recap of vector algebra: the space of real numbers
- Recap of tensor algebra.
- The eigenvalue problem.

Lecture - 5 and 6

- Recap of differential calculus.
- Recap on Differential equations

Lecture - 7 and 8

- Kinematics of three-dimensional bodies.
- Change of configuration of 3D bodies within the small displacements theory.
- The effect of infinitesimal rigid rotations.

Lecture - 9 and 10

- The small strain tensor E
- Mechanical meaning of the component of the small strain tensor E
- Principal strains and principal directions

Lecture – 11 and 12

- Recap of the kinematics of 3D bodies and of the small strain theory.
  Exercises on the kinematics:
  a) given a displacement field, prove its kinematic admissibility and calculate F, E, and \Omega
  b) calculate the component of the tensor E along given directions
- Eigenvalue problem associated with the small strain tensor E.
  Exercises on the eigenvalue problem:
  a) calculate principal strains
  b) calculate principal directions
- Further exercises on the eigenvalue problem associated with the small strain tensor E
  a) calculate principal strains
  b) calculate principal directions

Lecture – 13 and 14

- Introduction to the equilibrium problem of three-dimensional bodies
- The Euler axioms
- Introduction to the Cauchy stress model
- The Cauchy stress vector

Lecture – 15 and 16

- Properties of the Cauchy stress vector
- The Cauchy theorem
- The Cauchy stress tensor and its properties
- Local equilibrium equations (from the first Euler axiom)

Lecture – 17 and 18

- Symmetry of the Cauchy stress tensor (from the second Euler axiom)
- Equilibrium equations at the boundary
- Mechanical meaning of the component of the Cauchy stress tensor T and their representation
- Principal stresses and principal directions associated with the Cauchy stress tensor T
- Exercises: calculate the principal stresses and the associated principal directions

Lecture – 19 and 20

- Octahedral stresses: the octahedral normal stress and the octahedral shear stress
- Plane stress states and the calculation of the associated principal stresses and directions
- The Mohr's circles, graphical method to calculate principal states and associated directions. The case of the plane stress state.

Applications:
calculation of principal stresses
calculation of octahedral t, sigma and tau
plane stress state solve via Mohr method and analytically

Lecture – 21 and 22

- Constitutive behaviors: Linear elastic homogeneous isotropic behavior
- The Hooke law direct and inverse forms
- The Hooke law through the "Engineering parameters"
- The meaning of "Engineering parameters"
- How to calculate the "Engineering parameters"

Lecture – 23 and 24

- Linear Elastic Limit State and Strength Criteria
- The ideal normal stress
- The Von Mises strength criterion
- The Tresca strength criterion
- Application on strength criteria

Lecture – 25 and 26

- The De Saint Venant problem.
- Geometrical and mechanical assumptions
- The semi-inverse method and the De Saint Venant postulate
- New symbols convention
- Local equilibrium equations
- Integral (weak) form of the boundary conditions

- Definition of the force and moment resultants at the cylinder terminal cross-sections

Lecture – 27 and 28

- Integral form of the local equilibrium equations
- Variation along the cylinder axis of the force and moment resultants
- Hooke's laws and small strain tensor E in the De Saint Venant problem
- Strain-displacements differential relationships in the DSV problem
- Expression of the elongation along z in the De Saint Venant problem
- The shear strain vector and its differential relationship with the elongation along the z-axis
- The four De Saint Venant sub-problems

Lecture – 29 and 30

- Geometrical properties of surfaces
- Definition of area, first moments of area, and second moments of area
- Application to the rectangular section
- Calculation of a centroidal frame (i.e., the position of the center of area)
- Calculation of a principal inertia frame (i.e., the orientation of the frame)
- Classification of the cross-sections: compact and thin-walled
- Geometric properties of thin-walled cross-sections

Application: T-shaped cross-section calculated in the cases of compact and thin-walled cross-section

Applications
I-shaped thin-walled cross-section
C-shaped thin-walled cross-section
L-shaped thin-walled cross-section

Lecture – 31 and 32

- The DSV problem of uniform elongation (axial force)
- theory and application to the case of the rectangular cross-section
- The DSV problem of uniform bending (bending moment)
- The neutral axis, the moment axis and the curvature axis
- The case of moment Mx, about the x-axis
- The case of moment My, about the y-axis
- The case of moments Mx and My
- The DSV problem of axial force and bending moments
- The case of the statically equivalent axial force not passing through the center of area C
- The normal form of the equation of the neutral axis

Lecture - 33 and 34

Application to the I-shaped cross-section subjected to N not passing through C

Applications: the case of axial force and bending moment acting on
T-shaped thin-walled cross-section
I-shaped thin-walled cross-section having flanges of unequal length

Lecture – 35 and 36

- DSV problem: uniform torque: problem statement
- Uniform torque problem for the thin-walled rectangular cross-section
- Uniform torque problem for the thin-walled cross-sections

Applications:
T-shaped cross-section having different thicknesses (the case of torque moment)
C-shaped cross-section subjected to torque and a non-centroidal axial force

Applications: the case of bending moment and torque acting on
L-shaped thin-walled cross-section having different thicknesses

Lecture – 37 and 38

- The shear problem (non-uniform bending problem)
- The Jourawsky theory (approximate solution for the shear problem)
- Theory: the shear center in the Jourawsky theory

Applications: calculation of the shear center of a thin-walled cross-section
C-shaped thin-walled cross-section

Applications of the Jourawsky theory: calculation of the shear stresses of a thin-walled cross-section
I-shaped thin-walled cross-section having unequal flanges and thicknesses

Applications of the Jourawsky theory:
I-shaped thin-walled cross-section having unequal flanges and thicknesses
- calculation of the shear center
- calculation of the shear stresses given by the shear force non-passing through the shear center

Lecture - 39 and 40

- Introduction to the beam theory
- kinematics of the straight beam
- elongation, shear strains and curvatures
- equilibrium of the straight beam
- constitutive laws for the beam

Lecture – 41 and 42

- The planar model of beams
- The elastic problem of planar beams
- Classification of the constraints
- Kinematic and mechanical boundary conditions
- The Euler-Bernoulli beam

Application: the elastic in-plane problem for the Euler-Bernoulli beam
Simply supported beam subjected to uniform transversal load

Application: the elastic in-plane problem for the Euler-Bernoulli beam
hyperstatic beam subjected to boundary force and moment
Solution of the global and local equilibrium problems of isostatic beams without solving the elastic problem

Lecture – 43 and 44

- Isostatic systems of beams
- The internal hinge
- Solution of an isostatic systems of beam

Applications: Isostatic systems of beams, discussion of the alternative method to calculate N, T, and M in isostatic beams
Further exercise: calculation of the constraints reactions and of N,T,M

Lecture – 45 and 46

Applications: Isostatic systems of beams
Further exercise: calculation of the constraints reactions and of N,T,M